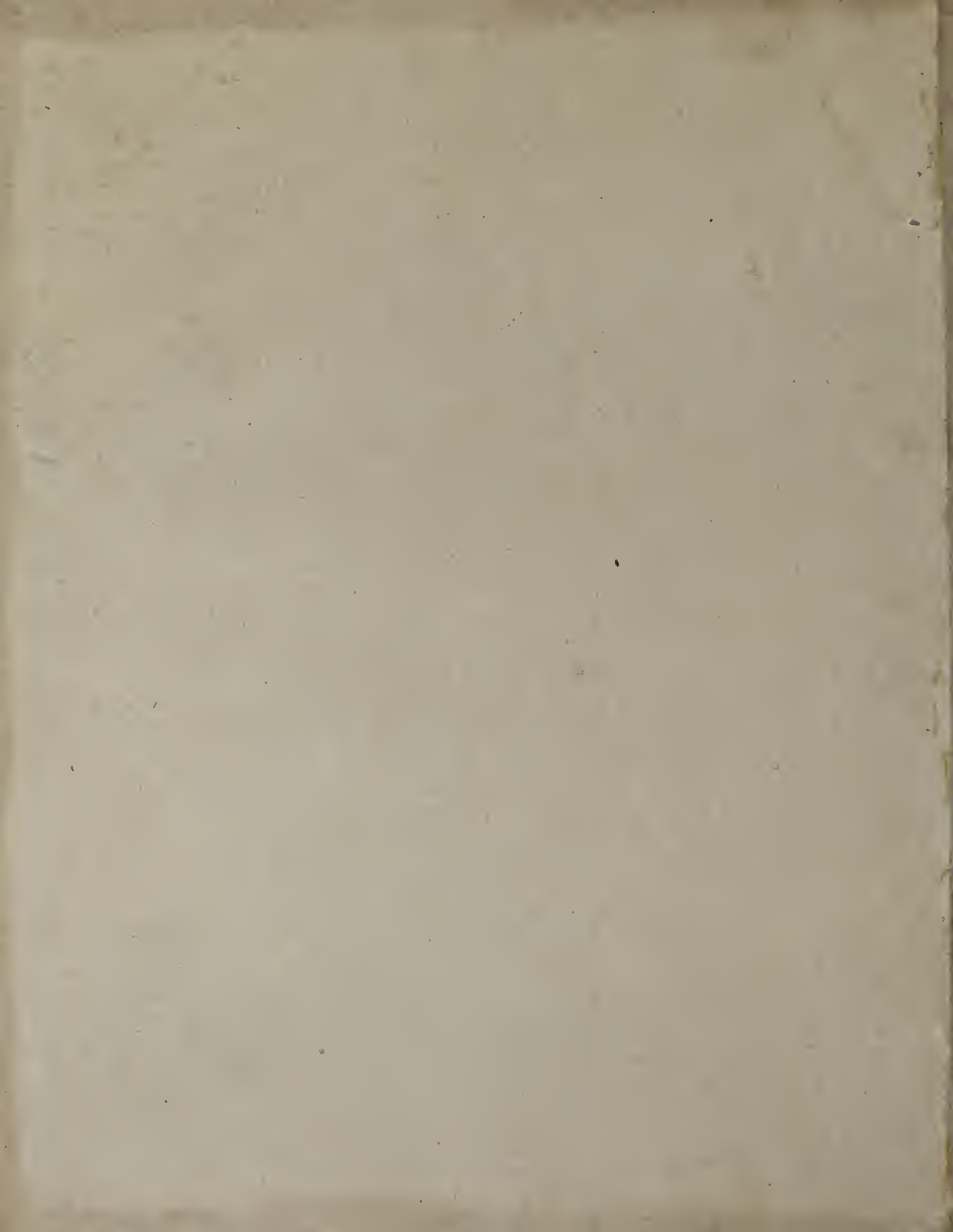
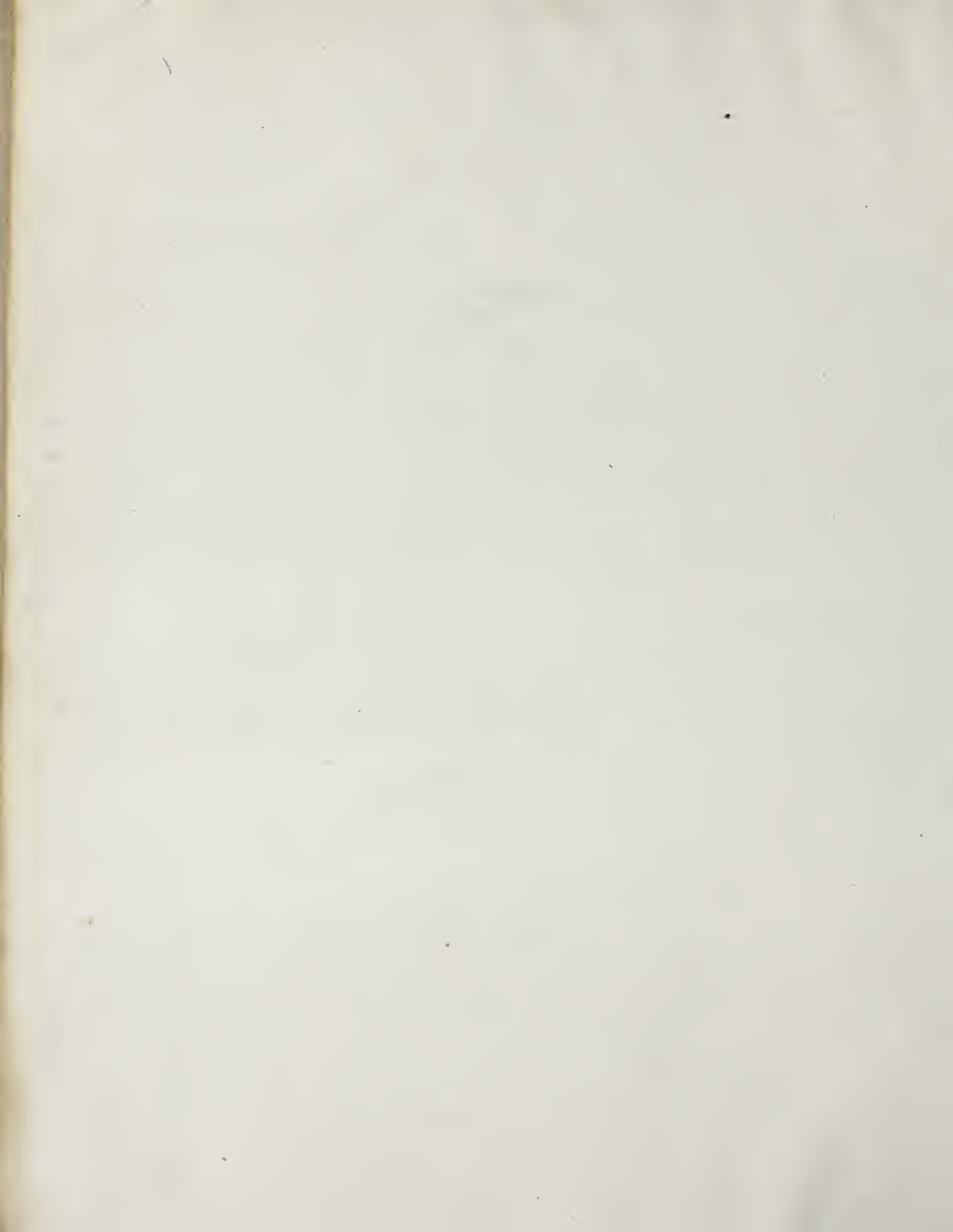
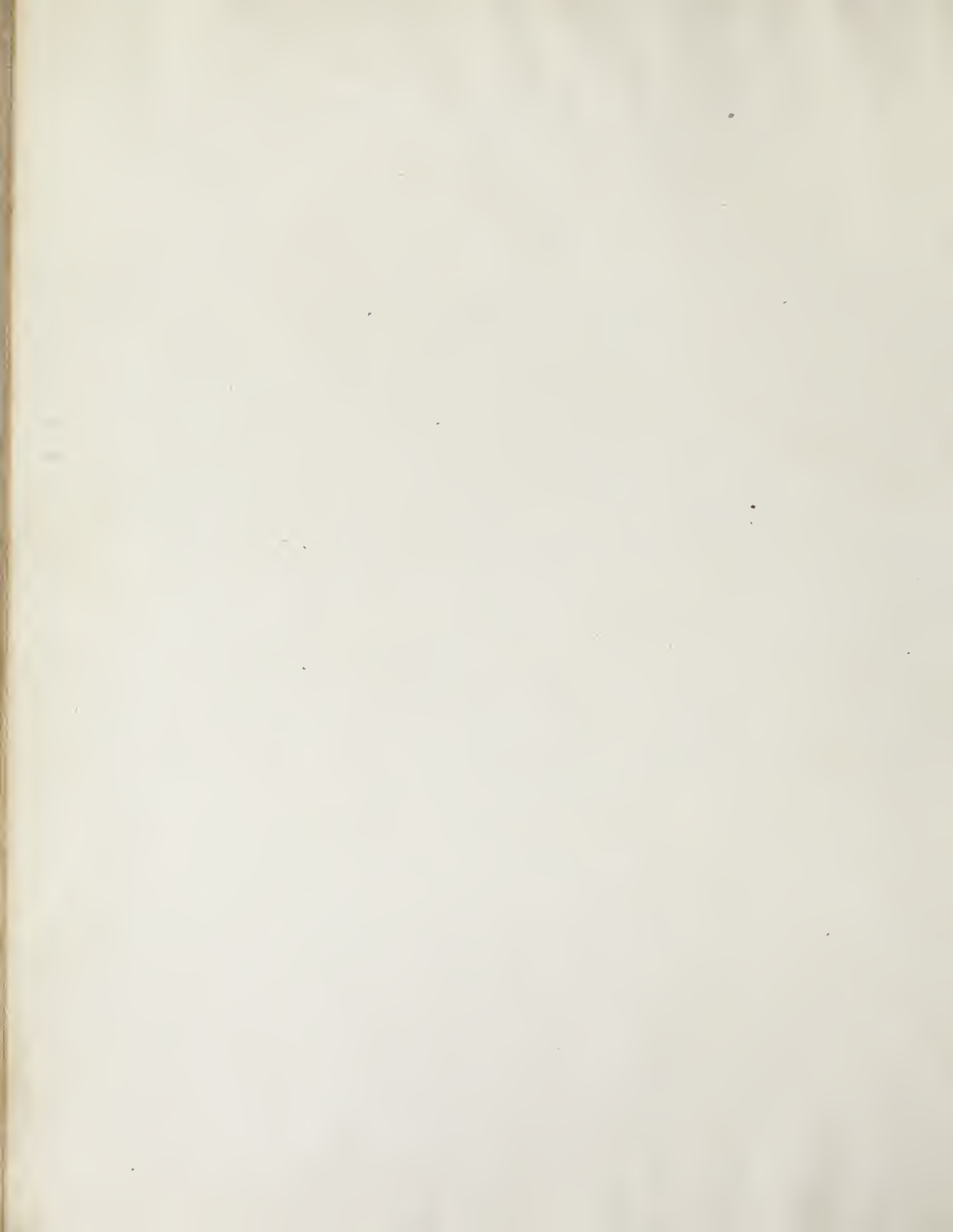
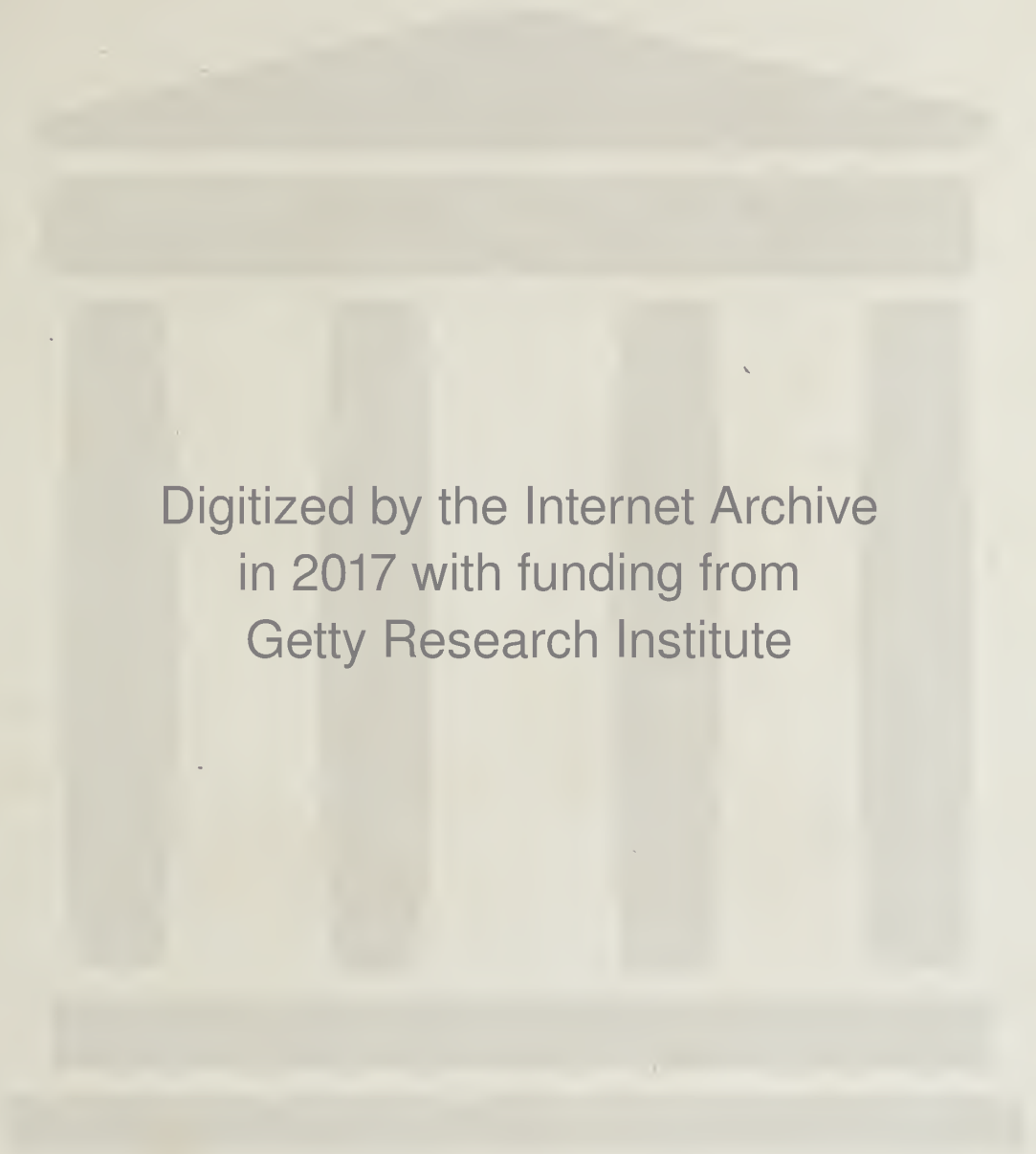








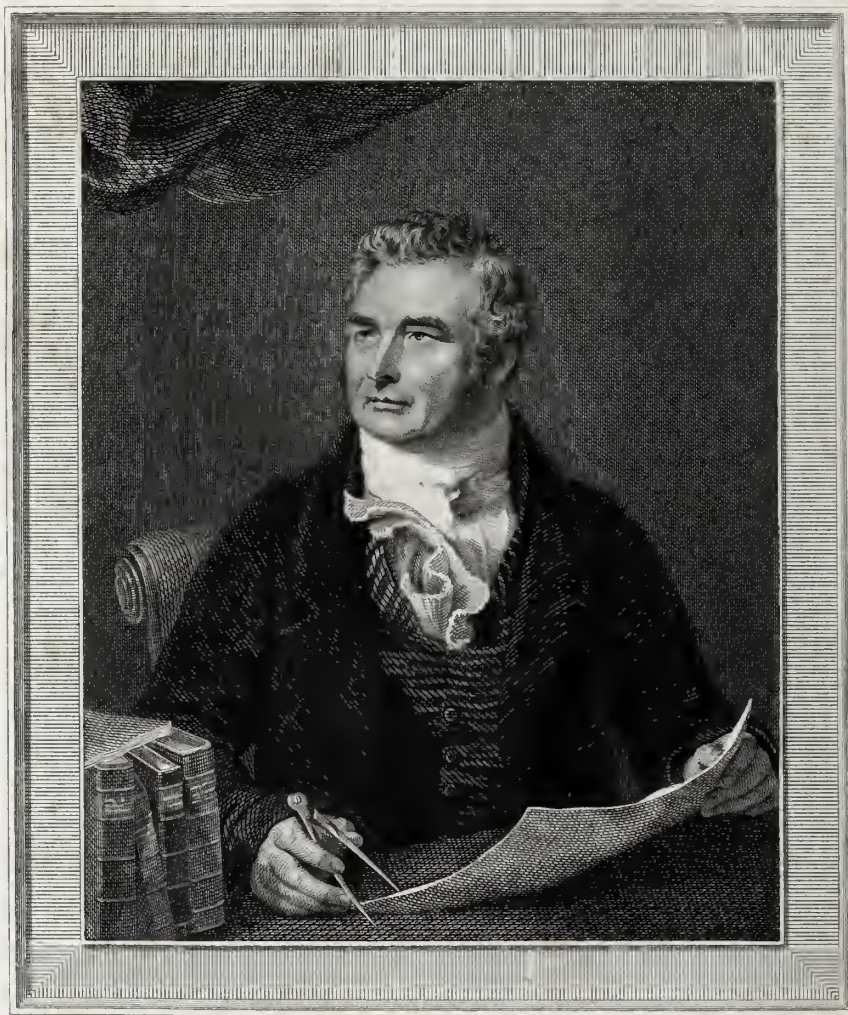




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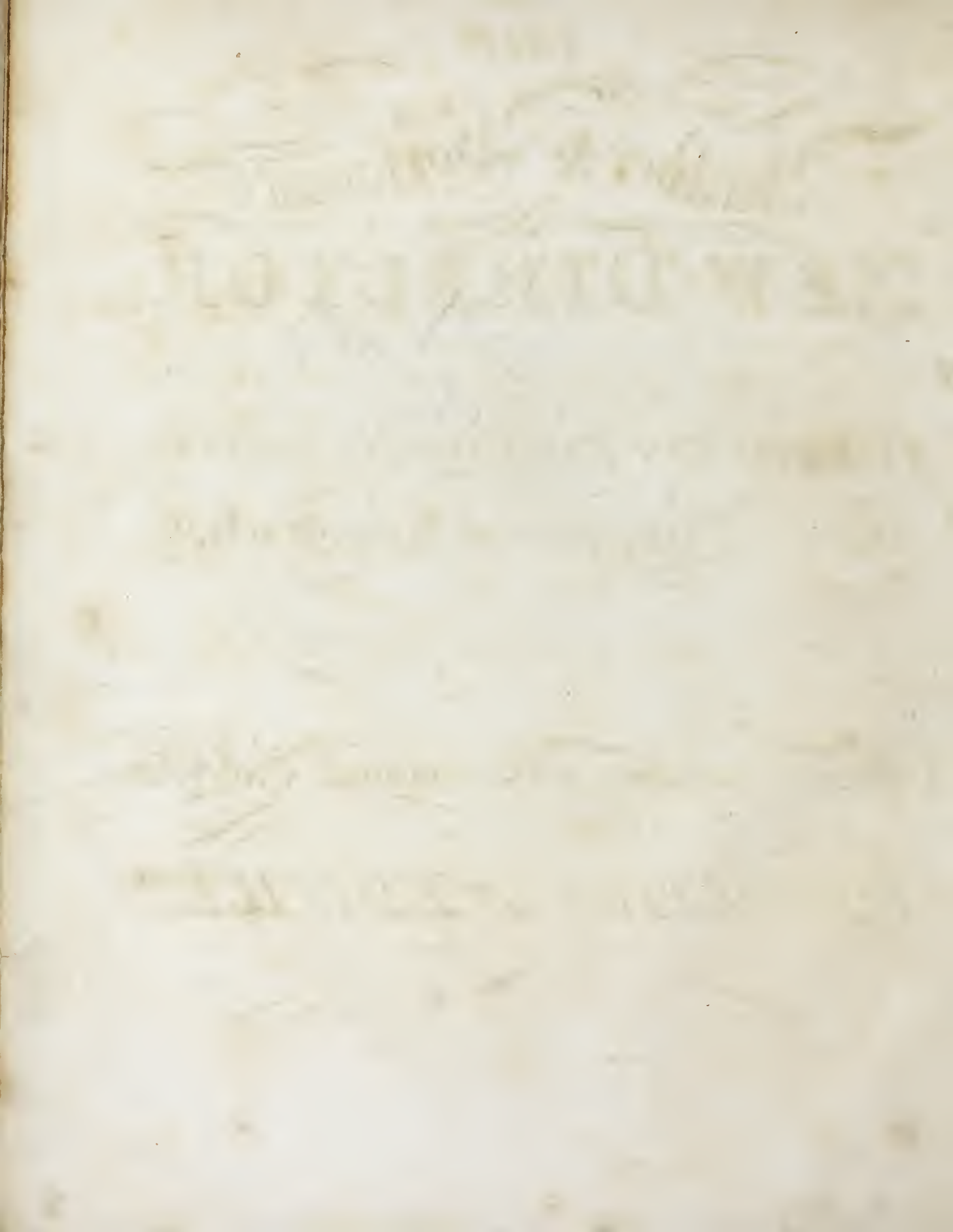
Engraved by C. Armstrong, from a Portrait by T. Heaphy, Esq.

MR. PETER NICHOLSON.

THE
Builder & Workman's
NEW DIRECTOR;
Comprehending Definitions and Descriptions
OF THE
COMPONENT PARTS OF BUILDINGS,
The Principles of Construction
AND THE
Geometrical Development of the principal Difficulties
THAT USUALLY OCCUR IN THE
Different Branches of Mechanical Professions,
Employed in the
FORMATION OF EDIFICES.

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PREFACE.

IN consequence of the great improvements that have recently been made in every branch of Architectural Science with regard to Geometrical Construction, the works formerly written on the subject are rendered deficient in elementary principles, and the methods which they contain have been superseded by others more elegant, and more expeditiously performed; therefore, in order to remove these defects, we have embodied in this work as much of Geometry, Conic Sections, Trigonometry, Projection, and Perspective, as will enable the student to comprehend with facility, and represent with accuracy, all the constructive departments of Building, with the application of these principles to Masonry, Carpentry, Joinery, and the Construction of Hand Railing.

The demonstrations are here annexed to the problems; for although a Workman may be taught to execute any part of a Building by rules only, he cannot be fully satisfied without a sufficient knowledge of the theory, which not only sharpens his intellectual powers, but renders him more ready of invention, and adds to his delight when ranging in quest of new speculations.

The useful subjects that have appeared in former publications are introduced, with a view of rendering this Work as complete as possible; but the reader will find them to be few, compared with what is now laid before him.

With regard to elementary principles, nothing is given that will not be found useful in its application to practice. The article on Practical Geometry contains more information than any thing of the kind formerly published, and the mode of treating it is very different from that usually adopted.

In the employment of the Engineer and Architect it is often necessary to draw lines to centres that are inaccessible; the methods for effecting this purpose are therefore given, with the arithmetical operations respecting the construction of large circles and their segments. The most useful properties of the Ellipse, Hyperbola, and Parabola, are demonstrated in a manner which is not only shorter, but much more easily perceived, by arranging the equations of the data in such a manner that all the

quantities are cancelled but those which form the equation expressing the property proposed to be demonstrated. Several useful Problems respecting these curves will be found in the work, and the construction of others much simplified. The Conchoid of Nichomedes and the Sinic Curve, on account of their utility, are here introduced; the former being frequently employed in diminishing the shafts of columns, and the latter generally appears in the formation of groins and domes. The method of forming Spirals is also inserted, because the best description of the Ionic Volute depends on the principles of the proportional Spiral, and the helices of the Corinthian capital are imitations of the reciprocal Spiral.

As every geometrical construction required in forming the various parts of an edifice consists in finding the sections and coverings of Solids, the articles on Stereotomy, and the Developement of the Surfaces of Solids, will be found of the greatest service; and because plans and elevations in all their generality depend on geometrical principles, the theory and practice of Orthopjection are given, which will also be found of the greatest use in the constructive departments of Masonry, Carpentry, and Joinery.

Trigonometry is here presented, both with regard to plane triangles and solid trhedrals, being highly useful to Engineers and Architects: by means of the trehedral, the angles of roofs and the oblique sections of solids can be ascertained, as well as the angles of the stones in masonic arches.

With regard to the practical subjects of Masonry, Carpentry, and Joinery, a greater variety of useful matter will be found than has ever been exhibited in any similar publication; and although the explanations in many instances are very short, it is hoped they will be found sufficient, as the principles have been explained at large in the theoretical part of the Work.

To make this Book generally useful, the Five Orders of Architecture are added, and drawn to a scale sufficiently large to display their forms and ornaments without further detail: the examples are selected from the most elegant remains of Grecian and Roman Antiquities.

Having thus given a summary of the articles that occur in the Work, more need not be said in this place. If the student attentively read the whole, and view the parts as they elucidate each other, an intimate and accurate knowledge of the subject will constitute an ample recompence.

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MEMOIR
OF
MR. PETER NICHOLSON.

MR. PETER NICHOLSON, who has produced so many useful, ingenious, and profound works on various branches of Mathematics, adds one to the number of those men, who, labouring under many difficulties in their youth, and without the aid of a scientific or even a liberal education, have distinguished themselves in a high degree, merely by the impulse of natural genius, accompanied by industry and sound judgment.

Genius, unaccompanied by industry, is of little utility, and is generally ruinous to its possessor; and even when so accompanied, unless directed by judgment to a proper purpose, is attended with but little advantage.

MR. NICHOLSON is a native of North Britain, and was born on the 20th of July, 1765, in the parish of Preston Kirk, in the county of East Lothian; and it deserves to be remarked, that the late Mr. John Rennie, the first civil engineer in this country since the death of Smeaton, and Mr. John Brown, perhaps the first scientific and practical agriculturist in Scotland, were all born within half a mile, and within a few years* of each other: they learned to read and write at the same school, and under the same master†.

The writer of this Memoir resided, in the early part of his life, not far from the parents of Mr. NICHOLSON, whose father, a stone-mason, much respected by his neighbours, was a man of peculiar integrity and simplicity of manners: his mother equally respectable, but distinguished from most others by superior manners, and an uncommon degree of quickness of parts and industry.

Young NICHOLSON at a very early age, even before he went to school, evinced a strong

* Mr. NICHOLSON being younger than either at the time he entered school, Mr. Brown had, for some time, completed his education, and Mr. John Rennie was then just about leaving it.

† The name of the schoolmaster was Richardson: he was brother of an architect of that name, who published several works, consisting of designs, a treatise on the different orders of architecture, &c.

mechanical genius, and also a turn for drawing whatever presented itself to him, whether of animated nature or otherwise.

On the opposite side of the small river on the northern bank of which his father's house stood, was a mill for dressing barley; and at no great distance on the same small river, were a saw, flour, and snuff mills; from which the ingenious boy was enabled to model many sorts of machines with considerable perfection. This sort of employment gave rise to an incident worth mentioning:—having finished a model of a saw-mill to be moved by the wind, he was trying how it would work on the road leading to a neighbouring village where the current of air was sufficiently strong, and where he expected to be uninterrupted. The machine had just begun to go to his mind, when the late Earl of Haddington passing, on horseback, the animal was frightened at the noise, and nearly threw his rider: NICHOLSON, afraid of what he had unintentionally done, escaped into an adjacent field; but the Earl, curious to know the cause, made his horse leap the fence, and followed him; when our youth, finding he must be overtaken, fell on his knees and begged pardon of his lordship, who, amused with the circumstance, gave him two shillings, and invited him to come to Tynningham Hall.

In those days, a lord in Scotland was an object of reverence, not to say of terror, to all the youth in his neighbourhood; a remnant of that feudal system, which till so lately existed in that country, and of which, in England, it is difficult to form any adequate idea.

It is to be regretted, that while every attention is paid to the education of some young persons, who undervalue opportunity and refuse instruction, others, who have both the capacity and inclination to learn, are often deprived of the means, or, at least, but very scantily supplied with them. Young NICHOLSON was in the latter situation: if his modesty or bashfulness had not prevented him from profiting by his casual introduction to Lord Haddington, he might probably have found a patron; as it was, however, three years' instruction was the most he ever had at the before-mentioned country school, which he left at the age of twelve. Although his scholastic instructions were very limited, this youth, bent on inquiry, and that decidedly of a mathematical nature, made by his own efforts considerable progress. Having borrowed from another boy, much older than himself, a copy of Commandine's Euclid, translated by Cunn, he soon made himself master of the 1st and 2d books, and got as far as the 18th Proposition in the 3d; but the plate which contained the diagrams to which the demonstrations refer being wanting, he constructed the figures from the problems, without the assistance of any copy. In performing this task, he made use of a pen tied to the foot of a pair of compasses (such as carpenters use), with which rude instrument and a rule he drew the twenty diagrams or figures that complete the 3d book with great neatness and accuracy. We have seen

the book, which, though rather worse for wear, is still entire, to serve as a proof and example of ingenious industry*.

At about the age of twelve, he assisted his father in his business; but as he never could relish that occupation, his parents had too much good sense and natural affection to force his inclination, and he was, at the end of a year, bound apprentice to a cabinet-maker, in the neighbouring village of Linton, for four years†, during which time he employed every spare moment in improving himself.

It may not be improper here to advert to a truth, perhaps not sufficiently attended to, namely, that to have worked at the business which forms the subject of his studies, is a great advantage to a man of science. There is a sort of knowledge obtained by the fingers' ends that is not otherwise to be got. It is seldom that men of science have laboured with their hands; but when they have, and afterwards devote their time to study and inquiry, their ideas are more accurate, and they are more likely to become inventors. Had Sir Isaac Newton, great as he was, been a practical mechanic in his youth, he never would have given the Theory of the Communication of Motion in the manner that he has done: he would not, at least, have given it without a qualification; for though with bodies perfectly hard, or perfectly elastic, the theory might hold true, yet, as no such bodies exist, it leads to great error. Every practical mechanic knows that a small hammer, moving with great velocity, will produce on the body it strikes a very different effect from that produced by a heavy hammer moved slowly; though their weights and velocities, multiplied by each other, may be equal. This is but one example of the difference between theoretical and practical knowledge; but hundreds might be given; and may account for the fact, that the greatest inventors in mechanics have, at one time or other, laboured with their hands. Mr. NICHOLSON had, at least, that advantage, and he has turned it to good account; for there is not any man at this day to whom the building art owes more, as will hereafter be seen.

Having served to the full extent of his time‡, he went to Edinburgh to work as a

* In recent editions of Euclid, the figures are from wood cuts, on the same page with the letter-press; but in Cunn's translation they are engraved on plates.

† In Scotland, three years is the usual period of apprenticeship, as seven is in England. Whether this is owing to the impatience of the Scotch to be turning the penny, or to greater confidence in their abilities, the reader must be left to judge.

‡ During his apprenticeship, he studied Algebra assiduously from Maclaurin; and from daylight in the summer mornings till six, when he went to work, as well as in the evenings, when his labours were over, he practised drawing from Salmon's London Art of Building, and occasionally by measuring heights and distances in the fields, making himself conversant with Trigonometry, as applicable to real business, which, it is to be observed, he had always in view.

journeyman, where he studied mathematics more generally from Ward's Introduction. As a reference from some of the books he had read excited an uncommon desire to acquire a knowledge of the fluxionary calculus, he went to a then noted bookseller, Mr. Bell, in Parliament-square, and looked over the prefaces and contents of several books; when the Introduction to Emerson's Fluxions determined him to give it a preference; but the scanty wages which he received for his labour being scarcely sufficient for necessaries, he had not the means of paying for it. Mr. Bell, however, wishing to give every encouragement to a young man in pursuit of knowledge, although a mere stranger, told him to take the book, and pay for it at his convenience, which accordingly was done.

Emerson's Treatise on Fluxions has been generally allowed to be the most difficult on this branch of analytical science; it was not long, however, before Mr. NICHOLSON obtained a complete knowledge of the first principles; and the pleasure which resulted from their application to so many sublime problems, so encouraged him, as to make him surmount every other difficulty, in whatever part of the work his inclination led him to study.

Mr. NICHOLSON came to London about the age of twenty, and for some time pursued his favorite studies of the mathematical sciences at his leisure hours in the evenings; at length, however, he dropped the pursuit of analytical subjects, and continued to study Geometry only, as being more congenial to his views in life.

About his 24th year, he had given such proofs among his fellow workmen of his knowledge in the geometrical constructions of Carpentry and Joinery, that many of them solicited to become his pupils; he, therefore, opened a school in Berwick-street for the instruction of young workmen. His success in teaching soon brought him a very great influx of pupils, raising him above the level of a mere journeyman, and this gave him leisure to invent, and arrange the materials for a new and original Treatise on Carpentry and Joinery, called the "CARPENTER'S NEW GUIDE," and published in 1792. The plates of this work were engraved by himself. Besides the improvements in various kinds of groins which is to be found in this work, he introduced the construction of spherical niches, both upon straight and circular plans. Before that publication appeared, no work on the Practical Parts of Building had shewn, generally, how the sections and coverings of solids were obtainable from their definitions. The principles went only to find the section of a right cylinder perpendicular to a given plane parallel to its axis, and to the covering of such a cylinder and that of the frustum of a right cone. Some attempts to obtain the same result had, indeed, previously been made; but they were erroneous and abortive: and, as they had not succeeded with the plane sections and coverings of simple solids, they could scarcely be expected to give rules for the construction of the intersection of any two surfaces or curves of double curvature, of

which the variety is almost infinite. Besides Mr. NICHOLSON'S own inventions, he has both simplified and generalised the old methods, which were only applicable to particular circumstances. His rules for finding the section of a prism, cylinder, or cylindroid, through any three given points, whether in or out of the surface of the body to be cut, enable workmen to execute hand-rails without difficulty, and from the least possible quantity of stuff. His principles on the Intersection of Solids extend to groins and arches of almost every description. The coverings of polygonal and circular domes had been exhibited in several prior publications on Carpentry and Building; but no author had ever shown how these coverings were to be formed without the actual plan, which might be very inconvenient to draw, on account of the great space it required: nor had any method for covering domes upon an elliptic plan been given. The "CARPENTER'S NEW GUIDE" is one of his first and most useful works: it has gone through seven editions, and is still as much in request as ever.

The "STUDENT'S INSTRUCTOR" was published immediately after the "CARPENTER'S NEW GUIDE," and was followed by the "JOINER'S ASSISTANT," a work abounding with useful information, either to those who might be employed to make working drawings, or in the superintendence of buildings. Here several of the articles which were slightly treated of in the "CARPENTER'S GUIDE" are more fully explained and extended to sufficient length. Besides the designs of roofs actually executed, which it contains, it was the first work that treated on the methods of forming the joints, hinging and hanging doors and shutters. It has now reached to the fourth edition, which is an indisputable proof of its utility.

The "PRINCIPLES OF ARCHITECTURE," in 3 volumes, octavo, appeared in 1799; till which time all the publications on that subject were greatly deficient in mathematical principles. In the latter publication was introduced a System of Practical Geometry, containing many new problems of great utility to builders.

Mr. NICHOLSON was the first who noticed in this publication that Grecian mouldings are conic sections; and that the volutes of the Ionic capital ought to be composed of logarithmic spirals, for which he accordingly gave rules for the describing of arches, mouldings, and spirals, of so general a nature, as to be applicable in all cases. He was the first, at least as far as we know, to show how to describe any number of revolutions between any two given points in a given radius in which the centre of the spiral was given. And thus he generalized the principles of De l'Orme and Goldman, whose methods were limited to three revolutions, and the eye of each of their volutes to one eighth part of the whole height. This work was also the first that contained universal methods for projecting plans and elevations, with the method of finding their shadows.

About the same time that this work appeared in England, a work nearly similar was published in France, by M. Monge ; but the methods employed for the purpose are widely different.

All those works have been highly esteemed by the best judges, and have passed through various editions. On account of the engravings, they are expensive, and the purchasers are not wealthy ; so that the sale to a large amount is an indisputable proof of their utility.

MR. NICHOLSON returned to Scotland in the year 1800, and, after staying a few months in his native village, went to Glasgow, where he practised with honor and reputation as an architect. Among the numerous edifices executed in and about that city from his designs, and which are esteemed classical models of taste, by those who are best qualified to judge, were Carleton Place, an addition to the College buildings, the wooden bridge over the river Clyde, and the town of Ardrossan in Airshire, designed as a bathing-place, at the request of the Earl of Eglington.

During his residence in Glasgow he wrote numerous articles for Rees's and the Edinburgh Encyclopedias, the two most celebrated works then publishing, together with various articles for other publications.

In the year 1808, MR. NICHOLSON removed to Carlisle, in Cumberland, having been appointed architect for the new court houses then begun, from designs given by Mr. Telford, the celebrated civil engineer, who recommended him to that situation, and as an architect for the county of Cumberland. MR. NICHOLSON returned to London in 1810, when he commenced his "ARCHITECTURAL DICTIONARY," and continued to write both for Rees's and Dr. Brewster's Encyclopedias.

His book of "MECHANICAL EXERCISES" was finished soon after his return to London ; the object of which work is to give a familiar description of such parts of a building as are susceptible of being explained without the aid of geometrical lines : and it is but doing justice to MR. NICHOLSON to say, that it has given rise to other publications of the same nature, though without his name.

In April 1814, the Society of Arts voted to MR. NICHOLSON their Gold Isis Medal, for a new improvement in hand-railing ; and in May, the same year, that Society rewarded him with the sum of Twenty Guineas, for the invention of the *Centrolinead*.

In the year 1815, he was rewarded with the Silver Medal of the Society, for the

invention of another *Centrolinead*, which is now brought into general use amongst those artists who make perspective drawings in architecture or machinery.

The numerous articles which his "ARCHITECTURAL DICTIONARY" contained, led him to many curious investigations, and induced him once more to turn his attention to analytical science, the result of which was, that during the publication of that work he produced a small tract on the "METHOD OF INCREMENTS," "ESSAYS ON THE COMBINATORIAL ANALYSIS," and his "RUDIMENTS OF ALGEBRA," all within the short space of two years. The last of these three works was published on the 1st of July, 1819, and the "ARCHITECTURAL DICTIONARY" was completed about two months afterwards.

MR. NICHOLSON'S "ESSAY ON INVOLUTION AND EVOLUTION" was honoured with the approbation of the French Institute and Royal Academy of Sciences:—that work, and his "ANALYTICAL AND ARITHMETICAL ESSAYS," were published in the year 1820.

His last book is entitled the "RUDIMENTS OF LINEAR PERSPECTIVE;" but, as his works are very generally known, and widely circulated, it is unnecessary to enter into any discussion of their merits, which are generally acknowledged, and were noticed in the Reviews and Publications of the time, particularly in the *Monthly Review* for December 1820, where there is a very perspicuous and luminous discussion on his Analytical Tracts.

The whole of Mr. NICHOLSON'S active and scientific labours have been directed towards the applying science to useful purposes; a thing generally too much neglected by men who are given to study.

In the month of February of the present year 1822, was published a "POPULAR COURSE OF MATHEMATICS," by Mr. N., consisting of Algebra, Euclid's Elements, Differential Calculus, Fluxions, Conic Sections, Doctrine of Curves, Trigonometry, Mensuration, Land Surveying, Gauging, Perspective, Dialing, Spherics, Mechanics, Dynamics, Hydrostatics, Optics, Physics, and Astronomy, with Tables of Logarithms, and numerous questions for exercise, illustrated by several hundred engravings.

To improve the union of industry and science, has been the object of the books he has written and the lessons he has taught: and if, in aid of those exertions, there were public

institutions here, as in other countries, to facilitate the attainment of those objects without expense, their progress would be still greater* than it is.

As an individual, Mr. NICHOLSON has done much; and he has had to do with pupils not insensible to his merit, and the advantage to be derived from his instruction: nor should it be omitted to be observed, that the Carpenters and Joiners of this country, for excellence of workmanship, and knowledge of the principles by which they are directed in their operations, surpass all others in the world.

* In Paris, there is a building, nearly as large as the Bank of England, where there is a collection of all the best tools and machines, which is open two days in the week, to all persons indiscriminately, without expense. And there are always numbers of workmen examining the whole with great care and attention. Such institutions are highly honourable to the government, and useful to the country.

THE
BUILDER AND WORKMAN'S
NEW DIRECTOR,

&c.

PLANE GEOMETRY.

GEOMETRY is the science which demonstrates the properties that obtain between angles, lines, surfaces, and solids.

Practical Geometry shews the methods for constructing angles, lines, and figures, according to certain given conditions called data, which are things given in order to discover others.

DEFINITIONS OF TERMS.

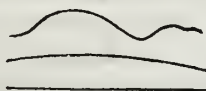
(1) *Def. 1.* A *solid* is that which fills a certain portion of space, so as to prevent every other solid from occupying that space.

(2) *Def. 2.* A *surface* or *superficies* is the boundary or boundaries of a solid; or the surface of a solid is that part of it that may be seen and felt.

(3) *Def. 3.* A *point* is that which indicates a certain situation on a surface, but is no portion either of that surface or of the solid (1).

(4) *Def. 4.* If a point be drawn in any manner upon a surface, the trace which it leaves behind it upon that surface is called a *line*.

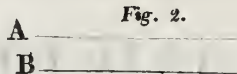
Fig. 1.



Coroll. Hence a line has continuity, but, if divided into two parts, each extremity thus cut is a point.

(1) The smallest mark that can be made so as to be seen, is practically considered to be a point. A visible point is usually formed with a pen or pencil, or in drawing with the one extremity of a compass.

(5) *Def. 5.* When one line A can be always so applied to another B, that, when any three points of the one line coincide with the other line, each of these lines is a *straight line* (2).



Coroll. Hence two straight lines cannot enclose a space, nor have a common segment; that is, they cannot coincide in part without coinciding altogether (2).

(6) *Def. 6.* If on a surface any two points be taken, and if a straight line be applied to these two points, and if every point in the part of the straight line between the points be found to coincide with the surface, such a surface is called a *plane* or *plane surface*.

We have given this kind of analysis of a solid, because no idea of a surface can be obtained without the previous knowledge of a solid; nor can any idea of a line be abstractly formed without having the knowledge of a surface; nor can any notion be formed of a point without the previous notion of a line: for no solid can possibly exist without a surface, nor on the contrary can a surface exist without a solid. As the termination of a solid is its surfaces, so the termination of surfaces are lines, and the termination of lines are points: points are therefore to be considered as the first elements of magnitude.

Euclid begins his definitions with a point; which, however, has been objected to by many eminent writers, and therefore they define the solid first, and say that a "solid is that which has both length, breadth, and thickness." This definition is, however, very improper, as it tacitly implies not only the notion of lines, but that of position also; for what idea have we of length, without the idea of a straight line? and what idea have we of length and breadth, without the idea of a right angle? and moreover, what idea have we of length, breadth, and thickness, without the idea of a plane, and a straight line perpendicular to that plane? And as these are subsequently defined, nothing is gained by beginning the definitions of geometry with a solid.

(7) *Def. 7.* The *distance* between two points is the straight line reaching from one of the points to the other (3).

(2) Thus suppose two very thin boards are set together in contact face to face, and their edges shot; and if any two points in the edge of the one board be each brought into contact with the edge of the other board; then, if no light appears between the two edges, each edge is considered to be straight, and is therefore called a *straight edge*.

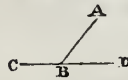
Another method of proving a straight edge upon the same principle, is as follows: shoot the edge of a board as before, and lay its face upon the planed face of another board, and draw a line along the edge thus shot upon the planed face of the board under it. Mark two points in the line so drawn, and upon the edge of the board over the line at the same two points. Lay the upper side of the upper board upon the planed face of the under board, and upon the other side of the line, and bring the two marks or points in the planed edge to the two points in the line; then, if the edge coincide with the line, the line is straight; but if it does not, the operation must be repeated.

The term *straight edge* is, however, applied by workmen to any board which has one straight edge, and which is used for the purpose of drawing straight lines; since the edge cannot exist without the board, which must be used as a handle in order to apply its straight edge in drawing.

(3) It may be proved (though we shall here take it for granted) that a straight line is the shortest line that can be drawn between two given points.

(8) *Def. 8.* One line AB is said to meet another CD when the extremity B of the one falls in the other line CD.

Fig. 3.



(9) *Def. 9.* One line is said to cut another, when either of these two lines divides the other into two parts.

(10) *Def. 10.* An *intersection* is the cross formed by two lines cutting each other.

Fig. 4, No. 1. Fig. 4, No. 2. Fig. 4, No. 3.



Once for all, it is only necessary to state, that, when lines are mentioned without any other word to restrain their signification than the simple word *line*, a straight line is implied; and thus all lines in the definitions or construction of diagrams are understood to be straight lines.

When any other kind of lines are meant, a word of distinction will always be introduced.

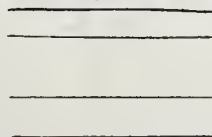
(11) *Def. 11.* Two straight lines are said to be parallel when they cannot meet, neither upon the supposition of their being produced at one end nor at the other.

Fig. 5.



(12) *Def. 12.* When any two lines of any number are parallel, the whole number are called *parallels*, and any one of them is called a *parallel*.

Fig. 6.



(13) *Def. 13.* If any given number of straight lines, when produced, all meet in one point, they are called *converging* lines, or *centrolineals*.

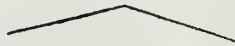
Fig. 7.



(14) *Def. 14.* An angle is the space contained between two straight lines which meet each other, but not in the same straight line; and the point where the two straight lines meet is called *the point of the angle*, or *point of meeting*, or *point of concurrence*, or simply *concurrence*.

Fig. 8, No. 1.

Fig. 8, No. 2.



(15) *Def. 15.* Each of the two straight lines which form an angle is called a *leg*, and sometimes a *side*.

(16) *Def. 16.* One angle A is greater than another B, if when A be applied to B, so that the point of the angle A may be upon that of B, and one of the legs of A upon one of the legs of B, and when the remaining leg of B falls without the space contained by the legs of A (4).

Fig. 9, No. 1.

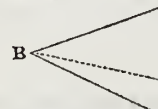
Fig. 9, No. 2.



(17) *Def. 17.* One angle A is less than another angle B, if when A is applied to B, so that the angular point of A may be upon that of B, and one leg of A upon one leg of B, when the remaining leg of A falls within the space contained by the legs of B (4).

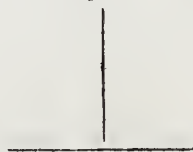
Fig. 10, No. 1.

Fig. 10, No. 2.



(18) *Def. 18.* When the two angles formed by a straight line passing through one extremity of another straight line are equal, each of the equal angles is called a *right angle* (5).

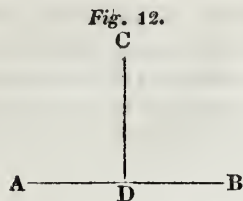
Fig. 11.



(4) The magnitude of an angle is not dependent on the length of its legs (as young beginners are apt to suppose), but upon the opening.

(5) This definition is the criterion to prove the truth of the instrument called a square, which Carpenters and Joiners use for making the faces of their stuff at right angles with each other. The workman applies the inner edge of the stock of the square to the straight edge of a board, so that the under side of the blade may

(19) *Def. 19.* In two right angles formed by a straight line passing through one extremity of another, the line which divides the other is called a *perpendicular* to the divided line.



Thus the straight line which has C at one end, and D at the other, is called the perpendicular; and the other straight line which has A at one end and B at the other, is sometimes called the base.

(20) *Def. 20.* In two right angles formed by a straight line passing through the one extremity of another, the dividing line is said to stand at right angles to the divided line.

So that when it is said that such a line is at right angles to such another line, it is equivalent to say, that that line is perpendicular to the other line.

(21) *Def. 21.* An *obtuse* angle is that which is greater than a right angle.

Fig. 13.



(22) *Def. 22.* An *acute* angle is that which is less than a right angle.

Fig. 14.



(23) *Def. 23.* A figure, or plane figure, is that which is enclosed by one or more lines.

be coincident with the surface of that board; and by one of the edges of the blade of the square, with a sharp-pointed instrument, draws a line on that surface: he then turns the square, so that the other side of the blade may be coincident with the surface of the board, and the inner edge of the stock of the square coincident with the edge of the board, and the same edge of the blade before used, upon some part or point of the line thus drawn; then, if this edge of the blade of the square coincide entirely with the line, the said square is adjusted, and the inner edge of the stock and that edge of the blade used will form a right angle; but if the edge of the blade cross the line, the square is not true, and the operation must be repeated.

When the two faces of a piece of stuff form a right angle, the one face is said to be square to the other; and when every two adjoining faces are made square, the stuff is said to be squared.

(24) *Def. 24.* The line or lines which enclose a figure, are called the sides of that figure.

The side of a figure next to the bottom of the paper is generally called the base, by way of distinction from the other sides; though occasionally any side may be considered as the base.

In this treatise, every side of a figure is considered to be a straight line, unless otherwise specified.

In every subsequent definition, a figure is supposed to be drawn upon a plane surface, according to the conditions specified in the definition; or, if the figure limit the extension of the surface, it is supposed to be such as will coincide with a plane.

(25) *Def. 25.* Equilateral or equal-sided figures are such as are enclosed by equal straight lines.

(26) *Def. 26.* Equiangular figures are those which have the angles contained by their sides equal to each other.

(27) *Def. 27.* Opposite sides of a figure are such two sides as have the same number of intermediate sides connecting each of their extremities.

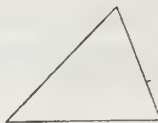
(28) *Def. 28.* Opposite angles of any figure, are any two angles that have an equal number of sides on each side of the straight line joining their angular points, or supposed to join them.

(29) *Def. 29.* An angle is said to be opposite to a side, when the figure has an equal number of sides joining each extremity of that side and the angular point.

(30) *Def. 30.* A figure bounded by three sides is called a *triangle*.

(31) *Def. 31.* A triangle which has no two equal sides, is called a *scalene* triangle.

Fig. 15.



(32) *Def. 32.* A triangle which has only two equal sides, is called an *isosceles* triangle.

Fig. 16.



(33) Def. 33. The side of an isosceles triangle, which is unequal to either of the other two, is called the *base*.

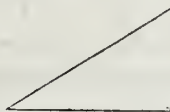
(34) Def. 34. A triangle which has every two of its sides equal, is called an *equilateral* triangle.

Fig. 17.



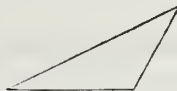
(35) Def. 35. A triangle which has a right angle, is called a *right angled* triangle.

Fig. 18.



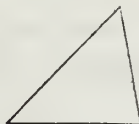
(36) Def. 36. A triangle which has an obtuse angle, is called an *obtuse angled* triangle.

Fig. 19.



(37) Def. 37. A triangle, which has all its angles acute, is called an *acute angled* triangle.

Fig. 20.



(38) Def. 38. A figure enclosed or bounded by four sides, is called a *quadrilateral* or *quadrangle*.

Fig. 21, No. 1.



Fig. 21, No. 2.

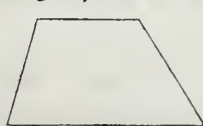


Fig. 21, No. 3.

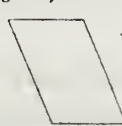
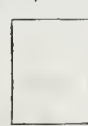
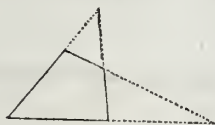


Fig. 21, No. 4.



(39) Def. 39. A quadrilateral, which has each pair of its opposite sides converging, is called a *trapezium*.

Fig. 22.



(40) *Def. 40.* A quadrilateral, which has only one pair of its opposite sides parallel, is called a *trapezoid*.

Fig. 23.



(41) *Def. 41.* A parallelogram is a quadrilateral, which has each pair of its opposite sides parallel to each other.

Fig. 24, No. 1.

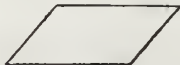
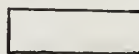


Fig. 24, No. 2.



(42) *Def. 42.* A rectangle is a parallelogram, which has one right angle (6).

Fig. 25, No. 1.

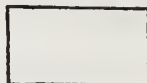


Fig. 25, No. 2.



(43) *Def. 43.* A rectangle, which has the two sides containing the right angle unequal, is called an *oblong*.

Fig. 26.



(44) *Def. 44.* A rectangle, which has two of its adjoining sides containing the right angle equal, is called a *square*.

Fig. 27.



(45) *Def. 45.* The *length* of an oblong is the distance between the extremities of the greatest side of the two that contain the right angle.

(46) *Def. 46.* The *breadth* of an oblong is the distance between the extremities of the least side of the two that contain the right angle.

(6) It may, however, be demonstrated, that if a parallelogram have one right angle, it will also have each of its other three angles a right angle.

(47) Def. 47. A *polygon* is a figure enclosed by more than four sides.

Fig. 28, No. 1.

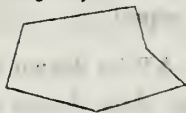


Fig. 28, No. 2.

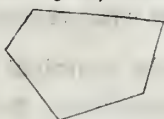


Fig. 28, No. 3.



(48) Def. 48. A *regular polygon* is a figure which is both equilateral and equiangular, or that which has all its sides equal, and all its angles equal.

Fig. 29, No. 1.



Fig. 29, No. 2.



Fig. 29, No. 3.



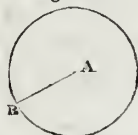
Fig. 29, No. 4.



(49) Def. 49. A *pentagon* is a polygon of five sides; a *hexagon*, a polygon of six sides; a *heptagon*, a polygon of seven sides; an *octagon*, a polygon of eight sides; and so on (7).

(50) Def. 50. If a point A be assumed in a plane, and if the end of a straight line of any given length be fixed in that point A, and if, when the other end B is carried entirely round, a pencil or point be held at that end B, so as to trace a line on the plane, the figure thus described by the moving point is called a *circle*.

Fig. 30.



(51) Def. 51. The fixed point A, is called the *centre* of the circle.

(52) Def. 52. The line described by the moving point B, is called the *circumference* or *periphery*.

(7) These names are composed from the Greek numerals *hise*, *duo*, *trice*, *teseras*, *penta*, *hex*, *hepta*, *octa*, *ennea*, which signify *one*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, and *nine*. Again, *deca*, *icosa*, *triaconta*, *teseraconta*, *penteconta*, *hexaconta*, *hepdomacont*, *ocdoeconta*, *enneneconta*, and *hecaton*, signify *ten*, *twenty*, *thirty*, *forty*, *fifty*, *sixty*, *seventy*, *eighty*, *ninety*, and a *hundred*, &c. Therefore, compounding the word which expresses the number of tens with that which expresses the number of units, we shall have the sum of both, or the particular number we desire: thus *icosa hise* signifies twenty-one.

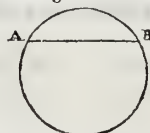
The other word which forms the last part of the name is derived from another Greek word *gonia*, which signifies an elbow or angle.

(53) *Def. 53.* Any straight line AB, drawn from the centre of a circle to its circumference, is called a *radius*.

Coroll. Hence all the radii (8) of the same circle are equal.

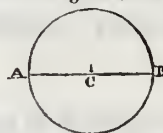
(54) *Def. 54.* If, in a given circle, a straight line AB be drawn so as to be terminated at each end A and B by the circumference, the line thus drawn is called a *chord* of that circle.

Fig. 31.



(55) *Def. 55.* A chord AB, which passes through the centre C of the circle, is called the *diameter*.

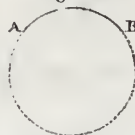
Fig. 32.



Coroll. Hence the diameter of a circle is equal to twice its radius.

(56) *Def. 56.* The arc of a circle is any portion AB of the circumference.

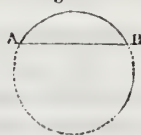
Fig. 33.



Thus, if the arc of a circle be half its circumference, it is called a *semicircular arc*; or, if it be one quarter, it is called a *quadrantal arc*.

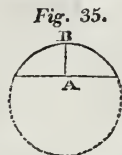
(57) *Def. 57.* A straight line AB, joining the extremities of an arc, is called the *chord* of that arc.

Fig. 34.



(8) The term radii is the plural of radius, and is pure Latin. It is used in order to avoid the disagreeable pronunciation of the word *radiuses*, formed according to English construction.

(58) *Def. 58.* A straight line AB drawn from the middle of a chord, perpendicular to that chord to meet the arc, is called the *versed sine* of that arc.



(59) *Def. 59.* A *segment* of a circle is a portion of a circle bounded by a chord, and the one arc of the circle intercepted by the chord.

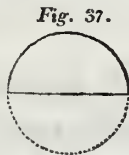
Fig. 36, No. 1.



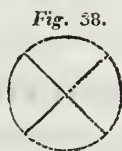
Fig. 36, No. 2.



(60) *Def. 60.* A *semicircle* is a segment terminated by the diameter and the semi-circumference cut off by the diameter.



(61) *Def. 61.* A *quadrant* of a circle is that portion of a whole circle which is contained by two radii ^{at right angles}, and the part of the circumference comprehended between the extremities of the radii.



(62) *Def. 62.* A *sector* of a circle is the figure ABC comprehended between two radii CA, CB, and the portion AB of the circumference terminated by these radii.

Fig. 39, No. 1.

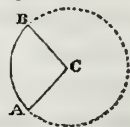
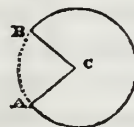


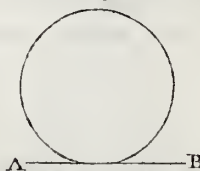
Fig. 39, No. 2.



Coroll. Hence a quadrant is the sector of a circle comprehended by two radii at a right angle with each other, and the arc between these radii.

(63) *Def. 63.* A straight line AB is said to touch a circle when the line is produced on each side of the point where the line meets the circle, without cutting the circle.

Fig. 40.



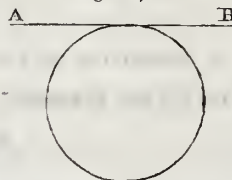
(64) *Def. 64.* The point where a straight line touches a circle is called the *point of contact*.

(65) *Def. 65.* A straight line AB, which touches the circumference of a circle or any arc, is called a *tangent*.

Fig. 41, No. 1.

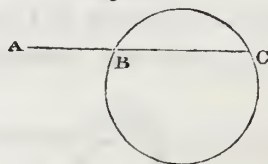


Fig. 41, No. 2.



(66) *Def. 66.* A straight line AC, drawn from any point A without a circle to cut the circumference and to meet it in another point, is called a *secant*.

Fig. 42.



(67) *Def. 67.* One circumference of a circle is said to touch another, when both circumferences meet each other in one point only.

Fig. 43, No. 1.

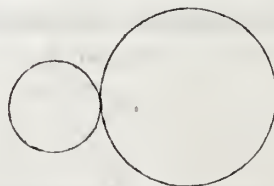
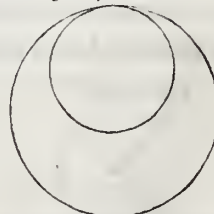


Fig. 43, No. 2.



(68) *Def. 68.* A Diagonal is a straight line drawn entirely within a figure, from one angle to another.

(69) *Def. 69.* One angle B is to another E, as the magnitude of the arc *ac*, described between the legs of the one angle, is to that *df*, described between the legs of the other, with the same radius.

Fig. 44, No. 1.

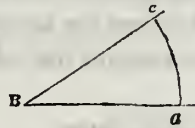
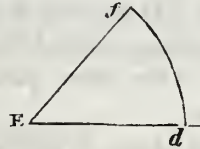


Fig. 44, No. 2.



Thus, if the arc *df* be twice the arc *ac*, then the angle E is twice the angle B; or, if the greater are be divided into equal parts, then, whatever number of parts the lesser are contains of the parts of the greater arc, the lesser angle will be the like parts of the greater: this will be evident by drawing lines from each point of concurrence to every division of each arc; then each angle will contain as many smaller angles, as the arc between the legs of that angle contains the smaller arcs between the points of division.

Thus, if the arc *df* be divided into five equal parts, and if the arc *ac* contain three of these parts, the angle E will be divided into five equal angles, and the angle B three equal angles: the angle B will, therefore, be three-fifths of the angle E.

(70) *Def. 70.* If, from a given point in any line, a given distance be set off upon that line, the other extremity thus found by the extent of the compass, is called the *point of distance*, or *point of extension*.

(71) *Def. 71.* To *rectify* or *develope* the arc of a circle is to find a straight line equal in length to that arc, supposing the arc to be extended or drawn between its extremities into a straight line.

(72) *Def. 72.* A straight line, equal to the arc of a circle when stretched out into a straight line, is called the *rectification* or *developement* or *length* of that arc.

(73) *Def. 73.* The distance between a point and a straight line, is the perpendicular drawn from that point to the straight line.

NOTATION.

A point is either given in position with respect to other points or lines, or found by the intersection or meeting of lines in an operation; and its situation is indicated by placing a letter as near to the point, or intersection, or meeting, as can be written.

When any number of letters indicating points are written in succession, each of the distinguishing letters is separated from the following by a comma.

A line which stands alone is sufficiently distinguished by placing a letter as near to one end of it as possible; but when a line is required to be distinguished from others crossing it in one or more points, it is necessary to place a letter at each of its extremities: in this

case, the letters which indicate the line are brought together in the text, without the intervention of any mark of division.

When any two letters are thus placed together in the text, without having the word line or straight line, or arc, immediately preceding these letters, a straight line is implied. Thus, if it is said, draw CD parallel to AB ; the meaning is, to draw the straight line denoted by CD parallel to that denoted by AB ; or to draw the straight line which has C at one of its ends, and D at the other, parallel to the straight line which has A at one of its ends and B at the other.

An angle which stands alone, is indicated by a single letter.

When two or more angles meet together at the same point, in order to distinguish any particular angle from the rest, it is necessary to place a letter somewhere in each line, and another letter at the point where the angles meet each other and in the text: the angle thus denoted will be expressed by three letters, without any mark of division between any two of them.

When three letters are placed together, without any word to denote the thing they mark out, they mean an angle. Thus, if it were said, make ABC equal to EFG , the meaning is, to make the angle denoted by ABC equal to the angle denoted by EFG ; or make the angle formed by the two straight lines AB , BC , equal to the angle formed by the two straight lines EF , FG .

A figure is denoted by placing a letter at the meeting of every two sides, and these letters follow each other in the text, without being separated by any other mark.

A circle, or an arc of a circle, is denoted by placing three or more letters either upon or as near to the circumference as can be written, and in the text by placing them in succession without any mark between them.

All given points and lines, whether straight or circular, are expressed by roman; and those points and lines which are either assumed, or found by the operation, are denoted by italics; given points, unconnected with lines, are denoted by capitals. The extremities of lines are also denoted by capitals, and the intermediate points of division in lines, by small letters.

The constructive lines of a diagram are either dotted or drawn very thin, in order to distinguish them from given and required lines, which are much more strongly marked.

When the word *join* immediately precedes two letters, the meaning is, to draw a straight line between the points where these two letters are placed; thus, if it be said join AB , we are to understand that a straight line is to be drawn between the point at A and the point at B ; and this amounts to the same thing as if it had been said draw AB .

When the word *bisect* is used with regard to a straight line, or the arc of a circle, or an angle, it means that the straight line is to be divided into two equal parts, or the arc is to be divided into two equal arcs, or that the angle is to be divided into two equal angles.

And when the word *trisect* is used, it means to divide the thing to which it is applied into three equal parts, whether it be a straight line, the arc of a circle, or an angle.

A *Proposition* is when something is either proposed to be done, or to be demonstrated, and is either a theorem or a problem.

A *Theorem* is a proposition in which something is proposed to be demonstrated.

A *Problem* is a proposition in which something is proposed to be done.

A *Corollary* is a consequent truth gained from some preceding truth or demonstration.

GEOMETRICAL PROBLEMS,

AS REGARDS PLANE FIGURES.

POINTS are merely assumed, that is, taken at pleasure, or are given in position previous to the commencement of any operation (9), and are either made with a pen or pencil; but, in nice operations, they are formed with the pointed extremity of an instrument.

The only instruments allowed in practical geometry, are a compass, and ruler, which is a bar of wood or metal, with one straight edge. The compass or dividers, as it is called, serves to measure the distance between any two points, or to transfer the distance between any two points to any other place, and also round a given point to describe a circle which may have any given radius. The ruler is employed in directing the motion of a point in a straight line.

(9) In practice, we must absolutely begin with a point: then, if we wish to represent a line on a given surface, we must assume or give two points, through which the line is to pass, and draw the line through them: and if we would represent a surface, we must draw lines round it; but though a drawing may be made on any surface, it is convenient to use a plane, or plane surface.

Since a point, a straight line, and a surface, only exist in the imagination, we must therefore use the surface of a solid for drawing upon; the straight edge of a ruler instead of a straight line; and the end of a sharp-pointed instrument, pencil, or pen, instead of a point.

Now, if a point is made with a sharp-pointed instrument through the surface of a solid, it will displace a small portion of the solid, and form a cavity of equal capacity; and if formed by a pen or pencil, it will leave a small solid upon the surface; so that in directing practical operations, as a point must be visible, it is either a small void, or a small solid. For the same reason as a line must be seen, it must have both breadth and thickness, whether considered as a small solid or void. From what has now been said, it is evident that, though geometrical operations are accurate in a theoretical point of view, they are only very near approximations in practice; therefore, the smaller we can assume points, and the finer we can draw lines, the more accuracy we shall obtain in our operations.

PROBLEM I.

Between two given points A and B, to draw a straight line.

Lay the straight edge of a ruler upon the point A ; then if the edge of the ruler thus applied to the point A coincide also with the point B, draw the point of a pencil or pen from A to B, keeping it always close to the straight edge ; then the trace which it leaves upon the surface is the straight line required.

But if, when the straight edge coincides with the point A, and not with the point B, move the ruler round upon A, until the straight edge arrive at the point B ; then draw the straight line from A to B as before.

Fig. 45.

**PROBLEM II.**

To produce or extend a straight line to any distance from one of its extremities.

Apply the straight edge of a ruler to the extremity of the straight line which is to be produced, and take any other point in the said straight line, so far from the end thus applied, that the distance may be less than the extent of the straight edge ; then slide the edge of the ruler upon the two fixed points, until one end of the straight edge arrive at the point taken in the given line ; then draw a line close to that part of the straight edge which is not upon the line from the end to be continued, and the given line will be produced as required. This operation may be repeated so as to lengthen a straight line to any extent.

PROBLEM III.

Round a given point A to describe the arc of a circle, or the whole circumference of that circle, with any given radius.

Press the legs of the compasses, so that the distance between their extremities may be equal to the radius given ; then place one of the extremities of the legs upon A, carry the other leg round upon the surface ; and the trace which it leaves will be the arc or circumference, according as the line traced out by the moving point has two extremities or none. In this last case it will enclose a space, and will therefore be a plane figure.

PROBLEM IV.

Through a given point C to draw a perpendicular to a given straight line AB.

Fig. 46, No 1.

From the given point C, with any radius, describe an arc, so as to cut the line AB in two points *d* and *e*. From the points *d* and *e* with the same radius, or any other equal radii, describe arcs cutting each other in *f*, and draw the straight line Cf; then Cf is perpendicular to AB.

This problem supposes the line AB to be sufficiently long, that one of the points *d* may be on the one side of the perpendicular, and the other point *e* on the other side of it; but the point C, through which the perpendicular passes, is not limited to any situation, and consequently the problem divides itself into two cases; one where the point C is out of the line AB, and the other where it is in the line AB; but the same description serves alike for each case. In No. 1, the point C is in the line AB; but in No. 2 it is out of it.

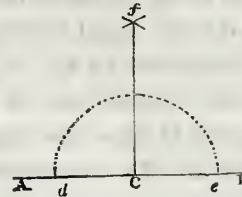
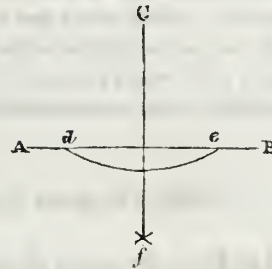


Fig. 46, No. 2.



PROBLEM V.

From a given point B, at or near the end of a straight line AB, to erect a perpendicular.

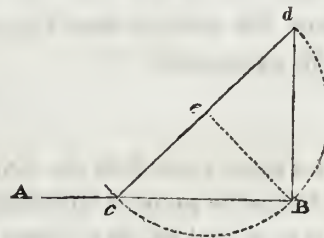
METHOD 1.

Take any point *e* above the line AB, and with a radius or distance *eB* describe an arc *cBd*, cutting the straight line AB in the point *c*: draw a straight line through the point *c* and the centre *e*, so as to meet the arc in *d*; draw the straight line *Bd*, and *Bd* will be perpendicular to AB, as required to be done.

This problem may be performed on the ground, in the following manner:—

Take a tape of any convenient length, and double it; fasten one end in the point B, and the other end in *c*; take hold of it by the middle where it is doubled, and stretch each half, and put a pin in the point *e*. Loosen the end B, and carry it round the arc to *d*, so that the points *c*, *e*, *d*, may be in a straight line, and the line *cd* stretched between *c* and *d*: this may be adjusted by the eye of the operator when it comes into the straight line with *c* and *e*; then draw the line *dB*, which will be perpendicular to AB, as required.

Fig. 47.

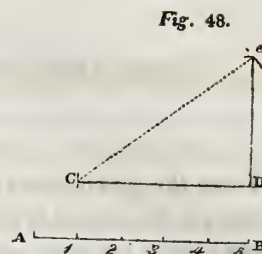


METHOD 2.

Let AB be a five foot rod, or a line consisting of five equal parts, and let it be required to draw a perpendicular from the end D of the straight line CD .

Make DC equal to four parts; upon D with a radius of three parts describe an arc at e ; from C , with a radius of five parts, describe another arc at e ; then through the intersection e and the point D draw the straight line eD , and eD will be perpendicular to CD , as required.

This method depends on the 47th proposition of the first Book of Euclid. It is commonly called the Pythagorean theorem, being discovered by Pythagoras, who, it is said, sacrificed a whole betacomb of sheep on the occasion. Euclid shews that the square of the hypotenuse, or longest side of a right angled triangle, is equal to the sum of the squares of the other two sides which contain the right angle. Now the side Ce being 5, its square is 25, which is equal to the sum of the squares of 4 and 3, which are 16 and 9, making together 25; agreeably to that celebrated theorem.



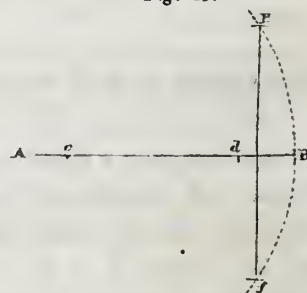
METHOD 3.

When the point through which the perpendicular is to pass is above the line.

Let E be the point through which the perpendicular is to pass, and AB the line to which it is to be drawn.

In AB take any two convenient points c and d ; from c , with the radius cE , describe the arc Ef ; and from d , with the radius dE , describe an arc cutting the former arc in f , and draw the straight line Ef , which will be perpendicular to AB , as required.

Fig. 49.



PROBLEM VI.

At a given point E in the straight line DE , to make an angle equal to a given angle ABC .

From the point B with any radius, describe an arc cutting AB at g , and BC in h ; from the point E with the same radius describe an arc ik , cutting DE in i ; make ik equal to gh , and through the points E , k , draw the straight line EF ; then the angle DEF will be equal to the angle ABC , as proposed to be done.

Fig. 50, No. 1.

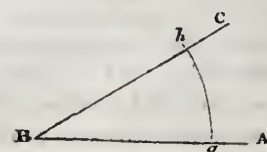
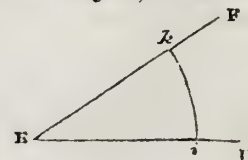


Fig. 50, No. 2.



PROBLEM VII.

Through any given point C, to draw a straight line parallel to a given straight line AB.

METHOD 1.

In AB take any point *f*, the more distant from C the better. Join Cf: from *f* with any radius describe an arc cutting AB at *g*, and fC at *h*; from C, with the same radius, describe an arc ik cutting Cf at *i*; make *ik* equal to *gh*; through C and *k* draw the straight line DE, and DE will be parallel to AB as required.

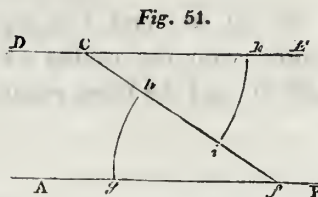


Fig. 51.

It may be here observed, that this problem depends upon the last.

METHOD 2.

Through C draw Cf perpendicular to AB, cutting AB in *f*; take any point *g*, the more remote from *f* the better, and draw *gh* parallel to fC; make *gh* equal to fC; and through the points C and *h*, draw the straight line DE, which will be parallel to AB as required.

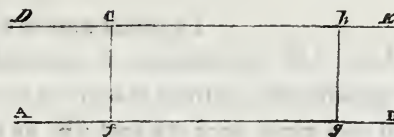


Fig. 52.

N.B. Instead of fC and *gh* being perpendicular, they may be drawn at any equal angles; but as the intention of this Treatise is to assist the workman or builder, who is most always provided with a square, parallel ruler, and a right angled triangle, it is better to draw them perpendicular.

PROBLEM VIII.

To draw a straight line at a given distance parallel to a given straight line AB.

In AB, take any two points *c* and *d*, the more remote the better: from each of these points, with a radius equal to the distance required, describe arcs *e* and *f*, and draw the straight line GH to touch the arcs *e* and *f*; then GH is parallel to AB.

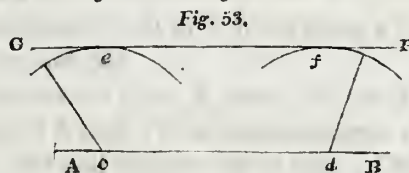


Fig. 53.

PROBLEM IX.

To divide a given line AB into two equal parts by a perpendicular.

From the point A, with any radius greater than the half of AB, describe an arc; from the point B with the same radius, describe another arc cutting the former in the points *e*, *f*, and draw the straight line ef, which will be perpendicular to AB as required.

N.B. This operation is called bisecting a given line AB by a perpendicular.

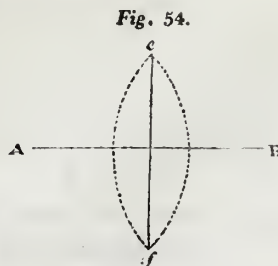


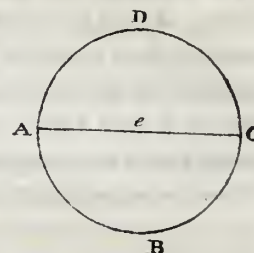
Fig. 54.

PROBLEM X.

Upon a given straight line AC, as the diameter of a circle, to describe the circumference of that circle.

Divide AC into two equal parts in the point e : from e as a centre, with the radius eA or eC , describe the circumference ABCD, and the thing required is done.

Fig. 55.

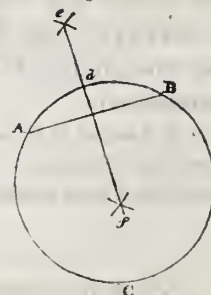


PROBLEM XI.

To bisect any arc AB of the circumference of a circle.

Join AB, or suppose it to be joined, and bisect AB by a perpendicular, cutting the given arc AB in d ; then AB is divided into two given arcs dA and dB , as required to be done.

Fig. 56.

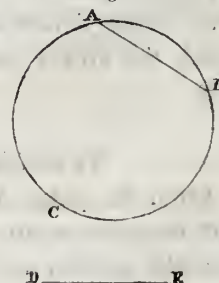


PROBLEM XII.

At a given point A in the circumference of a circle, to apply a straight line DE less than the diameter, as a chord to that circle.

From the point A, with the radius DE, describe an arc cutting the circumference in B; join AB, and AB is a chord to the circle ABC, equal to the straight line DE, as required to be done.

Fig. 57.

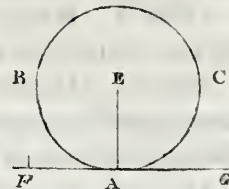


PROBLEM XIII.

Through a given point A in the circumference or arc ABC of a circle, to draw a tangent.

Method First. When the centre of the circle is given:— Fig. 58.

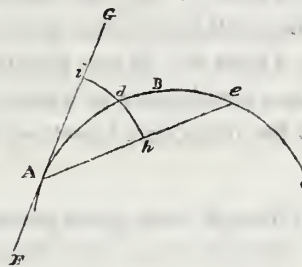
Join the given point A to the centre E, and through A draw FG perpendicular to EA, then FG is the tangent required.



Method Second. When the centre is inaccessible or not within reach:—

From the given point A take any two contiguous arcs Ad, de, and with the chord of the arc Ad, as radius, describe the arc hi; draw the chord Ae, cutting the arc hi at h: make di equal to dh, and through the points A and i draw the straight line FG, which is the tangent required.

Fig. 59.

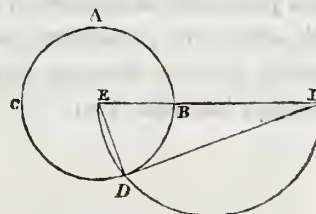


PROBLEM XIV.

Through a given point P, without the circumference of a circle ABC, to draw a tangent to that circle.

Through the centre E and the given point P, draw EP; and on EP as a diameter describe the semicircle EDP cutting the circumference of the given circle ABC in D: join DP, and DP is the tangent required.

Fig. 60.

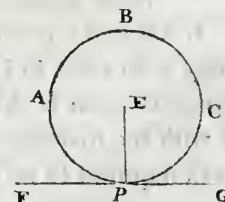


PROBLEM XV.

Given a straight line FG, touching the circumference of a given circle ABC, to find the point of contact.

Through the centre E draw EP, cutting FG in P; then P is the point of contact required.

Fig. 61.



AXIOMS.

Axiom 1. If a circle touch a given straight line at a given point, the centre is in another straight line, drawn from that point perpendicular to the given straight line.

Axiom 2. If one circle touch another, the centres of the two circles and the point of contact will be in one straight line.

Axiom 3. If a circle pass through two given points, the centre is in the perpendicular bisecting the straight line terminated by these points.

Axiom 4. If the centre of a circle is in two straight lines, it will be in the point where the two straight lines meet each other.

Axiom 5. If any two given points be joined, and the straight line which is extended to these points be bisected by a perpendicular, any point in the perpendicular will be equally distant from each of the two given points.

Axiom 6. If a straight line be drawn parallel to one of the sides of a triangle, the straight line will cut the other two sides of the triangle in the same proportion.

Axiom 7. If two straight lines cut any number of given parallels, the two straight lines will be divided in the same proportion.

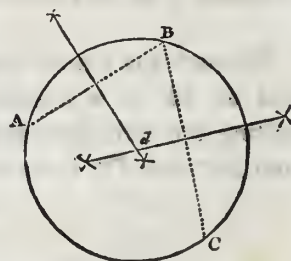
Axiom 8. If two parallel straight lines be cut by any number of converging lines, or lines terminating in a point, the intercepted parts of the one parallel will be proportionals to the intercepted parts of the other.

PROBLEM XVI.

Through three given points A, B, C, not in a straight line, to describe the circumference of a circle.

Join AB and BC, or suppose them to be joined, and bisect each of the lines AB and BC by a perpendicular, and let the two perpendiculars meet in d ; from the point d with the radius dA , dB , or dC , describe the circumference ABC of a circle, which, passing through any one point, will necessarily pass through the other two.

Fig. 62.

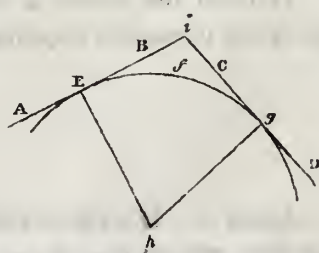


PROBLEM XVII.

Two straight lines AB and CD being given in position, but not in a straight line, to describe the arc of a circle that shall touch them both, and one of them AB in a given point E.

If AB and CD do not meet, produce each, so that they may both meet in i ; make ig equal to iE ; and draw Eh perpendicular to AB, and gh perpendicular to CD: from h with the distance hE or hg , describe the arc Efg , which was required to be done.

Fig. 63.

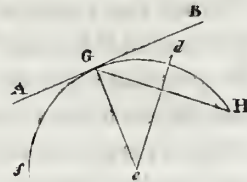


PROBLEM XVIII.

To describe the arc or circumference of a circle that shall touch a straight line AB in a given point G , and that shall pass through a given point H , not in the same straight line with AB .

Draw Gc perpendicular to AB , and join GH , or suppose it to be joined: bisect GH by a perpendicular cd : from the point c , where the two perpendiculars Gc and dc meet, and with the radius cG , describe the arc fGH , and it will pass through the point H as desired.

Fig. 64.



DEMONSTRATION.

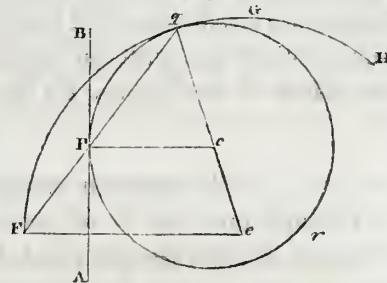
Because Gc is perpendicular to AB , the centre of the circle that will touch the straight line AB is in the perpendicular Gc , by Axiom 1; and because the perpendicular cd bisects the straight line GH , the centre of a circle passing through the points G and H is in the line cd , by Axiom 2; therefore, since the centre of the circle is in the two straight lines Gc and dc , it is in the point c of their intersection.

PROBLEM XIX.

To describe any portion of the circumference of a circle that shall touch a straight line AB in a given point P , and that shall also touch a given circumference or arc FGH .

Draw Pc perpendicular to AB , and through e the centre of the arc FGH draw Fe parallel to Pc , cutting the circle FGH in F : join FP , and produce FP to meet the arc FGH in q . Join qe , cutting Pc in c , and from c as a centre, with the radius cP , describe an arc which will touch the given circumference or arc FGH at q ; then Pqr is the arc or Pqr the circumference required.

Fig. 65.



DEMONSTRATION.

Because Pc is drawn from the point P perpendicular to AB , the centre of the circle which will touch the line AB in P is in the straight line Pc ; and because Fe is parallel to Pc , the triangles qeF and qcP are similar; but because F and q are in the circumference of the circle FGH , Fe and qe are equal, being radii of the circle; therefore, since qe is equal to eF , the portion qc of eq must be equal to cP : whence a circle described from c with the radius cP will pass through the point q , and touch the straight line AB at P . Now, since the centres e and c of the two circles, and the point q in which they meet, are in one straight line, the two circles will also touch each other at q .

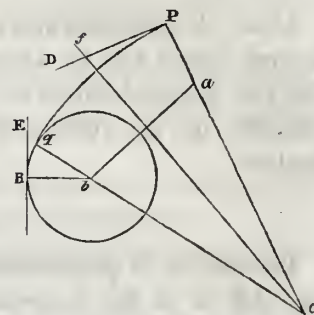
PROBLEM XX.

To describe two arcs that shall meet each other in the line of their centres, and that shall touch two straight lines DP and ER, at a given point in each line; and that the arc belonging to the lesser circle shall have a given radius.

Let R be one of the points of contact, and P the other.

From the point of contact P, draw Pa perpendicular to the tangent PD; and from the other point of contact R, draw Rb perpendicular to the other tangent line ER. Make Pa and Rb each equal to the given radius, and join ab: bisect ab by a perpendicular fc, cutting Pa produced at c: join cb, and produce cb to q. From c, with the radius cP, describe the arc Pq, and from b, with the radius bq, describe the arc qR: then will the arcs Pq and qR meet each other at q in the same straight line with their centres c and b, and touch the line ER at R, and PD at P.

Fig. 66.



DEMONSTRATION.

Because Rb is perpendicular to ER, the centre of the circle which touches the line ER at R is in the line Rb; for the same reason, because Pa is perpendicular to PD, the centre of the circle which touches the straight line PD at P is in the line Pa, or Pa produced.

Again, because ab is bisected by the perpendicular fc, the points a and b will be equally distant from any point in fc; therefore ca and cb are equal to each other: and because bq is equal to bR, and bR is equal to aP, bq is equal to aP; whence cq is equal to cP: therefore a circle described from c with the radius cP will touch the circle qR at q in the same straight line with the centres b and c; likewise the arc qR will touch the straight line ER at R, and the arc Pq the straight line PD at P.

PROBLEM XXI.

To divide a straight line AB into any number of equal parts.

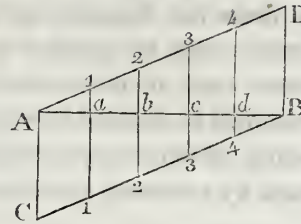
Through one end B of the straight line AB, draw BC, making any given angle with AB, and through the other end A draw AD parallel to BC; set as many equal parts upon each of the parallels, beginning at the point in the given line, as the given line AB is to contain equal parts, and join the one extremity of the given line to the remote extremity of the line which forms an angle with it, and join every two corresponding points in succession of the parallel lines; and the lines thus joining will cut the given line AB into the number of parts required.

EXAMPLE.

Divide the line AB into five equal parts.

Through B draw BC, making any angle with AB; and through A draw AD parallel to BC; from either end A of the line AB set five equal parts upon the parallel AD that joins that end, and from the other end B set five equal parts upon the parallel BC connected with the given line AB at B, and let C be the point where the last part reaches to: join AC; 1, 1; 2, 2; 3, 3, &c., and AB will be divided into equal parts at the points *a*, *b*, *c*, &c.

Fig. 67.



PROBLEM XXII.

To divide the circumference of a circle into any number of equal parts.

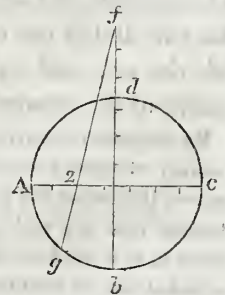
It is rather unfortunate for the practice of geometry, that this division can only be effected in a few cases (10), and that each of these cases cannot be resolved by the same method. The number of primary cases is five, viz.:—a circle may be divided into two, three, four, five, and six equal parts, but we cannot continue the progression any further; it is however evident, that when any of these divisions is once found, any multiple of that division, by any term of the geometrical series of numbers 2, 4, 8, 16, &c. may be found by continual bisection; so that though we may find certain divisions without limitation, the intervals to be filled will be much more numerous than those which can be ascertained by rule: but as these rules, though thus limited, are of the greatest use to the workman, we shall proceed to show the particular method for effecting each case.

(10) The following method may be seen in many of our modern publications of practical geometry; it is general, and in all cases very near the truth, but not exact in any case:—

To divide the circumference of a circle into parts which shall be nearly equal to each other from a given point A.

Draw the diameter *Ac* and the diameter *bd* at right angles to *Ac*. Produce *bd* to *f*, and make *df* equal to three quarters of the radius: divide the diameter *Ac* into as many equal parts (by Prob. XXI), as the number of equal parts required in the circumference; and from the point *f*, and through the second point 2 of division, draw the straight line *fg*; then the part *Ag* of the circumference will be nearly equal to the part required.

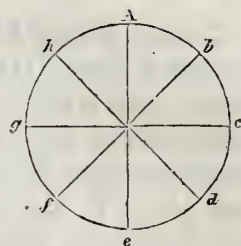
Fig. 68.



Rule 1. *For dividing the circumference of a circle into 2, 4, 8, 16, 32, &c. equal parts, from a given point A in the circumference,*

Draw the diameter Ae , and the circumference will be divided into two equal parts, Ae and eA : bisect each of the arcs Ae , eA , or draw the diameter gc at right angles to Ae , and the circumference will be divided into four equal parts Ac , ce , eg , gA : bisect each of these arcs of the fourth part, and the circumference will be divided into eight equal parts Ab , bc , cd , de , &c., and so on, as long as we please to double the last number of equal parts by continual bisection.

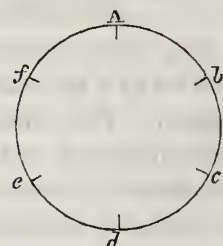
Fig. 69.



Rule 2. *For dividing the circumference of a circle into 3, 6, 12, 24, &c. equal parts, from a given point A in the circumference.*

From the point A, with the radius of the circle, describe an arc cutting the circumference in b ; from b , with the same radius, describe an arc cutting the circumference in c , and so on continually one after another is found, and the circumference will be divided into six equal parts, at the points A, b , c , d , &c.

Fig. 70.



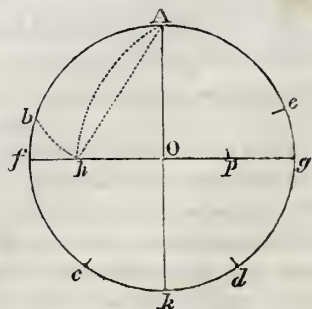
The circle may, by this means, also be divided into three equal parts, by passing over every other division.

By bisecting each arc of the sixth part, the whole circumference will be divided into twelve equal parts; and if each arc of the twelfth part be bisected again, the whole circumference will be divided into twenty-four equal parts; and so on, as often as we please to double the last number of equal parts.

Rule 3. *To divide the circumference of a circle into 5, 10, 20, 40, &c. equal parts, from a given point A in the circumference.*

Fig. 71.

Draw the diameter Ak and the diameter fg at right angles to Ak ; bisect the radius Og in p , and from p , with the radius pA , describe the arc Ah cutting fg in h : then the straight line Ah will divide the circumference into five equal parts Ab , bc , cd , de , ea ; and by continually bisecting the arcs, we shall arrive at the number of equal parts as proposed by the rule.



By the three rules given, we can divide the circumference of a circle into any number of equal parts between one and seven: and it may be generally observed, that, when the primary number is effected, the circumference may be divided into any multiple of the parts which it obtains by any term of the series 2, 4, 8, 16, &c.: thus, when the primary number is found, that number may be doubled, quadrupled, octupled, &c. by continual bisection of the arcs.

Rule 4. *To divide the circumference of a circle into any number of equal parts, by trials.*

Set the points of the compasses or dividers as near to the chord as possibly can be conjectured or guessed at; then apply this distance as successive chords round the circumference of the circle, and, if the circumference is divided into more parts than is required, the points of the dividers must be made to subtend a greater distance; but if the circumference contain fewer parts, the points of the compass must be made to subtend a less distance. Proceed in this way by continually correcting the distance last found of the points of the dividers, until the number of equal parts are found.

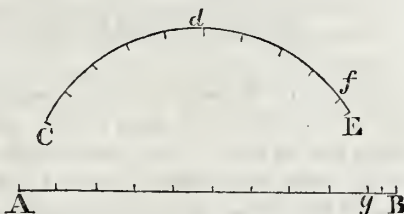
We have no occasion to apply this rule for any number of equal parts less than seven. The greater number that is required, the more difficult it will be to approximate to the true distance.

PROBLEM XXIII.

From a given point A, in a given straight line AB unlimited towards B, to find the length of any arc CdE of a circle.

Fig. 72.

Take any small extent in the compass, and repeat it upon the arc CdE as often as it can be contained, and let *f* be the remote extremity of the last of these parts; which being done, repeat the same extent from A towards B upon the straight line AB, as often as it has been contained upon the arc Cf, and let the extremity of the last be at *g*. In the straight line AB make *gB* equal to the remainder *fE* of the arc; then AB is very nearly equal in length to the arc, being somewhat less.



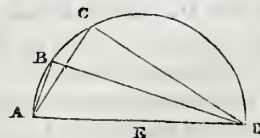
It is obvious, that the greater number of chords or parts is contained in the arc, the more exact will its length be obtained: but though the result of this operation is evidently defective, it is easier, and more to be depended upon, than any other method. It will be satisfactory to show the degree of accuracy obtained by this process.

Since 3.1393 is the sum of the sides of a 48 sided polygon inscribed in a circle, of which the diameter is unity (11); therefore this number is less than the true circumference. Now, Archimedes of old has shown,

(11) It is proved in a Lemma, page 172, Simpson's Geometry, Coroll. the new Edition, 1821, that, *if the diameter of a circle is 2, then if the supplemental chord of any arc be added to the number 2, the square root of the sum will be the supplemental chord of half that arc.*

Fig. 73.

Now, if ACD be a semicircle, and AC the chord, then CD is the supplemental chord; and if AC be equal to the radius, it will be equal to 1, because AD is equal to 2; therefore in the right angled triangle ACD we have the hypotenuse $AD = 2$, and one of the sides $AC = 1$; but it is proved in Euclid 47, book 1, that $AD^2 = AC^2 + CD^2$; therefore $CD^2 = AD^2 - AC^2$, that is, $CD^2 = 2^2 - 1^2 = 4 - 1 = 3$; whence $CD = \sqrt{3} = 1.7320508075$, which



that the diameter and circumference of a circle are to each other in the ratio of 7 to 22; whence, as $7 : 22 :: 1$, the fourth number, which is 3.142; therefore, when the diameter is 1, the circumference will be 3.142, which is too much for the diameter; consequently, when the diameter is unity, the circumference of the circle will

is the supplemental chord of $\frac{1}{3}$ of the semi-circumference; then, by the preceding corollary,

$$\left. \begin{aligned} \sqrt{2 + 1.7320508075} &= 1.9318516525 \text{ for supplemental chord of } \frac{1}{6} \\ \sqrt{2 + 1.9318516525} &= 1.9828897227 \text{ for supplemental chord of } \frac{1}{12} \\ \sqrt{2 + 1.9828897227} &= 1.9957178465 \text{ for supplemental chord of } \frac{1}{24} \\ \sqrt{2 + 1.9957178465} &= 1.9997322757 \text{ for supplemental chord of } \frac{1}{48} \end{aligned} \right\} \text{ of the semi-circumference.}$$

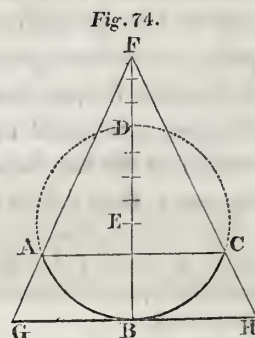
&c. &c.

Now, subtract 3.9828897227, which is the square of the supplemental chord of 24th part of the semi-circumference, from 4, gives 0.0171102773, the square of the chord itself. The square root of this last number is 0.1308062, which is therefore the chord of the 48th part of the whole circle: whence $0.1308062 \times 48 = 6.27869$, which is the sum of the sides of a polygon inscribed in a circle consisting of 48 sides, the diameter being 2; therefore, when the diameter is 1, the sum of all the chords will be 3.1393 nearly.

To find very nearly the length of the arc of a circle geometrically, ABC being the given arc.

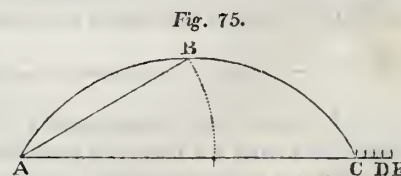
METHOD 1.

Join AC, and bisect AC by a perpendicular BF; find the centre E if not given. Make ED equal to EB; divide the radius DE into four equal parts, and set three of these parts from D to F; through B draw GH parallel to AC; from F, through the points A and C, draw FG and FH: then the straight line GH will be the development or length of the arc very nearly.



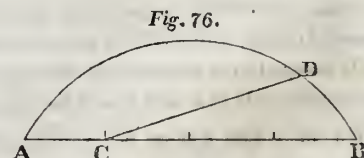
METHOD 2.

Produce the chord AC to E, and bisect the arc ABC in B; set twice the chord AB of the half arc from A to D; divide CD into three equal parts, and make DE equal to one of the parts; then AE will be nearly equal to the length of the arc ABC nearly.



METHOD 3.

Divide the chord AB into four equal parts; set one part on the arc from B to D; join the remote extremity C of the third part from B to the extremity D of the arc BD; then CD will be nearly equal to half the length of the arc.



The first and second of these geometrical methods are much nearer to the truth than the third and last method; but neither of them is so correct, and so convenient for practice, as that recommended in the text.

be between 3.1393 and 3.142. Now let us suppose the diameter of a circle to be one foot, and, as workmen's rules are generally divided into inches and eighth parts, multiply the decimal part of each of these numbers by 12, and point off the decimals, and the remainder will be inches: the decimals of the inches of each number being multiplied by 8, and the decimals of each new product pointed off, we shall have the number of eighth parts; thus,

3.1393	3.142
12	12
1.6716	1.704
8	8
5.3728	5.632

Therefore the sum of the sides of a polygon of 48 sides, inscribed in a circle whose diameter is 1, is three feet, one inch, and five eighth parts of an inch; and the circumference of the same circle, expressed in the like denominations of feet, inches, and eighths, contains equal parts of each to those of the polygon; therefore, by only dividing the circumference of a circle into 48 equal parts, or each quadrant into 12, we come sufficiently near the truth for the purpose of the workman, and the loss of length does not amount to the 16th part of an inch; and if we divide an arc of a circle not exceeding a quadrant of the same diameter into twenty-four parts, and extend them upon a straight line, we should not lose one twentieth part of an eighth part of an inch. And thus we see how far the rule may be depended upon. The methods generally given for this purpose, in books of Practical Geometry, are not only erroneous, but the same rule gives the length of the arc both in excess and defect, according to the proportion that obtains between the chord and the versed sine of that arc.

APPLICATIONS.

Description of the Plate of Exercises to Problems XIX and XX.

The first figure on the right is termed a *lancet arch*. It is composed of two curves, meeting in a point at one of each of their extremities, and joining a straight line at each of the other extremities in such a manner, that the two straight lines are parallel to each other, and tangents to each curve; and that every portion of the two curves, terminated by equal chords from their point of meeting, are equal and similar arcs.

Suppose now that it were required to draw each curve by means of circular arcs; and suppose the two arcs next to the point of meeting to be formed to the fancy of the draughtsman, how must the other part be described?

The problem to be performed is this:—

To describe the circumference of a circle that shall touch a straight line AB in a given point P, and that shall also touch a given arc FGH.

To execute this problem, the reader is required to read Problem XIX, page 23, with reference to the figure now explained.

The second figure on the left is what is commonly denominated an *oval*. An oval, generally speaking, is a curve line which returns to itself: from this definition of an oval it is evident that there may be an infinite variety of curves so called. The oval which we

are about to describe, is a figure resembling an ellipse compounded of four circular segments, in such a manner, that the opposite segments may be equal portions of equal circles, and that the radii of the two arcs, at the point where they meet, may be in the same straight line.

In order to describe this kind of oval,

Draw the line PK equal in length to the longest dimension of the figure, and bisect PK by a perpendicular HL, cutting each other in p : make pH and pL each equal to half the breadth of the figure; then describe a circle or the arc of a circle FGH, in such a manner that the centre may be in the line HL, or in HL produced, and that the circumference may not cut the line PK, but may fall as much without as the draughtsman may think proper; and draw AB perpendicular to PK, by Problem V. Then, by applying Problem XIX successively to each end, we shall be enabled to complete the figure, for we have only *To describe such a portion of the circumference of a circle that shall touch a straight line AB in a given point P, and that shall also touch a given circumference or arc FGH.* One end being described, the other will be found in the same manner, and thus we shall have three quarters of the figure.

Let these three arcs thus found be qHo , the part first described, and qPm , oKn the two parts now drawn; these three arcs are terminated by drawing lines through every two of their centres so as to meet the arcs: the remaining segment being similar to the opposite one, qHo , must be described with the same radius, so as to have its centre i in the straight line LH, and that its curve may pass through the point L.

Having now shown some applications of Problem XIX, we shall point out some uses of Problem XX, which applies to all the remaining figures in the plate.

The second figure downwards on the right hand is another *lancet arch*, supposed to have the same properties as that defined; but here the angle GPI at the vertex is given; it must, however, always be greater than the angle formed by the lines extending from each extremity of the chord RH to the summit P.

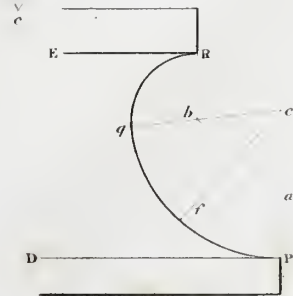
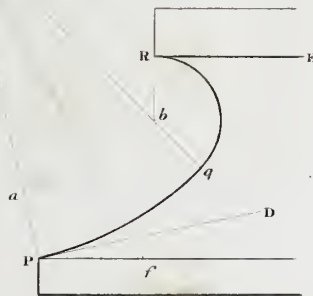
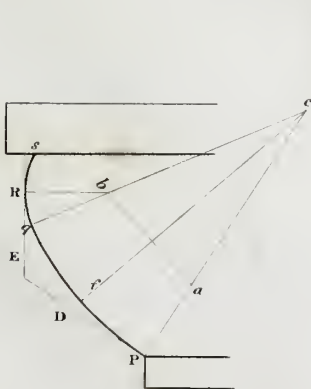
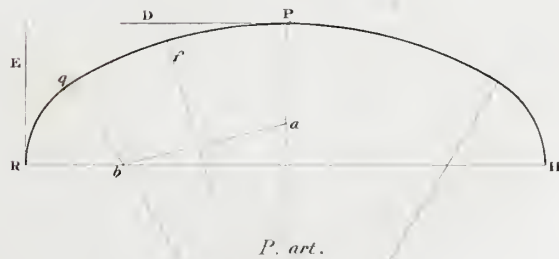
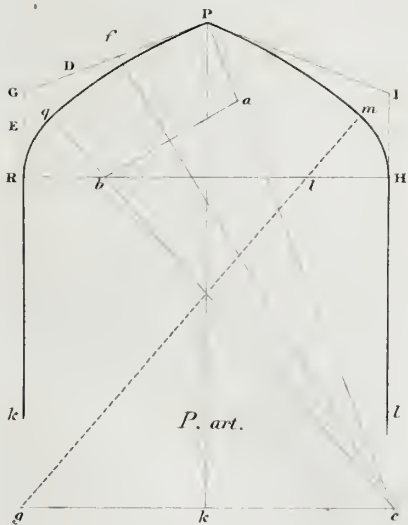
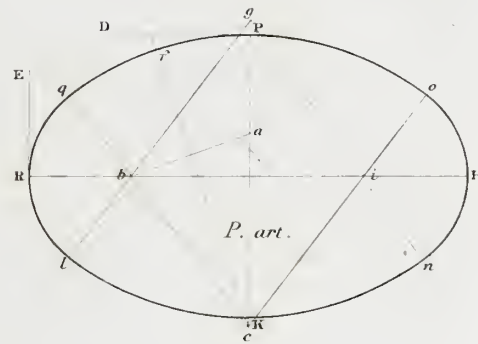
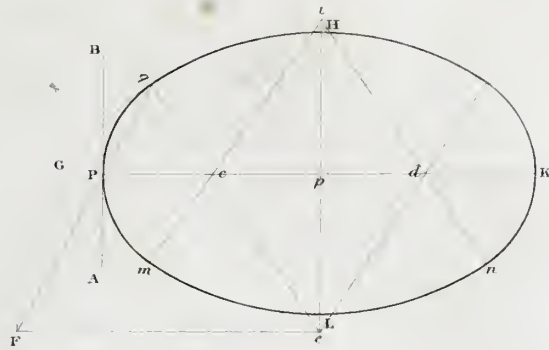
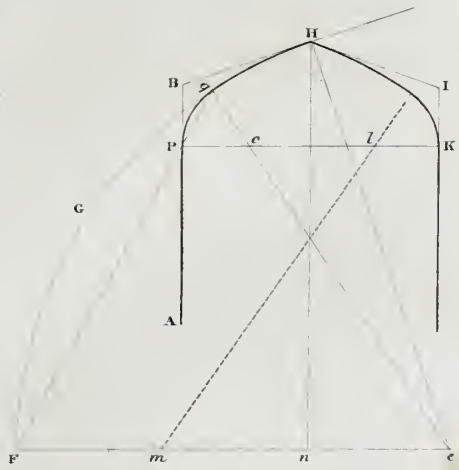
To prepare the figure for the application of the above Problems,

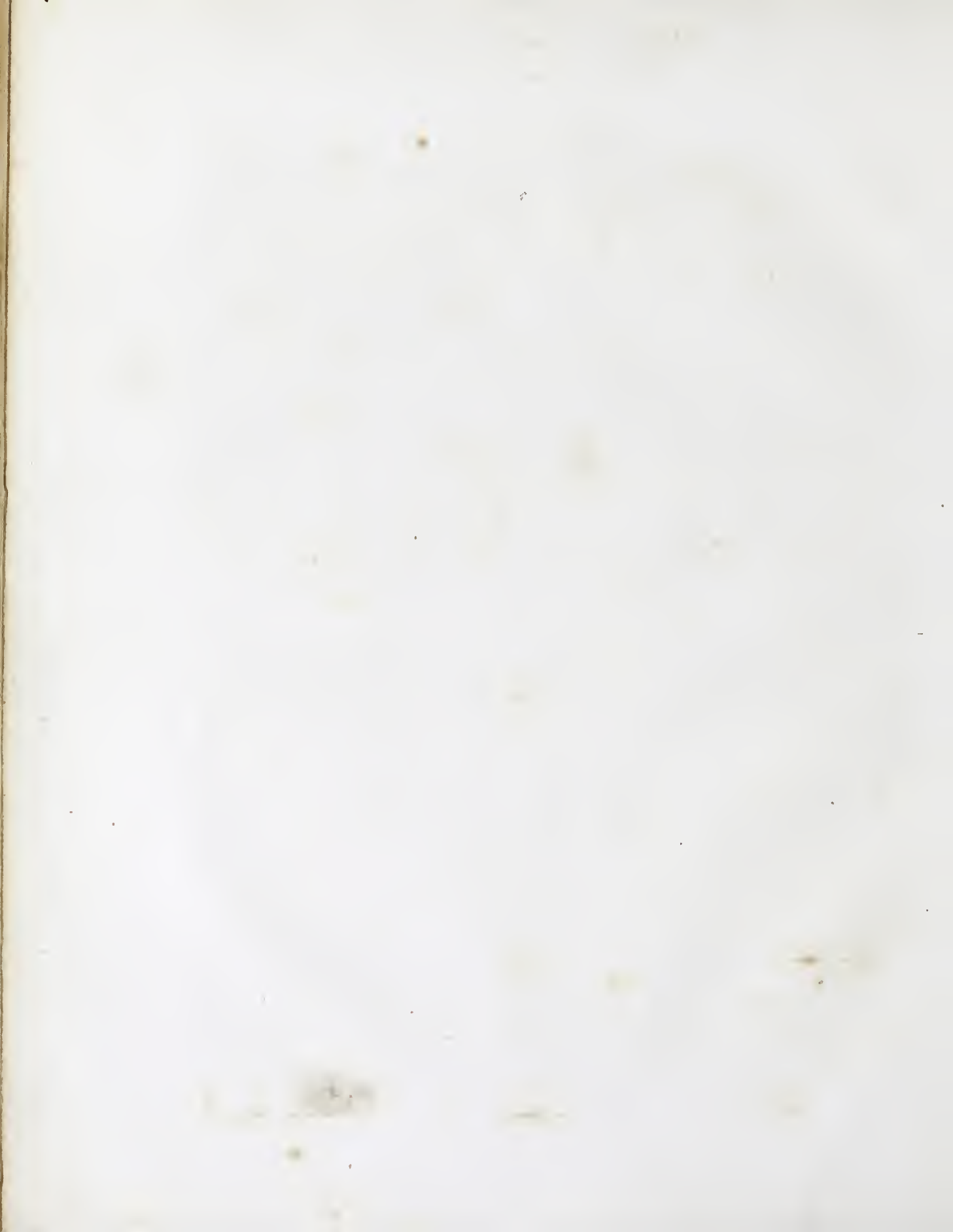
Draw Pc perpendicular to PG, by Problem V, and let R be the point which divides the arch from the straight line on one side, or what is denominated one of the springing points of the arch, and let RH meet the tangent RE in R: then, to describe the half of the arch PqR, we have only, by Problem XX, *To describe two arcs which shall meet each other in the line of their centres, and that shall touch two straight lines DP, ER, at a given point in each line, and that the arc belonging to the lesser circle shall have a given radius.*

The other half is described in the same manner; or through c , the centre of the arc Pq, draw eg parallel to RH the base of the arch. From the summit P, draw Pk parallel to Rk or Hl, cutting gc in k . Make kg equal to kc , and make Hl equal to Rb, and draw glm : then g is the centre for the arc Pm, and l the centre for the arc mH.

PLANE GEOMETRY.

EXERCISE TO PROBLEMS, 19, & 20.





The second figure downwards on the right hand side of the plate is another oval, having the same properties as that which stands above it.

To prepare this figure: Draw the straight line RH , equal to the length of the oval. By Problem IX, bisect RH by the perpendicular PK : then set off half the breadth on each side of the centre, and the remote extremities P and K are the extremities of the breadth. In order that our oval may be a good representation of the mathematical curve called an ellipse, divide the difference between half the length and half the breadth into two equal parts, and set three from the centre upon the line RH on each side of it to b and i ; then, by Problem XX, *Describe two arcs that shall meet each other in the line of their centres, and that shall touch a straight line DP in P , and ER in R , and that the arc belonging to the lesser circle shall have a given radius.*

Having finished one end, the other will be described in the same manner, and the remaining side by the same radius as the first side opposite.

The third figure from the top on the right hand side is a *semi-oval*, drawn in the same manner as the figure above it.

The curves of the three figures at the bottom of the plate are each composed of two circular arcs, and, supposing the number of the figures to proceed from left to right in the usual manner of reading, the first is an imitation of the section of a Grecian ovolo; the second and third figures are in imitation of the section of a scotia. In both these figures the lower edge of the fillet is a tangent to the upper part of the curve; and in the third figure, the upper edge of the fillet is a tangent to the curve at its lower extremity.

Suppose now that the arc Rq is described to fancy; then, if it touch the lower edge RE of the fillet at R , and if Rb be perpendicular to RE , the centre of the circle which will touch the line RE at R will be in the line Rb ; therefore, from a centre in the line Rb , with any radius which the operator may think proper, describe the arc. Draw Pc perpendicular to DP . In Pc cut off Pa equal to the radius of the lesser circle, and join ab . Bisect ab by a perpendicular fc . Draw cq through the point of intersection c and the centre b . From c with the radius cq describe the arc qP , and this will complete the scotias: but without describing any figure particularly, as the lines DP and ER are supposed to be given, and the points of contact R and P , therefore in all the figures of this plate, except the two uppermost, we have only to execute Problem XX, which is,

To describe two arcs that shall meet each other in the line of their centres, and that shall touch two straight lines DP and ER at a given point in each line, and that the arc belonging to the lesser circle shall have a given radius.

PROBLEM XXIV.

To divide an angle into any number of equal angles.

This problem, like that of the division of the circle, cannot be generally effected except by continual bisections; in this case we may divide an angle geometrically into two,

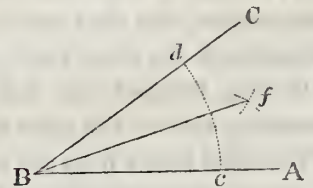
four, eight, sixteen, &c. equal parts, by first describing an arc, and, if we wish to divide the angle into two equal parts, we must bisect that arc; and if it were required to divide the angle into four equal parts, we must bisect each half arc; and if it were required to divide the angle into eight equal parts, we must bisect each quarter of the arc, and so on to the proposed number of parts required.

EXAMPLES.

Ex. 1. *Bisect the given angle ABC.*

From the angular point B describe an arc cd cutting AB at d , and BC at e , and bisect the arc de by the straight line Bf: then will the angle ABC be divided into the two equal angles ABf and fBC.

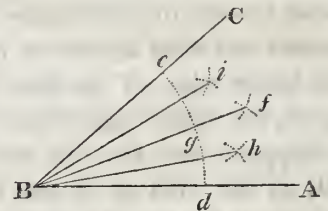
Fig. 77.



Ex. 2. *Quadrisect or divide the angle ABC into four equal angles.*

From B, the point of the angle as a centre, describe the arc de , cutting AB at d , and CB at e ; and bisect the arc de by the line Bf, cutting the arc de in g : bisect the arcs dg and ge by the lines Bh and Bi, then will the angle ABC be divided into the four equal angles ABh, hBf, fBi, and iBC.

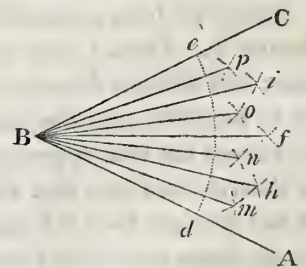
Fig. 78.



Ex. 3. *Divide the angle ABC into eight equal parts.*

From the angular point B describe the arc de , cutting BA at d , and BC at e , and bisect the angle ABC by the straight line Bf, cutting the arc de in g : bisect the arcs dg and ge by the straight lines Bh and Bi, cutting the arc de in the points k and l : bisect the arcs dk , kg , gl , and le , by the straight lines Bm, Bn, Bo, Bp, and the angle ABC will be divided into the eight equal angles ABm, mBh, hBn, nBf, fBo, oBi, iBp, and pBC, as required to be done.

Fig. 79.



SCHOLIUM.

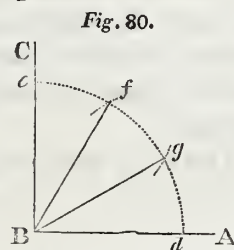
And thus we may proceed to double the last number of equal angles to any given number of angles in the geometrical progression 2, 4, 8, 16, 32, 64, &c.

Though an angle cannot be geometrically divided into any number of equal parts, the problem is not impossible, as it may be done by the resolution of algebraic equations of the third, fourth, fifth, &c. degree. Analytical principles are, however, too difficult to be acquired by the generality of readers; we must content ourselves by the method of approximation, as in the case of dividing a circle into equal parts. The practice of the operator will enable him to ascertain, by a few trials, the portion of the arc that will divide the whole into the required number of parts; and, consequently, to divide the angle into the same number of equal parts, by drawing straight lines from the angular point through every point of division. The trisection of a right angle is of considerable use, and, as it can be geometrically constructed, we shall show how it is to be done in the following

PROBLEM XXV.

To trisect or divide a right angle ABC into three equal parts.

From the angular point B describe the arc *de*, cutting BA at *d*, and BC at *e*. From *d*, with the radius of the arc, describe another arc, cutting the arc *de* at *f*, and from *e*, with the same radius, describe another arc cutting the arc *de* in *g*: join B*f* and B*g*, and the angle ABC is divided into the three equal angles AB*g*, *g*B*f*, *f*BC.



PROBLEM XXVI.

To inscribe a polygon of any given number of sides in a given circle, and from any point in that circle.

Divide the circumference of the circle into as many equal parts as the polygon is to contain sides: join any point to the next point of division, and proceed progressively, always joining the next point to the end of the chord last drawn, until one chord only remains to be drawn; then draw that chord from the extremity of the chord last drawn to that of the chord first drawn, and the polygon will be formed.

Examples in a hexagon, octagon, and dodecagon.

Fig. 81, No. 1.

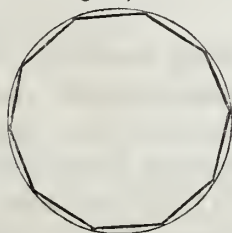
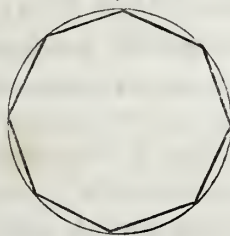
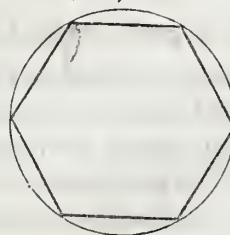


Fig. 81, No. 2.



G

Fig. 81, No. 3.



Examples in a pentagon, heptagon, and enneagon.

Fig. 82, No. 1.

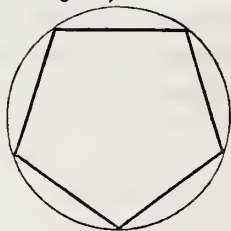


Fig. 82, No. 2.

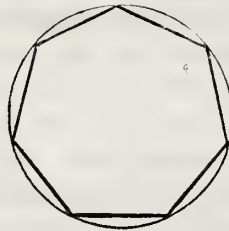


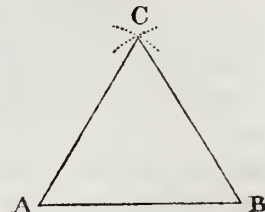
Fig. 82, No. 3.

**PROBLEM XXVII.***Upon a given straight line AB to describe an equilateral triangle.*

From the centre A, with the radius AB, describe an arc; from the centre B, with the radius BA, describe another arc cutting the former; from the point C, where the two arcs meet, draw the straight lines CA and CB, and ABC is the equilateral triangle required.

This is the first proposition in Euclid's Elements.

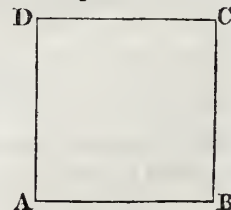
Fig. 83.

**PROBLEM XXVIII.***Upon a given straight line AB to describe a square.*

Draw the straight line BC perpendicular to AB; make BC equal to AB: from A, with the radius AB or BC, describe an arc; and from C, with the same radius, describe another arc, cutting the former in the point D: join AD and DC; then ABCD is the square required.

This is the forty-sixth proposition of the first book of Euclid's Elements. After having drawn the right angle ABC, Euclid directs that the straight line CD be drawn parallel to AB, and AD parallel to BC.

Fig. 84.

**PROBLEM XXIX.***Upon a given straight line AB to describe a polygon of any given number of sides.*

Produce the straight line AB to K, and on AK describe the semicircle ACK. Divide the arc ACK into as many equal parts as the number of sides proposed in the polygon: join the second point of division C from K to the centre B; bisect each of the sides AB and BC by perpendiculars meeting each other in I; from the centre I, with the radius IA, IB, or IC, describe a circle, which being made to pass through any one of the three points

A, B, C, will necessarily pass through the other two; therefore we may begin at C or A, and apply the chords each equal to AB or BC to the remaining part of the circumference (12).

The following are examples in a *pentagon*, *hexagon*, and *heptagon*.

Fig. 85, No. 1.

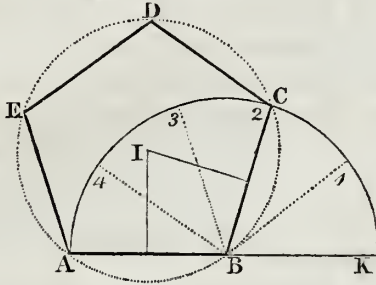


Fig. 85, No. 2.

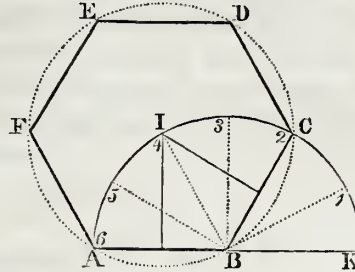
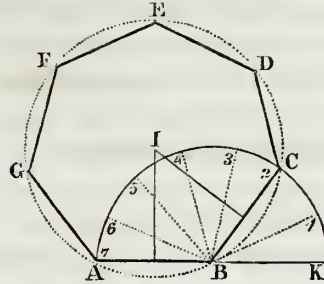


Fig. 85, No. 3.



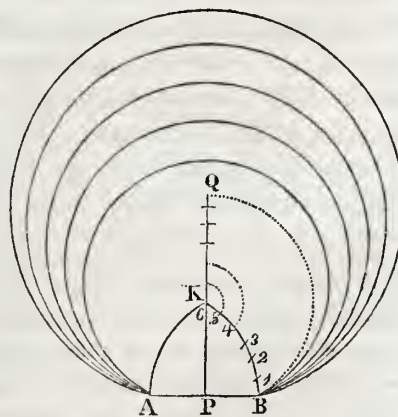
(12) There is another method of executing this problem; and though the principle is not correct, it may not be amiss to show the process and construction, as the beauty of the scheme has so frequently attracted the notice of the student.

Let AB be the given side.

Upon the centre A, with the radius AB, describe an arc BK, and from B, with the same radius, describe an arc AK: through K draw PQ perpendicular to AB: divide either of the arcs, as BK, into six equal parts. Then if it were required to describe a circle that should contain AB six times, there is nothing more required but to describe a circle from K with the radius KA or KB; in this case, the rule is exact.

But, if a greater or less number of parts were required, we must describe an arc from K with a radius equal to the chord of as many parts of the arc BK as the number required exceeds six, or equal to the chord of as many parts of the arc BK, as six exceeds the number of sides required, and cross the line PQ above or below K accordingly: then from the point thus found in PQ as a centre, with a radius extending to either of the points A or B, describe a circle, and it will contain the side AB as required.

Fig. 86.

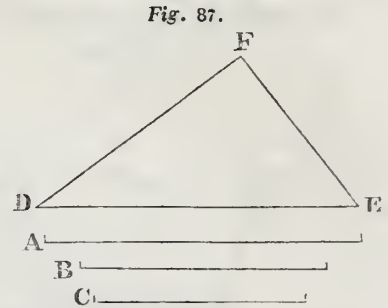


PROBLEM XXX.

To describe a triangle of which the three sides shall be each respectively equal to each of the three given lines, so that any two of them may be greater than the third.

Let A, B, C , be the three given straight lines.

Draw the straight line DE , and make it equal in length to A : with the radius B , from the centre D , describe an arc, and with the radius C from the centre E describe another arc, cutting the former arc at F : join DF and FE , and DEF is the triangle required.



PROBLEM XXXI.

To describe a polygon equal and similar to a given polygon upon a given straight line corresponding to one of the sides of the given polygon.

Divide the given polygon into as many triangles, wanting two, as the given figure has sides, so that no space may remain but what is entirely resolved into triangles. Upon the given side describe a triangle equal and similar to the triangle upon the corresponding side of the given polygon; then describe the remaining triangles to succeed each other in the same manner as in the given figure, and the figure thus drawn will be similar to the one proposed.

In the example here given, $ABCDE$ is the given polygon, and FG the side of the required polygon, corresponding to AB of the given polygon. Here the given polygon is resolved into the triangles ABC, ACD , and ADE ; then, in the polygon to be constructed, the triangle FGH is described by Problem XXX, equal and similar to the triangle ABC ; the triangle FHI , equal and similar to the triangle ACD ; the triangle FIK , equal and similar to the triangle ADE ; and thus the whole figure $FGHIK$ is described equal and similar to the given figure $ABCDE$ (13).

Fig. 88, No. 1.

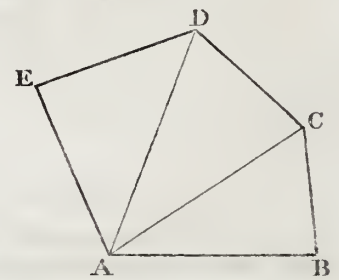
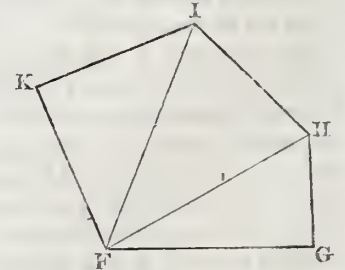


Fig. 88, No. 2.



(13) It is by the application of this problem that we are enabled to make a plan of any proposed place, or to take the dimensions for executing any piece of work. Suppose, for instance, that it were required to make the plan of a quadrangular room: a rough drawing must first be made of the four sides; then each side of the room must be measured, and the dimension placed upon each corresponding side of the draught: but these four sides will not be sufficient, as the figure may be moved out of its position; we must therefore take one of the diagonals also, and this being done, will resolve the figure into two triangles, which is comprehended in this problem.

PROBLEM XXXII.

To describe a triangle similar to a given triangle upon a given straight line corresponding to one of the sides of the given triangle.

Make an angle at each extremity of the given straight line equal to the angle at each extremity of the corresponding line (Prob. VI) of the given triangle, and on the same side of the given line as the corresponding angles of the given triangle: produce the leg of each angle which is not the given side till both meet, and the triangle thus formed will be similar to the given triangle.

Thus, let ABC be the given triangle, and DE the given straight line.

At the point D , with the straight line DE , make the angle EDF equal to the angle BAC , and at the point E , with the straight line ED , make an angle DEF equal to the angle ABC ; then, if the two legs of the angle meet in F , the thing required is done; but if not, produce these two other legs till they meet in F , and the triangle DEF will be the triangle required.

If the two first corresponding sides are parallel, every other pair of corresponding sides will also be parallel; and consequently in this case the required triangle may be constructed by means of parallel lines.

Fig. 89, No. 1.

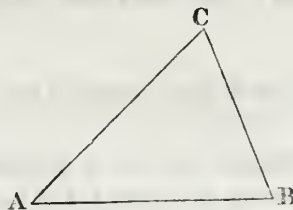
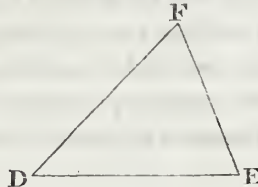


Fig. 89, No. 2.

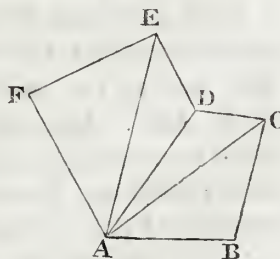


PROBLEM XXXIII.

To construct a rectilineal figure similar to a given rectilineal figure upon a given side, corresponding to a side of the given figure.

Describe a triangle upon the given side similar to that upon the corresponding side of the given figure: again, upon the side of the figure under construction corresponding to the common side of the similar triangle to that constructed, and the next similar triangle to be constructed in the given figure, describe the next triangle similar to that in the given figure, and so on, one triangle after another, upon its succeeding corresponding line, until the whole series are found; then the figure thus formed will be similar to the figure proposed.

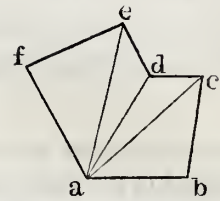
Fig. 90, No. 1.



EXAMPLE.

In the two figures here exhibited, $ABCDEF$ is that which is given; $abcdef$ is the figure required to be constructed; and ab is a side corresponding to the side AB . First, the triangle abc is described similar to the triangle ABC ; the triangle acd is described next similar to the triangle ACD , and so on.

Fig. 90, No. 2.



PROBLEM XXXIV.

Upon a given straight line AC to describe the segment of a circle that shall contain a given angle H .

Bisect AC by a perpendicular EF , and draw AG , making the angle CAG equal to H : draw AE perpendicular to AG . From the centre E , and distance EA , describe the arc ABC : then, if any point B is taken in the arc, and the lines BA and BC be joined, the angle ABC will be equal to the given angle H .

For the angle contained by a chord and a tangent at the extremity of that chord is equal to the angle in the alternate segment (*Euclid*, book iii, prop. 33).

Fig. 91, No. 1.

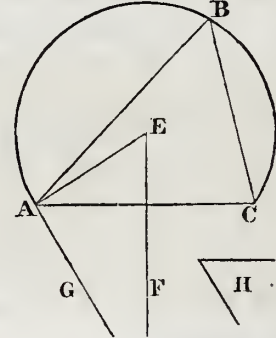
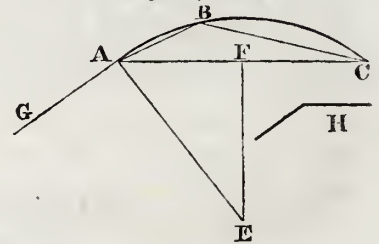


Fig. 91, No. 2.



PROBLEM XXXV.

In a given circle DEF to inscribe a triangle similar to a given triangle.

From any point D in the circumference draw any chord DE , and draw the chord DF , making the angle EDF equal to the angle ABC . Join EF ; then, if the angle FED be equal to the angle CAB , the thing is done; but if not, draw the chord EG , making the angle FEG equal to the angle CAB , and join GF ; then EGF is the triangle required.

Fig. 92, No. 1.

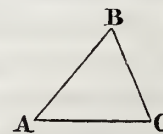
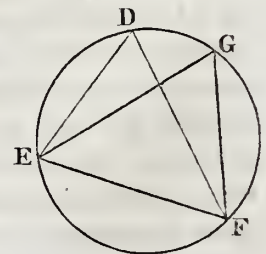


Fig. 92, No. 2.



SCHOLIUM.

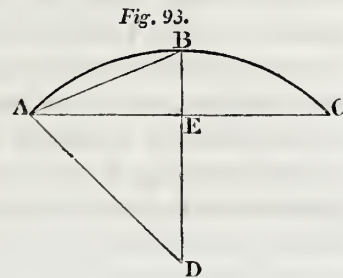
Though the problem of describing the segment of a circle is comprehended in that of making the circumference pass through three given points (Prob. XVI); yet, as it is more convenient in practice to take the chord and versed sine of the arc, than to find the position of three points, we shall here show how this is to be done.

PROBLEM XXXVI.

The chord AC and versed sine of the segment of a circle being given in magnitude, to describe the arc.

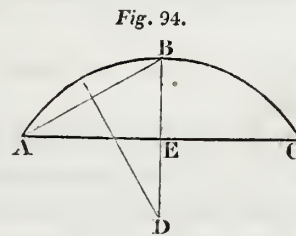
METHOD 1.

Bisect AC in E by a perpendicular BD; make EB equal to the versed sine, and join AB. Make the angle ABD equal to the angle BAD. From the centre D, with the distance of any one of the two lines DA, DB, describe the arc ABC, which will necessarily pass through the other two points, and therefore must pass through all the three points.



METHOD 2.

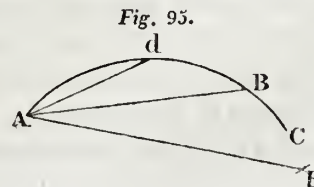
Bisect AC in E by a perpendicular BD, and join AB as before: but, *now the remaining part of the process is different.* Bisect AB by a perpendicular meeting BD in D: then with the radius equal to any one of the three distances DA, DB, DC, describe an arc ABC, which will necessarily pass through the other points, and therefore will pass through all the three points.



PROBLEM XXXVII.

To find the point of prolongation of one extremity C of the arc ABC of a circle, so as to meet a given straight line AE passing through the other extremity A, without making use of the centre.

From A draw any chord AB; and draw the line Ad to cut the arc in d, so that the angle BAd may be equal to the angle BAE. From the centre B, with the distance Bd, describe an arc cutting AE at E, and E will be the point required.



SCHOLIUM.

By this means, an arc, which is too small a portion of the circumference to answer the intended purpose, may be prolonged when the centre is inaccessible, or when it is so remote as to be inconvenient to use it. This is a case that frequently occurs in practice.

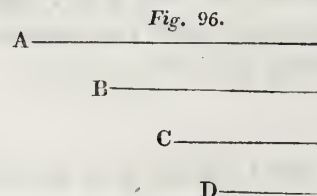
DEFINITIONS.

Def. 1. A *ratio* is the relation between two things, such as lines or magnitudes of any description or quantity, by considering how often the one is contained in the other, or how many equal parts the one may be of the other.

Def. 2. The first quantity which contains the other is called the *antecedent* or leading quantity; and that which is contained is called the *consequent* or following quantity.

Def. 3. An *analogy* or *proportion* is the equality of two ratios, or the comparison of four quantities, so that the first will contain the second, or the like parts of it, as often as the third contains the fourth or the like parts of it.

Thus, four lines A, B, C, D, are proportional when A is divided into equal parts, and when C is divided into the same number of equal parts as A : then, if B contain as many parts of the scale A, as D contains of the scale C, these four lines are proportional.



Def. 4. When four lines are proportional, the lines themselves are called *proportionals*.

Def. 5. Each of the four proportionals is called a *term*.

Def. 6. Of four proportionals, the first and last terms are called *the extremes*, and the two terms between are called *the means*.

Def. 7. When the two middle terms happen to be equal, the four terms having thus the two middle ones equal, are called *three proportionals*.

Def. 8. A fourth proportional to three given lines, is a line of such length that it may contain the like parts of the third which the second does of the first.

Def. 9. A third proportional to two given lines, is a line of such length that it may contain the like parts of the second which the second contains of the first.

This is, in fact, finding a fourth proportional to three lines, when the second and third lines are equal.

Def. 10. When four lines are proportionals, the first is said to be to the second as the third is to the fourth.

Def. 11. When three straight lines are proportionals, the first is said to be to the second as the second is to the third.

Def. 12. A mean proportional is one of the equal means in three proportional lines; that is, it is a line of such length, that the first line is to the mean, as the mean is to the third line.

SCHOLIUM.

A knowledge of the proportionality of lines is of the greatest use not only to practical mechanics, but to architects and to builders' clerks: for it is upon this principle that all scales are made, and objects enlarged or diminished, and that drawings and models are made to represent buildings.

PROBLEM XXXVIII.

Given three straight lines to find a fourth proportional.

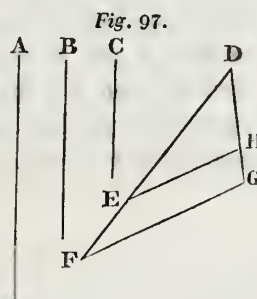
Observe first, that all lines applied to the legs of an angle are applied from the point of concourse, and the points set off on the legs are called points of extension; this being understood, we may proceed.

Draw two straight lines at any angle, calling one of the lines the first leg, and the other the second leg. Apply the first and second of the lines upon the first leg, and the third line upon the second leg: join the points of extension of the first and third lines; and through the point of extension of the second line, draw a line parallel to the line which connects the two legs of sufficient length as to reach the other leg; then the distance between the point of intersection now made and the point of concourse of the angle is the fourth proportional required.

EXAMPLES.

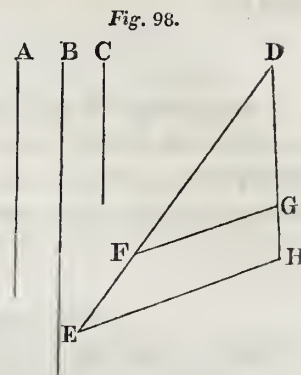
Ex. 1. *Given the three straight lines A, B, C, to find a fourth proportional.*

Draw any angle FDH; apply the straight line A from D to F, and the straight line B from D to E: apply C from D to G; join FG, and draw EH parallel to FG, cutting DG in H; then DH is the fourth proportional required.



Ex. 2. *Given the three straight lines A, B, C, to find a fourth proportional.*

Having, as before, made the angle EDH, apply the straight line A from D to F, and the straight line B from D to E, and the straight line C from D to G: join FG, the points of extension of the first and third terms, and through E draw EH parallel to FG, cutting DG produced in H: then DH is the fourth proportional.



PROBLEM XXXIX.

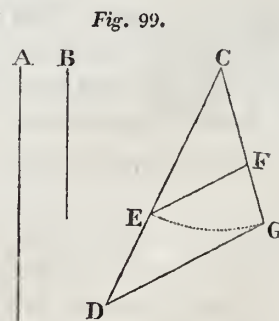
To find a third proportional to two given lines.

Having drawn an angle as in the case of four proportionals, apply the first and second of the two given lines upon the first leg, and the second line also on the second leg: join the point of extension of the first line to that of the second on the second leg; and through the point of extension of the second line on the first leg draw a line parallel to the line extending between the two legs to cut the second leg; and the distance of the intersection from the point of concourse of the angle is the third proportional required.

EXAMPLE.

Find a third proportional to the lines A and B.

Having, as before, drawn the angle DCG; apply the two given lines A and B respectively from C to D and E upon the first leg, and the line B upon the second leg from C to G: join DG, and through E draw EF parallel to DG, cutting CG in F, and CF is the third proportional.



SCHOLIUM 1.

The method of finding a third proportional, as has been observed, can hardly be called a distinct problem from that of finding a fourth proportional; the former being only a

particular case of the latter, when the two middle terms happen to be equal ; but from the frequent occurrence of three proportionals in practice, it is here given under a distinct head, as will be found in most mathematical works.

SCHOLIUM 2.

Proportion is not only useful in finding a third and fourth term, but also in dividing a line, so that the ratio of every two parts may be equal to the ratio of every two corresponding parts of a given line ; or, in other words, the dividing of a line into as many parts as another line, and in the same proportion as the corresponding parts of that other line ; which is shortly and generally expressed, the dividing of a line in the same proportion as another.

PROBLEM XL.

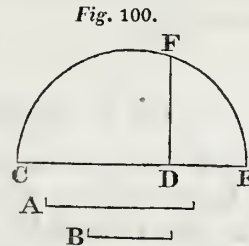
To find a mean proportional between two given straight lines.

Place the two straight lines in one straight line, and mark the point of division. On the line compounded of the two lines, as a diameter, describe a semicircular arc : from the point of division draw a perpendicular to the straight line to meet the arc, and the perpendicular will be the mean proportional required.

EXAMPLE.

Find a mean proportional between the two straight lines A and B.

Draw the straight line CE, on which set CD equal to A, and DE equal to B. On CE, as a diameter, describe the semicircle CFE, and from the point D, where the two lines A and B join, draw DF perpendicular to CE, cutting the arc of the semicircle in F: then DF is the mean proportional required.



PROBLEM XLI.

To divide a straight line into as many parts as another given line, so that every pair of corresponding parts, one and one from each line, shall have an equal ratio to any other such pair.

METHOD 1.

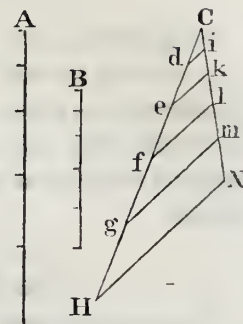
Draw two straight lines forming an angle as formerly ; place the given divided line upon the first leg, and mark the points of division, and place the undivided line on the second leg: join the unconnected extremities of the two lines, and through all the points of division in the first leg draw lines parallel to the connecting line to cut the undivided leg: then the leg now divided will have all its parts in the same proportion as the given divided line.

EXAMPLE TO METHOD 1.

Divide the line B in the same proportional parts on the line A.

Make the angle HCN, and place the given divided line A upon the leg CH with its divisions, and place the other undivided line B upon the leg CN; join HN, and through the points d, e, f, g of division, draw the lines di, ek, fl, gm, parallel to HN; and the line CN, equal to the undivided line B, will be divided in the same proportion as the divided line A.

Fig. 101.



METHOD 2.

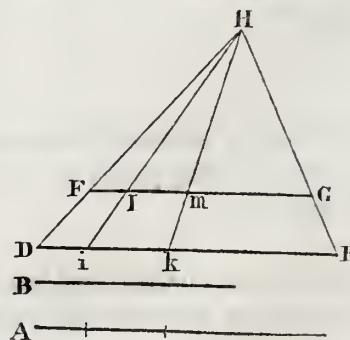
Place the two given lines parallel to each other; join each pair of their extremities, and prolong the lines thus joining, if necessary, till they meet or cross from the point of intersection; draw lines through all the points of division of the divided line, and prolong them, if necessary, to meet or cut the undivided line; and the undivided lines will thus be divided into parts which shall have the same proportion to each other as the parts of the divided line.

EXAMPLES.

Ex. 1. *Divide a line equal to B in the same proportion as another A.*

Draw DE equal to A, and FG parallel to DE equal B. Mark the points i, k, corresponding to the divisions of A: join DF and EG, and produce the lines DF and EG till they meet in H. Draw the lines Hi, Hk, cutting FG in l, m; then FG, which is equal to B, is divided in the same proportion as the line DE, equal to A. —See Note (13) in next page.

Fig. 102.

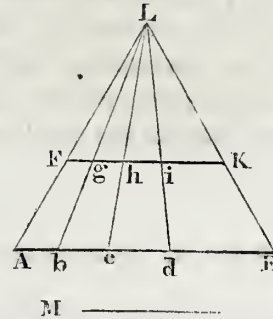


METHOD 3.

When the same proportion is frequently required in practice, the following method, which is founded on the same principle, will perhaps be the most eligible.

Describe an equilateral triangle LAE, of which the side is the greatest of any line that may be required to be divided; then suppose the side AE is divided in the proportion required, and the dividing lines bL, cL, dL, drawn: let it be required to divide the line M in the same proportion as the line AE. On the two other sides LA and LE, mark off LF and LK, each equal to the line M, and join FK; then FK will be divided in the same proportion as AE (13).

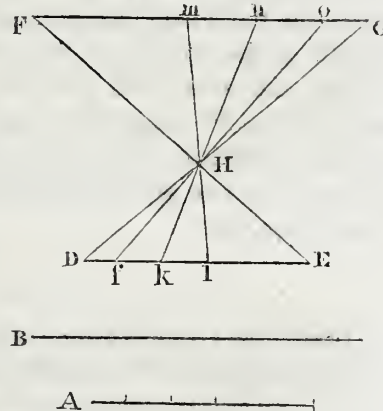
Fig. 103.



METHOD 4.

Draw the parallels DE, FG, respectively equal to the lines A and B, and mark the divisions of the line A on DE: join DG and EF, cutting each other in H; from the points f, k, l, draw lines through H to meet the line FG in o, n, m; then will the line GF, equal to B, be divided in the same proportion as the line DE equal to A.

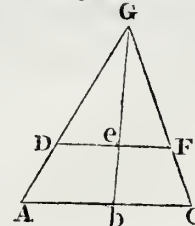
Fig. 104.

(13) *Demonstration of the second and third methods.*

Suppose the side AC of the triangle AGC to be divided in any given proportion Ab, bC; and the line bG drawn; and let DF parallel to AC, cutting AG in D and CG in F, be cut in the point e by bG; then will Ab be to bC, as De, eF.

For, by the similar triangles, GbA, GeD, $Gb : bA :: Ge : eD$,
and by the similar triangles, GbC, CeF, $Gb : bC :: Ce : eF$;
therefore, by equality of ratios, $Ab : bC :: De : eF$.

Fig. 105.



SCHOLIUM.

It will be observed, that, when the contrary ends of the lines are joined, the point of intersection falls between the parallels, and therefore the divisions on the parallels have contrary positions to each other; but this cannot be called an inconvenience, since the parts of the divided line can be made to proceed from either end of the line DE, so as to make the progression begin as may be required in FG. This method will be most convenient when the two lines are nearly of the same length; because, in this case, the space which contains the figure would be very great.

PROBLEM XLII.

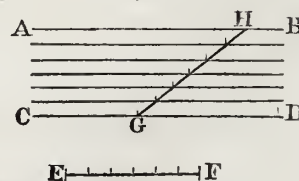
To divide the space between two parallel lines into any number of spaces of equal breadths by parallel lines.

Repeat any convenient distance upon a separate line as often as the space between the two given parallel lines is intended to contain one of its equal spaces, so that the length of all the parts may exceed the distance between the two parallel lines. Take the whole length of the parts, and from any point in one of the parallel lines describe an arc cutting the other, and join the centre and the point of intersection; then, upon the line thus joining the parallels, set off the divisions of the line of equal parts, and through the points of division draw lines parallel to either of the two given lines, and the distance between the two given parallel lines will be every where divided into the number of equal parts required.

EXAMPLE.

Divide the space between the two parallel lines AB, CD, into six equal parts; set off six equal parts upon the line EF, so that the whole of the parts E, F, may be greater than the distance between AB and CD. From any point G, in CD, describe an arc cutting AB at H, and join GH; transfer the equal parts of the line EF to GH, and through the points of division draw lines parallel to AB or CD, and these lines will divide the space as required.

Fig. 108.



SCHOLIUM.

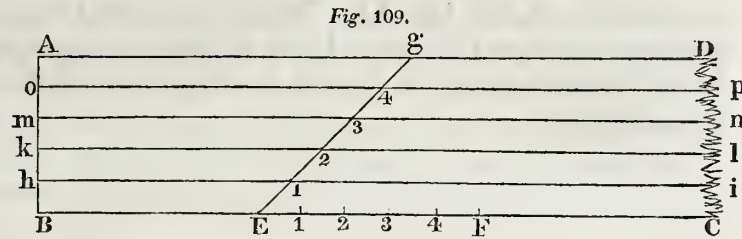
Upon this principle, a Carpenter may divide the breadth of a board into any number of parallel slips, all equal in breadth, or divide them in any given proportion.

Thus, let ABCD be a board, and let it be required to divide it into five laths or battens of equal breadth.

Upon the edge of the board set off five equal parts of any convenient length from E to F: from any

point E, with the radius EF, describe an arc cutting the opposite side of the board at g, and join Eg. Transfer the divisions of the line EF into the line EG; and through 1, 2, 3, 4, the points of division, draw the parallel lines hi, kl, mn, op, and the board will be divided into five equal slips as required. It is plain that the distance EF must always be greater than the breadth of the board; and if any other proportions are required, the process will be exactly similar.

Instead of using the line EF, the number of parts may be set off on a small slip of wood, or upon the edge of a two-foot rule, and transferred at once to the line Eg.



PROBLEM XLIII.

To find a series of lines in continued proportion from two given lines.

Having formed an angle, place one of the given lines from the point of the angle upon one of the legs, and place the other line from the angular point upon the other leg: join the points of extension.

From the angular point cut off a distance from the leg on which the first line was placed, equal to the second line; and through the point of division draw a line parallel to the line joining the two legs, cutting the leg on which the second of the two given lines was placed.

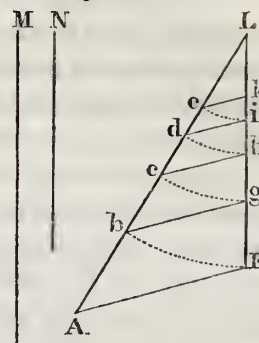
Proceed in the same manner, always cutting off a distance from the angular point upon the leg on which the first given line was placed, equal to the distance from the angular point to the point on the other leg cut by the parallel last drawn: then, from the point of intersection draw another parallel to cut the second side; and the first side will be divided into a series of continued proportionals.

EXAMPLE.

Find a series of lines in continued proportion from the two given lines M and N.

Having drawn the angle ALF, make LA equal to M, and LF equal to N, and join AF. Make Lb equal to LF, and draw bg parallel to AF, cutting LF in g. Make Lc equal to Lg, and draw ch parallel to bg, and so on, as far as may be necessary(14).

Fig. 106.



METHOD 4, BY PARALLEL LINES.

When the undivided line is greater than the divided line.

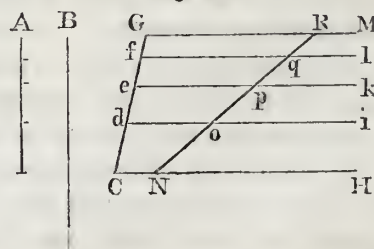
Upon any convenient surface place the divided line, and through its extremities and its points of division draw parallel lines: from any point in one of the extreme lines, with the radius of the undivided line, describe an arc cutting the other extreme line; then a line drawn from the centre to the point of intersection will be divided into the proportion required.

EXAMPLE.

Divide the line B in the same proportion as the line A.

Draw the line CG equal to the divided line, and mark its divisions at the points d, e, f: through the points C, d, e, f, G, draw the lines CH, di, ek, fl, GM, at any convenient angle parallel to each other. From any point N in CH, with the radius of the undivided line B, describe an arc cutting GM at R, and join NR, which is equal to the given line B; then will NR be divided at the points o, p, q, in the same ratio as the line A.

Fig. 107.



(14) When LA, Lb, Lc, Ld, &c., are in continued proportion, their differences Ab, bc, cd, de, &c. are also in continued proportion. The truth of this assertion will be shewn in the following demonstration:—

PROBLEM XLIV.

From a given straight line to cut off any part of it.

METHOD 1.

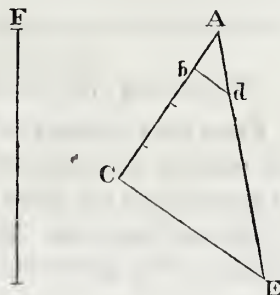
Draw two straight lines at any given angle as before. On one of the legs repeat as many equal parts of any convenient length from the fixed point, that is, the point of the angle as the part required of the given line is intended to divide the whole, and on the other leg set off the given length: join the two extreme points, and through the first point of division marked on the first leg draw a line parallel to the line extending between the extremities of the two legs to cut the other leg; then the distance between the vertex and the intersection will divide the given line into the number of parts required.

EXAMPLE.

Let it be required to take one fourth part of the line F.

On the leg AC of the angle CAE, set off four parts from A to C, and make AE on the other leg equal to the given line F. Join CE, and through b, the first point of division, draw bd parallel to CE, cutting AE in d; then Ad is one fourth part of the given line F.

Fig. 110.



METHOD 2.

Draw any line parallel to the given line; and on the parallel thus drawn, repeat any convenient distance as often as the portion to be cut off from the given line will divide its whole. Draw a straight line through each two corresponding ends of the given line and

Let $LA = a$, $Lb = b$, $Lc = c$, $Ld = d$, $Le = e$, &c.; then, by the definition of continued proportion,

$$a : b :: b : c$$

$$b : c :: c : d$$

$$c : d :: d : e$$

&c.

Then, by subtracting the alternate terms,

$$\left. \begin{array}{l} a : b :: a - b : b - c \\ b : c :: b - c : c - d \\ c : d :: c - d : d - e \end{array} \right\}; \text{ therefore } \left\{ \begin{array}{l} a - b : b - c :: b - c : c - d \\ b - c : c - d :: c - d : d - e \end{array} \right. \text{ \&c.}$$

But $a - b = La - Lb = Ab$; $b - c = Lb - Lc = bc$; $c - d = Lc - Ld = cd$, &c.; therefore Ab , bc , cd , de , &c. are in continued proportion, as well as La , Lb , Lc , Ld , Le , &c.

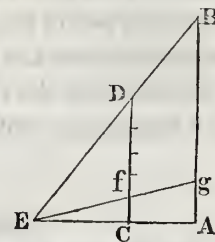
its parallel; and from the point where the two lines meet, draw another straight line through the first division or end of the first part of the parallel divided, to cut the given line; and the least part of the given line thus cut off, will be the portion required.

EXAMPLE.

Divide the straight line AB into five equal parts.

Fig. 111.

Draw CD parallel to AB; from C to D set off five equal parts: draw AC and BD meeting at E; from E through f, draw the line Efg, cutting AB at g; then will Ag be a fifth part of the whole line AB as required.



PROBLEM XLV.

To find any fractional part whatever of a given line, or to form an accurate scale.

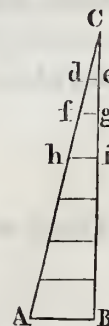
From the extremity of the given line draw a straight line at any angle with it. From the point of concurrence on the line thus drawn, set off as many equal parts as the number of parts which the given line is intended to contain, and join the unconnected extremities of the last part, and that of the given line: through the points of division draw lines parallel to the given line to cut the connecting line.

EXAMPLES.

Ex. 1. *Find any number of sevenths of the line AB.*

Fig. 112.

Draw BC, and mark off seven equal parts from B to C; join AC; through the points of division draw lines parallel to AB to meet the line AC; then de will be one seventh part of the line AB, fg two sevenths, hi three sevenths, &c.



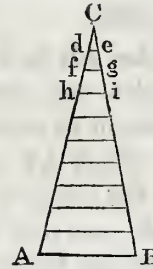
Ex. 2. *Find any number of tenths of the line AB.*

Draw AC, and set ten equal parts on the line AC; join BC, and through the points of division in AC, draw lines parallel to AB to cut the line BC; then de is one tenth, fg two tenths, hi three tenths, &c.

SCHOLIUM.

It is upon this principle that diagonal scales are divided.

Fig. 113.



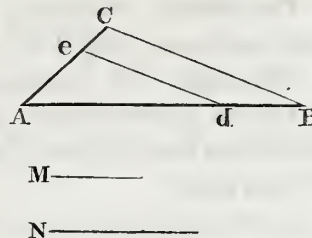
PROBLEM XLVI.

Given one side of a triangle, and one of the adjacent angles to that side to complete the triangle, so that the other two sides may have a given ratio.

Let AB be the given side of the triangle, BAC the given angle, and let the lines M and N be the ratio of the other two sides.

In the indefinite leg AC of the angle, make Ae equal to M; from the point e with the distance of N describe an arc cutting AB at d: join ed, and through B draw CB parallel to ed; then ABC is the triangle required (15).

Fig. 114.



(15) DEMONSTRATION.

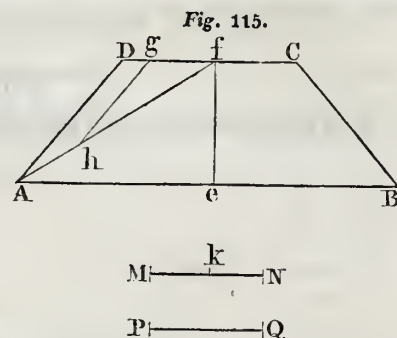
Because (in the figure given in the text) Ae is equal to M, and ed is equal to N; therefore Ae is to ed as M is to N; but because BC is parallel to de, Ae is to ed as AC to CB; therefore AC is to CB as M is to N; whence the triangle ABC is described as required.

PROBLEM XLVII.

Upon a given straight line to describe a trapezoid, so that the two sides which join the given side may be equal to each other, and the intermediate side may be parallel to the given side and at a given distance from it, and in a given ratio with either of its adjacent sides.

Let AB be the given straight line, and let the ratio which the middle side to be described is to have to each of its adjacent sides be as MN to PQ .

Bisect AB by the perpendicular ef cutting it in e , and make ef equal to the distance of the opposite side, and join Af : through f draw DC parallel to AB , and bisect MN in k . From the point f in the line CD make fg equal to Mk or kN , and from g with the distance PQ describe an arc cutting Af in h . Draw AD parallel to hg ; make fC equal to fD , and join BC ; then $ABCD$ is the trapezoid required.



SCHOLIUM.

This method, after having made the perpendicular ef , and having drawn DC parallel to AB , and having joined Af , is the same as in Problem XLVI; for here are given the base Af , the angle AfD , and the ratio of the sides Mk , PQ , to describe the triangle AfD .

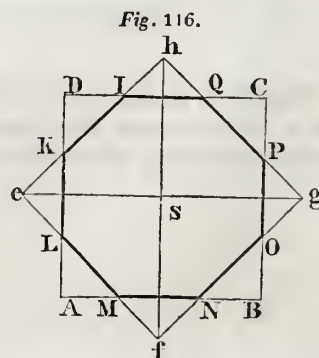
The use of this Problem is to describe the plan of a prismatic bow of a building which may either have its three visible sides equal to each other in breadth or in any given ratio to each other.

PROBLEM XLVIII.

A square being given, to inscribe a regular octagon which shall have four of its sides in common with the four sides of the square.

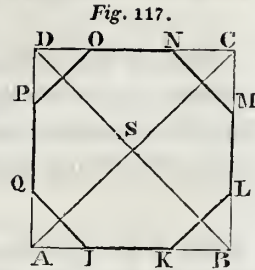
METHOD 1.

Let $ABCD$ be the given square; bisect any two adjacent sides AB and AD by the perpendiculars fh and eg , cutting each other in the centre s : make se , sf , sg , sh , each equal to sA , sB , sC , or sD : join ef , cutting the two adjacent sides AD in L , AB in M ; join fg cutting AB in N , and BC in O : join gh , cutting BC in P , and CD in Q ; and lastly, join he , cutting CD in I , and DA in K ; then will $IKLMNOPQI$ be the octagon required.



METHOD 2.

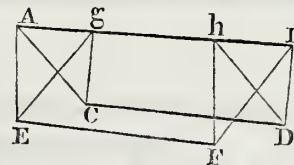
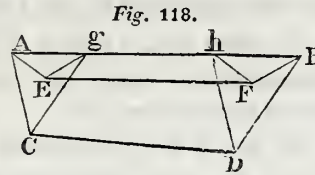
Draw the two diagonals AC, BD, cutting each other in s, as before; then each of the diagonals will be bisected by the point s. With a distance equal to As half of a diagonal, cut off from the four angular points A, B, C, D, the distances AK, AP, BI, BM, CL, CO, DN, DQ from each of the sides of the square, and join KL, MN, OP, QI; then will KLMNOPQ be the octagon required (16).



PROBLEM XLIX.

Given any two lines AB and CD tending to an inaccessible point, to draw a line through another point E, so that all the three lines may tend to the same point.

In the line AB, take Ag any convenient part of AB, and C any convenient point in CD, and join AC, gC, also join AE and gE. Through any point B at or near the other extremity of AB draw BD parallel to gC; and through the point D draw Dh parallel to CA, BF parallel to gE, and hF parallel to AE, and join EF: then AB, CD, EF, would all meet in the same point if produced.



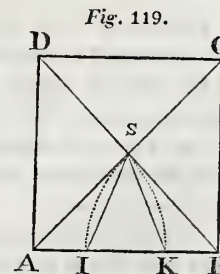
The operation is evidently the same whether the point E lies between or without the two given lines.

When there are two given lines and several points, either without or between the two given lines, the same process must be gone through for each point; but the same bases Ag and hB may be made to serve for every point, as is plain by inspecting *Plate I* of inaccessible lines.

See also *Fig. 1* and *2, Plate I* of inaccessible lines, where the diagrams are in a proportion which is likely to occur in practice.

(16) DEMONSTRATION.

Join Ks; then AK being equal to As, the two angles AKs, AsK are equal to each other; and, since the three angles of every triangle are equal to two right angles, and the angle KAs is half a right angle, each of the two angles AsK, AKs must be three quarters of a right angle; consequently the angle KsB must be a quarter of a right angle. For the same reason the angle AsI must also be a quarter of a right angle; therefore the two angles AsI, KsB, together make half a right angle; and since the angle AsB is a right angle, the angle IsK must be half a right angle, or the whole number of angles round the centre s equal to eight half right angles, that is four right angles.



PROBLEM L.

To draw a straight line through a given point that shall make equal angles on the same side of it with two other given straight lines.

Through the given point draw a perpendicular upon each of the two given lines, and produce one of the lines on the other side of the point; bisect the angle contained by the line thus produced and the other perpendicular, and the bisecting line, continued to meet each of the two given lines, will make equal angles with those two lines.

EXAMPLE.

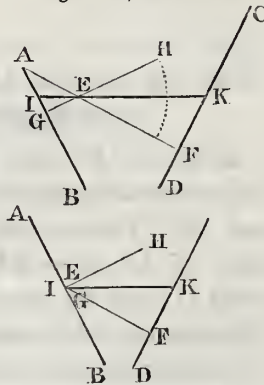
Draw a straight line through the point E to make equal angles on the same side of it with the two given lines AB and CD.

Through E draw EF perpendicular to CD, and EH perpendicular to AB; then draw IK bisecting the angle FEH, and the angles BIK and DKI will be equal to one another.

See also *Fig. 3 and 4, Plate I*, inaccessible lines, where the diagrams are nearer to the proportion in which they would occur in practice (17).

In No. 2, the given point E, and the two other points E, G, where the perpendiculars cut AB, coincide.

Figs. 120, No. 1 & 2.



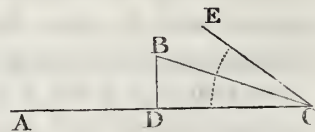
PROBLEM LI.

Given the chord and versed sine of a segment, to draw a tangent at either extremity of the chord, without making use of the centre or any part of the arc.

Let AC be the chord, and DB the versed sine; join BC, and draw the line EC making the angle BCE equal to the angle BCD (18).

See also *Fig. 5, Plate I*, of inaccessible lines, where the diagram is nearer to the proportion in which it would occur in practice.

Fig. 121.



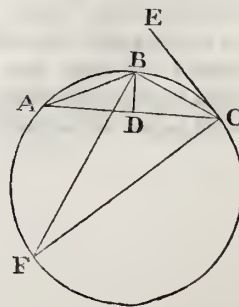
(17) DEMONSTRATION.

Because in No. 1, EG is perpendicular to AB, and EF perpendicular to CD, the two triangles EGI, EFK are right angled; and because the two opposite equal angles GEA, FEH are bisected, the two angles IEG, KEF must be equal; therefore in the two triangles EGI, EFK, the remaining angle EIG is equal to the remaining angle EKF.

(18) The above depends upon this proposition of Euclid, book iii, prop. 32, that the angle made by a chord and its tangent is equal to the angle in the alternate segment.

Now BC being a chord, BAC or BFC is the angle in the alternate segment; and since DB is perpendicular to AC, the isosceles triangle ABC is divided into two equal and similar triangles; therefore the angle BCA is equal to the angle BAC, and consequently the angle BCA is equal to the angle BCE.

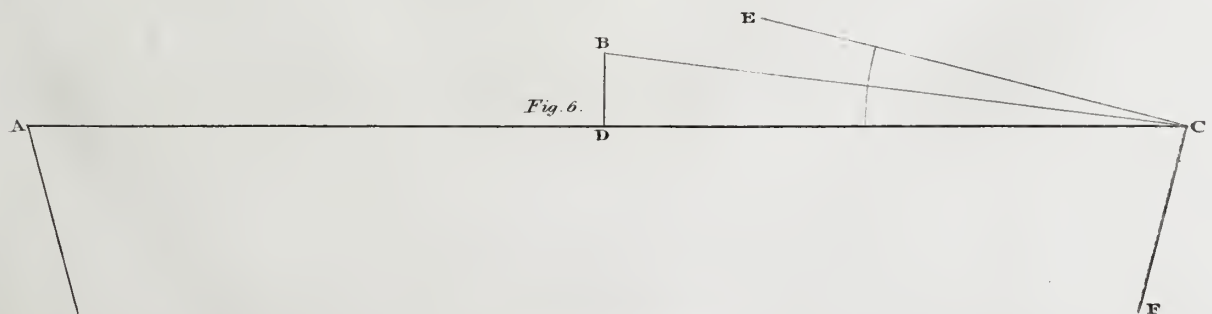
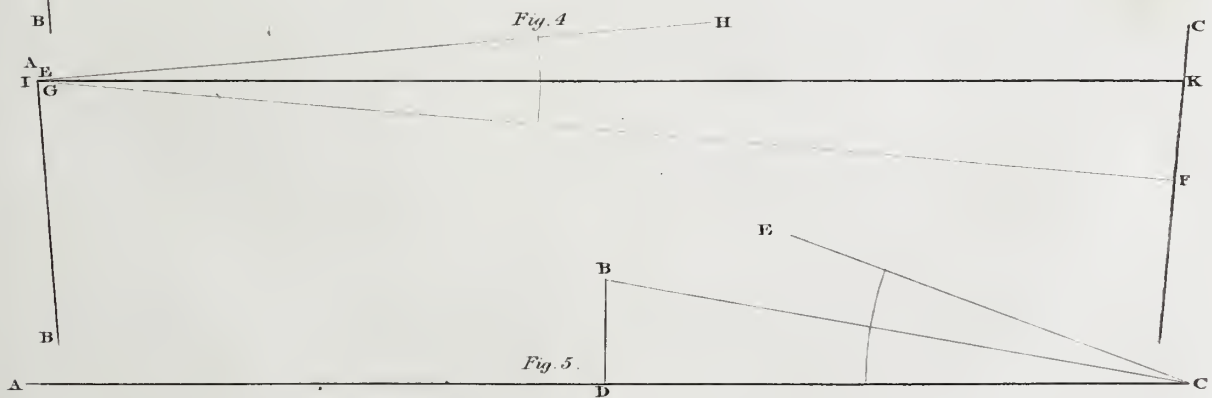
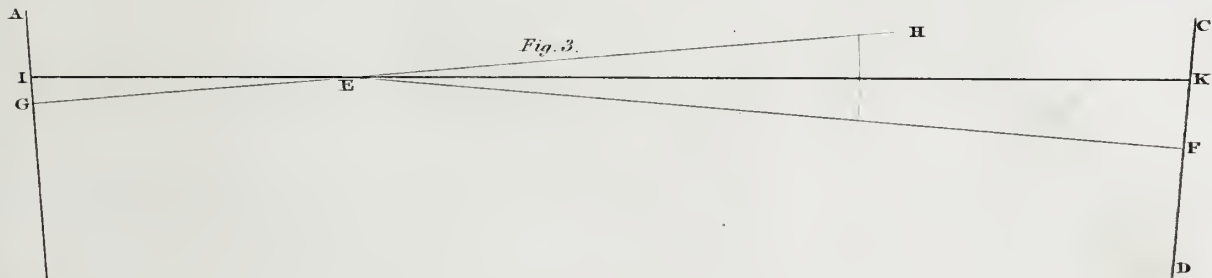
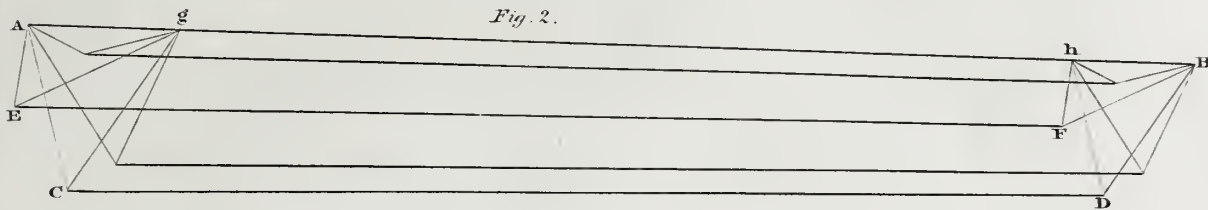
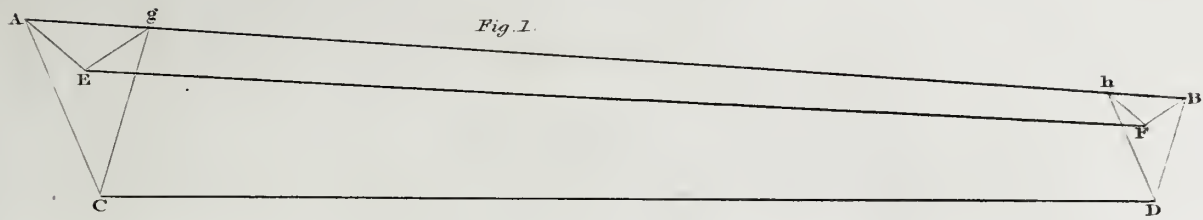
Fig. 122.



PLANE GEOMETRY.

INACCESSIBLE LINES.

PLATE 1



Invented by P. Nicholson.

PROBLEM LII.

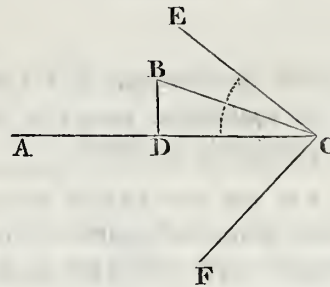
Given the chord and versed sine of the segment of a circle, to draw a line tending to the centre from the extremity of the chord, without making use of the centre.

Let AC be the chord, BD the versed sine given in position and magnitude.

Find the tangent CE as in the last problem, and draw CF perpendicular to CE; then CF will tend to the centre as required.

See also *Fig. 6, Plate I*, of inaccessible lines.

Fig. 123.



PROBLEM LIII.

To describe the segment of a circle when the extension of the radius reaches beyond the space where the operation is to be performed.

Bisect the chord AC (see *Fig. 1, Plate II*, inaccessible lines) by the perpendicular DB, making DB equal to the versed sine or height of the segment: join AB and BC.

Prolong BA to E, and BC to F. Fasten two slips of wood and BE and BF together, so that their outer edges may contain the angle EBF; then bring the angular point B to A, and move the instrument so, that, while the point B is proceeding from A to C, the edge BE may slide upon a point or pin fixed at A, and the edge BF upon a pin fixed at C; then the pressure of a pencil at the angular point B upon the plane of description will describe the segment required (19).

Here it is plain that both the legs BE and BF must not be less than the chord AC, otherwise the arc cannot be described at one movement of the instrument.

SCHOLIUM.

Though this method in the simple form now explained is exceedingly convenient in a variety of cases, there are many, however, that still occur where it cannot be applied for want of room to work the instrument, without moving the apparatus and board or table on which the arc or circular line is described, as is already evident from the problem now given, and will be still more so by inspecting *Fig. 2, Plate II*, inaccessible lines, where

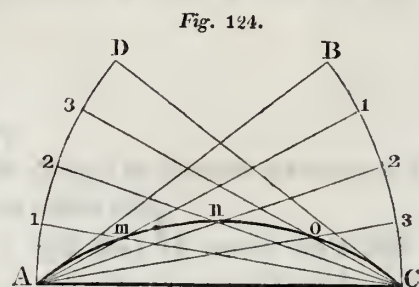
(19) The principle of the above method is founded on this theorem, that *all angles* in the same segment of a circle are equal to each other, and is demonstrated in proposition xxi, book iii, of Euclid.

it appears that the instrument requires much more space than the radius itself; therefore some modifications in its application will be necessary in order that it may be accommodated to every case that may occur in practice. This will be explained in the following problems.

PROBLEM LIV.

Given two tangents to the arc of a circle and their points of contact, to find any number of equidistant points in that arc without making use of the centre, or having the arc previously described.

Let AB and CD be the two tangents, A and C their points of contact: join AC , and divide the angles CAB and ACD each into as many angles as the number of equidistant parts required in the arc; then the points m , n , o , where the lines intersect, are the points required (20).



SCHOLIUM.

If the number of parts are even, this problem can be geometrically executed by continual bisections. The practice may be as follows:—Having drawn the chord AC ; from the centre A , with the distance AC , describe the arc CB , and from the centre C , with the distance CA , describe the arc AD : divide the arcs AC and BD , each into as many equal parts as the number of equidistances to be in the arc between the points of contact A and C , which are here four; then, numbering the parts at one end from A to D , and the other in the contrary order from B to C , every pair of lines drawn from the same numbers to the extremities A and C of the chord will be a point in the arc required.

Keeping the same principle in view, this method is still capable of receiving a more practical form, as will be shewn in the following

(20) This method depends on the equality of the arcs in the circumference, when the angles at the circumference are equal; and here, since the angles CAo , oAn , nAm , mCA , as also the angles ACm , mCn , nCo , oAC are equal, the points m , n , o must be in the circumference of the same circle in which the points A and C are. The theorem on which this principle depends is, “In equal circles, angles, whether at the centres or circumferences, have the same ratio that the arcs on which they stand have to one another;” which is demonstrated in proposition xxxiii, book iii, of Euclid.

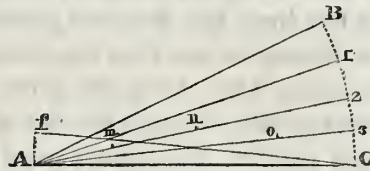
PROBLEM LV.

Given the two extremities of an arc, and a tangent at one of them, to find any number of equidistant points in the arc, without having any part of the circumference, or making use of the centre.

Let A, C, be the two extremities of the arc, and AB the tangent at the point A.

From the centre A, with the distance AC, describe the arc BC; divide the arc BC into as many equal parts, at the points 1, 2, 3, as the number of equidistances required in the arc. From the centre C, with the distance CA, describe the arc Af: make Af equal to one of the equal parts of the arc BC; draw 1A, 2A, 3A, and fC, which will intersect 1A at m; then from m, with the distance mA, describe an arc cutting 2A at n; from n, with the same radius, describe an arc cutting 3A at o; then the distances Am, mn, no, oC, will be all equal.

Fig. 125.



SCHOLIUM.

As the mechanical construction of a geometrical problem is seldom executed with the rigorous exactness of the theory, and very frequently insufficient to the end proposed, unless proved by trial; so in this instance it may happen, that the distance Am, when repeated upon the several lines to the number of times, may either extend beyond the point C, or fall short of it, as, from the obliquity of the intersection, the exact point m cannot be easily seen: in this case we must extend or contract the distance a very small portion, until the number required fall upon the point C. This may be easily adapted to the following

PROBLEM LVI.

Given the chord and versed sine of the arc of a circle, to find any number of points in each half arc of that segment.

Let AC (Fig. 3, Plate II, inaccessible lines) be the chord, and DB the versed sine.

Through B draw PQ parallel to AC. Here are now given the two extremities A, B, and the tangent BP, to find the number of equidistant points required: this being done by Problem LV, we shall have the arc.

In like manner for the other half, there are given the two extremities B and C, and the tangent BQ, to find the number of equidistant points required; which being executed by the same problem, we shall have the other half, and consequently the whole arc of which the chord is AC and the versed sine DB.

The diagram exhibited in the plate is in one of those proportions that is likely to happen in practice.

PROBLEM LVII.

Given the chord and versed sine of the segment of a circle, to describe the curve, without having recourse to the centre.

Find the intermediate points m, n, o (*Fig. 4, Plate II*, inaccessible lines) for each half of the arc, by Problem LV. Now, taking any three adjacent points m, B, m , draw the two lines Bm, Bm , and produce Bm to k , and the other Bm to i , so that neither Bk nor Bi may be less than the distance between m and m : form now the edge of a board to the angle kBi ; bring the point B of the angle of the board to m ; then in moving the board so that the edge Bk may slide upon a pin at m , and the edge Bi upon a pin at the other point m , the angular point B will trace the arc mBm by means of a pencil.

An arc being described in this manner for every two adjacent distances, will complete the whole segment.

SCHOLIUM.

The use of this problem will render the operation of describing the arc of the segment of a circle a most agreeable undertaking, when the centre is inaccessible. The same method ought still to be followed in very large arcs, supposing the centre to be accessible beyond the reach of a rod, as it would not be easy to move a chain or a line, so as to keep it straight.

PROBLEM LVIII.

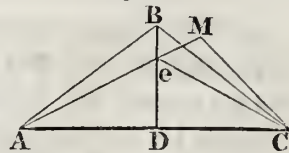
Given the chord of an arc, and any intermediate point whatever in that arc, to find the versed sine of the segment without having the centre given.

Let AC (*Plate II, Fig. 5*, inaccessible lines) be the chord of the arc, and M the intermediate point: bisect AC by a perpendicular DB , and join AM , cutting DB at e ; join eC and MC ; draw BC bisecting the angle eCM , and DB will be the versed sine required (21).

(21) DEMONSTRATION.

In addition to the constructive lines join AB ; we must here prove that the angle ABC is equal to the angle AMC : to this purpose, since all the angles of every triangle are equal to two right angles, and since all the angles in the same segment are equal to each other, the sum of the angles BAC, BCA at the base of the triangle ABC ought to be equal to the angles MAC, MCA at the base of the triangle AMC ; therefore each of the two angles BAC, BCA , ought to be equal to the less angle MAC of the triangle AMC , together with half the difference of the two angles MAC, MCA of the triangle AMC . Now the angle eCA is equal to the angle eAC , and the angle eAC is equal to the angle MAC ; therefore the angle eCA is equal to the angle MAC , and the two angles eCA, MCA are equal to the two angles MAC, MCA ; consequently eCM is the difference of the two angles MAC, MCA ; and since BC bisects the angle eCM , the angle BCA is equal to the angle MAC , together with half the difference of the two angles MAC, MCA .

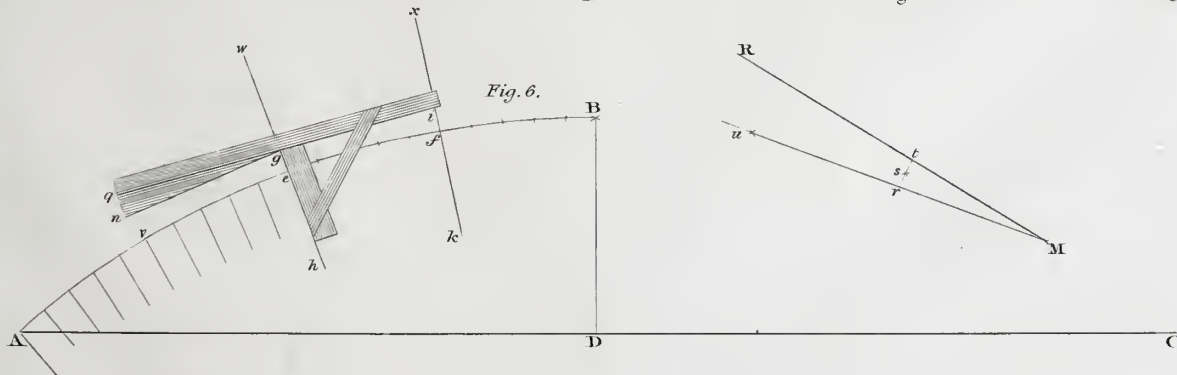
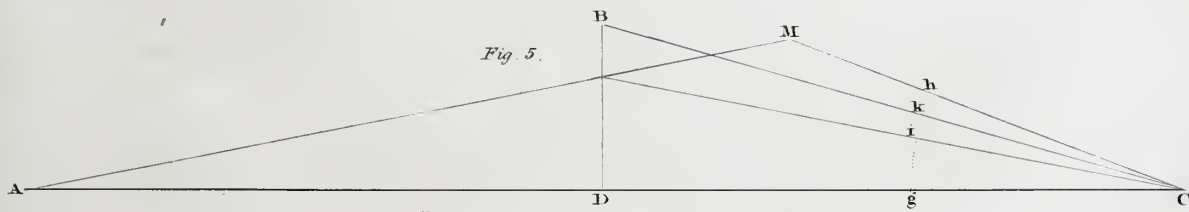
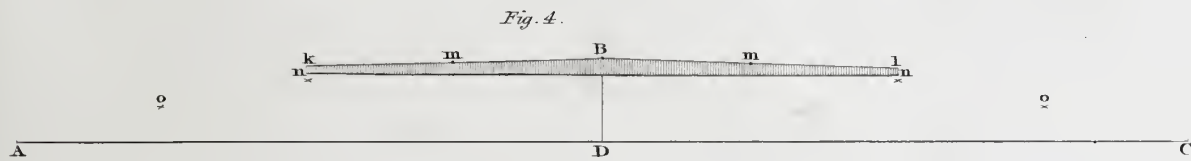
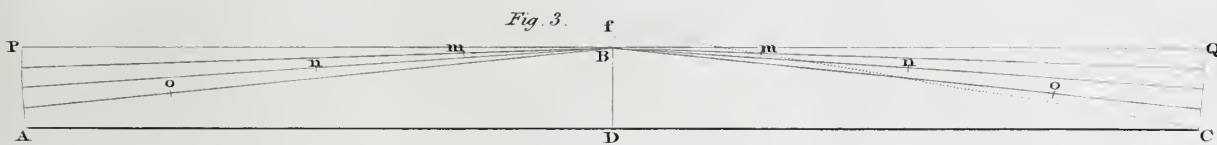
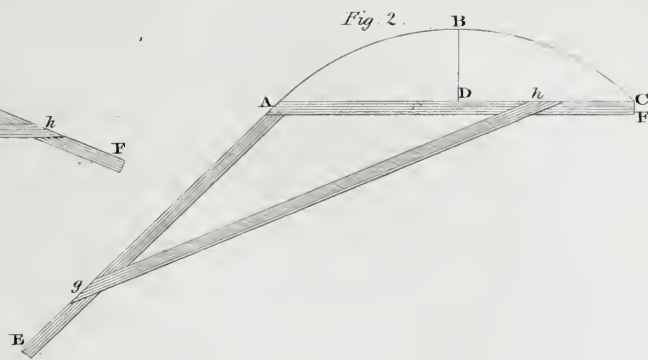
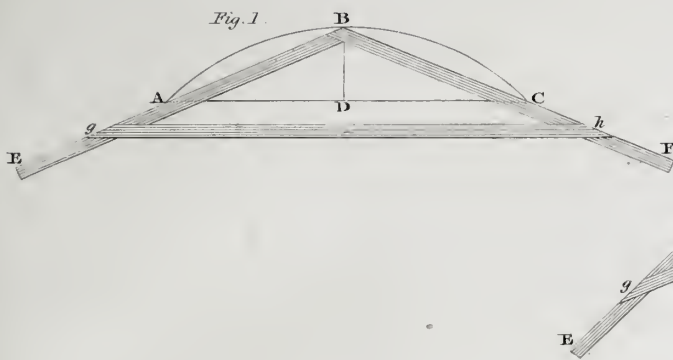
Fig. 126.



PLANE GEOMETRY,

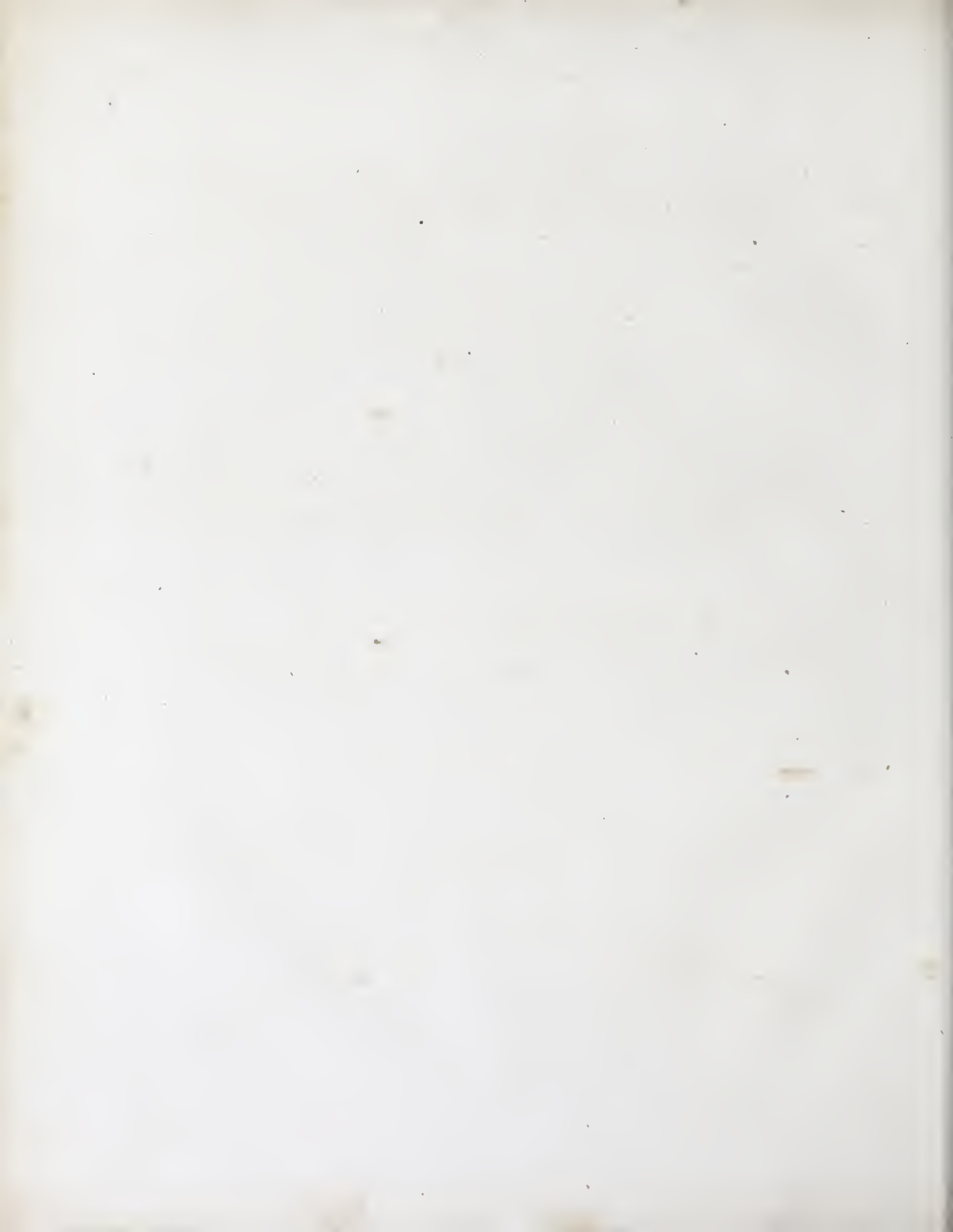
INACCESSIBLE LINES.

PLATE. II.



Invented by P. Nicholson

London, Published by John Day & Co. Oct^r 20. 1822.



PROBLEM LIX.

Given any number of equidistant points in the arc of a circle, to draw a line through any one of these points which shall be a tangent to the arc at that point, without making use of the centre of the circle.

Let M, s, u (Fig. 6, Plate II, inaccessible lines) be any three points, of which the middle one s is equidistant from each of the other two, u and M , and let it be required to draw a line through the point M that shall be a tangent to the curve. Join Mu , and from M with the radius Ms describe an arc rst cutting Mu at r . Make st equal to sr , and draw Mt ; then Mt is the tangent required (22).

PROBLEM LX.

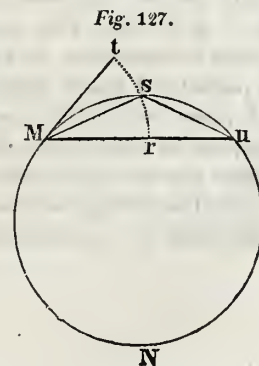
To draw lines through any given points in or out of an arc of a circle that shall tend to the centre, without knowing where the centre is.

Find the equidistant points $A, v, e, f, B, u, s, M, C$ in the arc, by Problem LV, at such a distance as to be within the reach of the instrument now about to be described; but before we can apply this instrument conveniently, it will be necessary to draw lines through the points v, e, f, B , &c. tending to the centre: for this purpose, bisect the distance between every two points which has only one intermediate point between them by a perpendicular, as vf by the perpendicular wh , and eB by the perpendicular xk ; wh will pass through the point e , and xk through the point f : the two perpendiculars thus drawn to their imaginary chords will tend to the centre.

Draw any line igq cutting xk at i , and wh at g , and draw ip parallel to gh : make the angle qgn equal to the angle kip ; then place several straight edges together in such a manner that the edge of one of them may be upon the line iq , another upon the line gh , and a third upon the line gn ; then fasten these together at the angle in which they are placed as one machine: put a pin in the point g , and another in the point i , and move

(22) DEMONSTRATION.

Let Msu be the arc of a circle, and let it be bisected in s , and join Ms, su . Draw the chord Mu ; from M with the distance Ms describe the arc rst , cutting Mu in r . Make st equal to sr , and join Mt ; then, because the arc st is equal to the arc sr , the angle sMt is equal to the angle sMr ; that is, equal to the angle sMu ; and because sM is equal to su , the angle sMu is equal to the angle suM . But sMt is the angle made by the chord Ms and the tangent Mt , and Mus is the angle in the alternate segment; therefore Mt is a tangent to the circle.



the instrument so that the edge gn may slide upon the pin at g , and the edge gi upon the pin at i ; then, when the edge gh falls upon any intermediate point between e and f , stop the motion of the instrument, and draw a line along the edge gh ; the line thus drawn will tend to the centre (23).

PROBLEM LXI.

Given any two lines tending to the centre of a circle, to describe an arc of the same which shall pass through a given point in one of the lines, and shall have a given chord, and the point of intersection of the two lines as a centre.

Let AB , CD (*Plate III, Fig. 1 and 2* inaccessible lines) be the two lines tending to the centre, and let C be the point through which the arc is to pass.

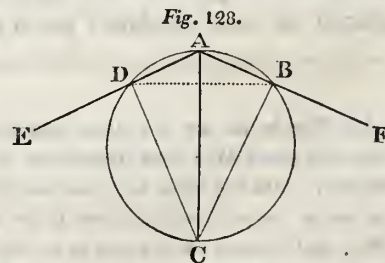
Draw AC , by Problem L, so as to make the angles DCA and BAC equal to each other (24). Divide AC into any convenient number of equal parts, as 3 in CA : make CE equal to a third of CA . Draw EG parallel to AB , meeting AC in E . From G , with the radius GC , describe the arc $ECHF$. From the point C , with a radius equal to a third of the extension of the given chord, describe an arc cutting the arc CHF at F . Join CF , and produce CF to I , making CI equal to three times CF . Bisect CI in K , and draw KL perpendicular to CI ; also bisect the arc CHF in H . Through the points C and H draw the straight line CL ; then, by Problem LVI, describe the arc of a circle, of which CI is its chord, and KL its versed sine, and the thing required will be done.

Fig. 1 has its centre at so very remote a distance, that the lines are not so intelligible for description as *Fig. 2*; but it is here shown on account of its being as likely to happen in practice as that exhibited in *fig. 2*.

(23) DEMONSTRATION.

The angle CBD is equal to the angle CAD , because they stand upon the same arc DC ; and the angle CDB is equal to the angle CAB , because they stand upon the same arc BC ; therefore, if D , C , B , be three fixed points, the angles CAD and CAB will always continue to be the same wherever the point A is situated in respect of A and B ; therefore, if AE , AC , AF , be three inflexible rulers always at the same angles, if the ruler AE slide upon the point D , and AF upon the point B , the middle ruler AC will always pass through the point C .

(24) This Problem may be very neatly performed, as in *Fig. 3*. Draw CE parallel to AB , and bisect the angle DCE by the straight line CA , and CA will make equal angles with AB and CD .

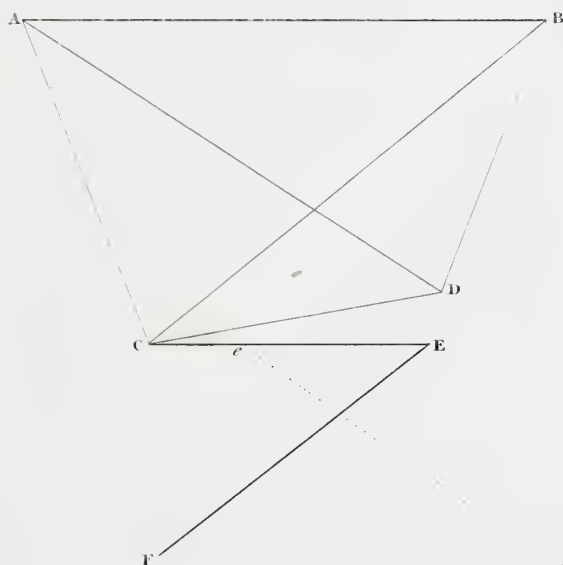
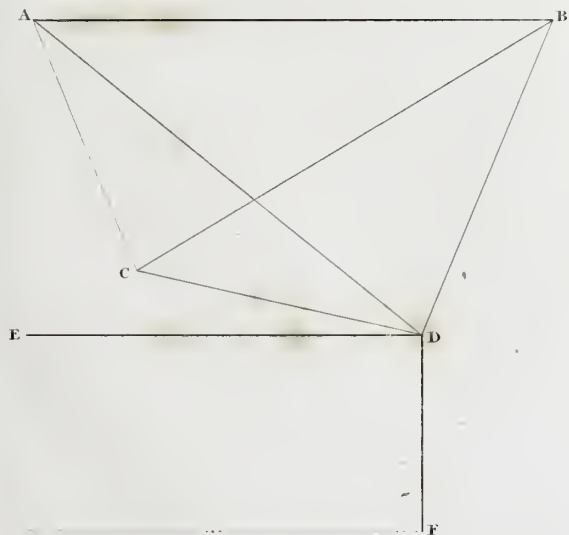
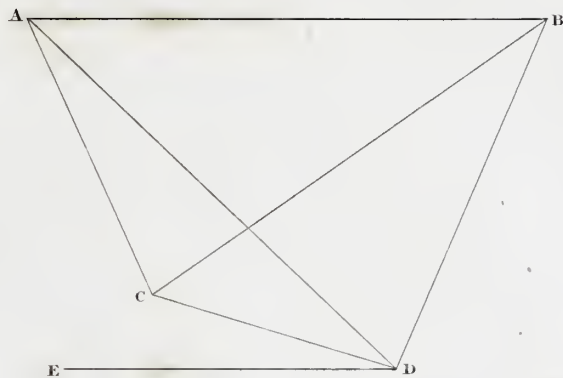
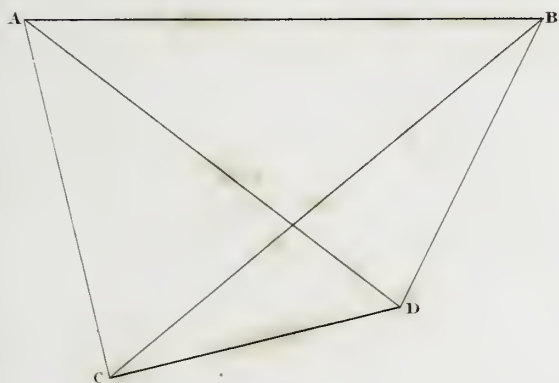
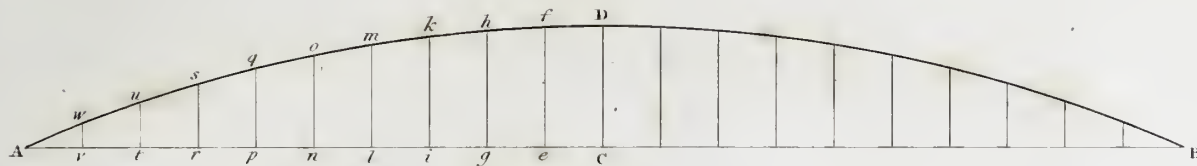


PLANE GEOMETRY,

INACCESSIBLE LINES.

PLATE III.

Fig. 1.



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PLANE GEOMETRY.

INACCESSIBLE LINES.

PLATE III.

Fig. 1.



Fig. 3.

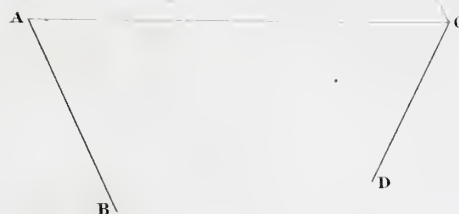
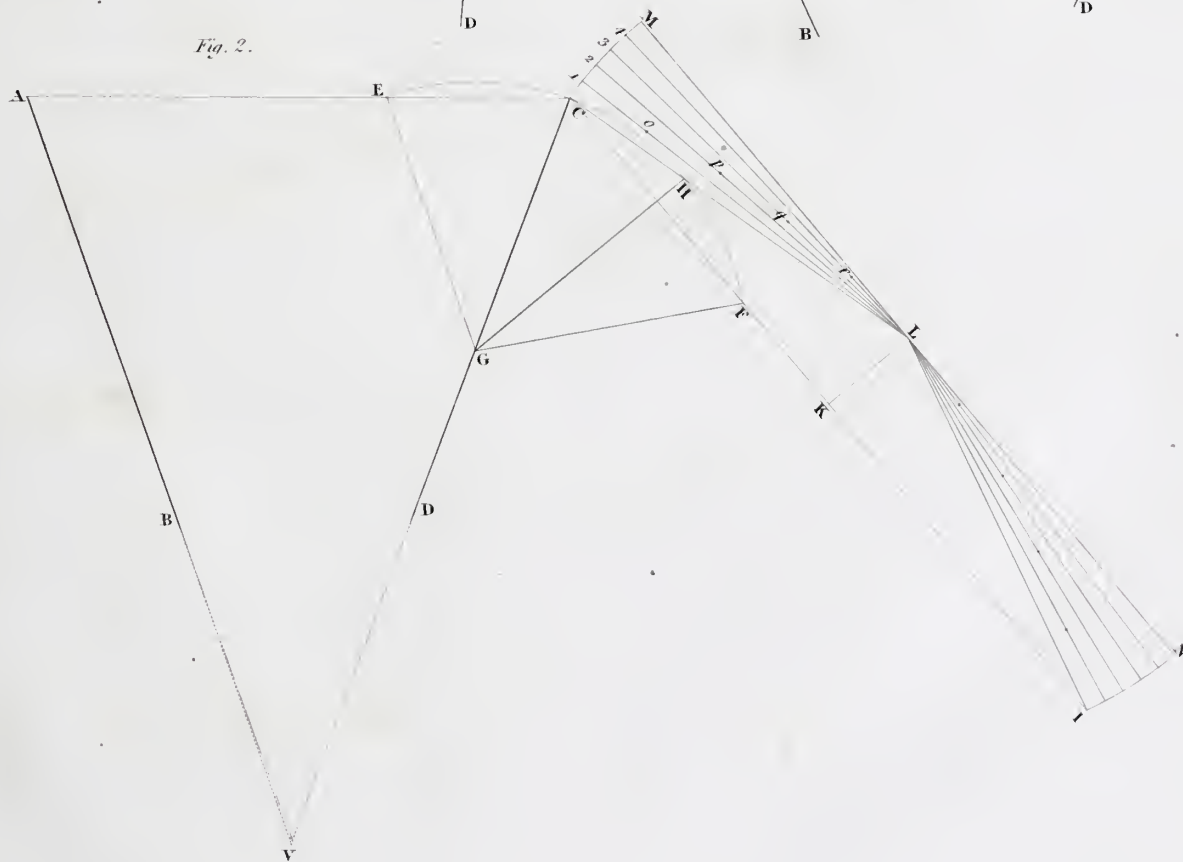


Fig. 2.



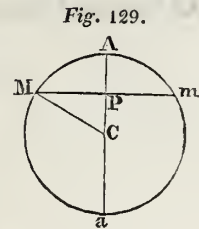
ARITHMETICAL OPERATIONS

RESPECTING THE CIRCLE.

PROBLEM I.

Given the chord and versed sine of the arc of a circle, to find the radius that will describe that circle.

Let Mm be a chord, and AP the versed sine, and let $AMam$ be the entire circle, C its centre. Produce AP to a : then, by the *Elements of Geometry*, Theorem xlii, page 11, $AP \times Pa = PM \times Pm$; but because $PM = Pm$, therefore $AP \times Pa = PM^2$.



Let $r =$ the radius $CM = CA$, $CP = x$, $PM = y$, $AP = v$:

then will $Pa = 2r - v$, and $(2r - v)v = y^2$

that is $2rv - v^2 = y^2$

therefore $2rv = v^2 + y^2$

whence $\therefore r = \frac{v^2 + y^2}{2v}$ which gives the following

RULE.

Divide the squares of the half chord and that of the versed sine by twice the versed sine, and the quotient is the radius.

EXAMPLE.

Required the radius of a circle that will describe an arc of which the chord is 16 feet and the versed sine 5 feet.

$$\begin{array}{l}
 64 = \text{the square of 8, the half chord} \\
 25 = \text{the square of 5, the versed sine} \\
 2 \times 5 = 10 \overline{)89} = \text{the sum of the squares} \\
 8.9 \text{ the radius required} \\
 \text{or 8 feet } 10\frac{1}{2} \text{ inches radius.}
 \end{array}$$

PROBLEM II.

The radius and half chord of the segment of a circle being given, to find the versed sine of the arc of that segment.

The same letters representing the same parts as in the immediately preceding problem,

the equation of the curve is $y^2 = 2rv - v^2$,

or, by transposition, $v^2 - 2rv = -y^2$

completing the square, $v^2 - 2rv + r^2 = r^2 - y^2$

and extracting the root, $v - r = \mp \sqrt{r^2 - y^2}$

therefore $v = r \mp \sqrt{r^2 - y^2}$, which gives the following

RULE.

From the radius subtract the square root of the difference of the squares of the radius and that of the half chord, and the remainder is the versed sine.

EXAMPLES.

Ex. 1. Required the versed sine to the segment of a circle of which the chord is 12 feet, and the radius of the circle 250 feet.

$ \begin{array}{r} 250 \text{ radius} \\ 250 \\ \hline 12500 \\ 50 \\ \hline 62500 \text{ square of the radius } 250 \\ 36 \text{ square of the 6, the half chord} \\ \hline 62464 (249 \cdot 927 \\ 4 \\ \hline 44) \overline{224} \\ 176 \\ \hline 489) \overline{4864} \\ 4401 \\ \hline 4989) \overline{46300} \\ 44901 \\ \hline 49982) \overline{139900} \\ 99964 \\ \hline 499847) \overline{3993600} \\ 3498929 \\ \hline 494671 \end{array} $	$ \begin{array}{r} \text{from radius } 250 \\ \text{subtract } 249 \cdot 927 \\ \hline 0 \cdot 073 \text{ the versed sine in decimals} \\ 12 \\ \hline 876 \\ 8 \\ \hline 7 \cdot 008, \text{ so that the versed sine required is about } \frac{7}{8} \text{ of} \\ \text{[an inch.} \end{array} $
---	--

Ex. 2. Supposing the radius of the plan of a circular bow in a building to be 8 feet $10\frac{3}{4}$ inches, and that that bow contains windows each four feet wide; how far will the arc of the sash-frame project from its chord?

$ \begin{array}{r} 8 \text{ f. } 10\frac{3}{4} \text{ in.} \\ 12 \\ \hline \text{radius } 106 \cdot 75 \text{ reduced to inches and tenths.} \\ 106 \cdot 75 \\ 53375 \\ 74725 \\ 64050 \\ 10675 \\ \hline 108195625 (104 \cdot 017 \\ 1 \\ \hline 204) \overline{0819} \\ 816 \\ \hline 208 \cdot 01) \overline{35625} \\ 20801 \\ \hline 208 \cdot 027) \overline{1472400} \\ 1456189 \\ \hline 16211 \end{array} $	$ \begin{array}{r} 106 \cdot 75 \\ 104 \cdot 017 \\ \hline 2 \cdot 733 \\ 8 \\ \hline 5 \cdot 864, \text{ versed sine as required.} \end{array} $
---	---

So that the versed sine, or the projection of the sash, is very nearly $2\frac{3}{4}$ inches.

Ex. 3. Required the versed sine to a segment of which the chord is 200 feet, and the radius of the arc 250 feet.

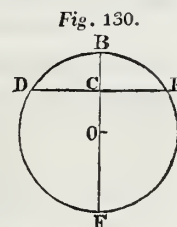
$$\begin{array}{r}
 250 \\
 250 \\
 \hline
 12500 \\
 500 \\
 \hline
 62500 \\
 10000 \\
 \hline
 52500(229 \cdot 128 \\
 4 \\
 42) \overline{) 125} \\
 \underline{84} \\
 449) \overline{) 4100} \\
 \underline{4041} \\
 4581) \overline{) 5900} \\
 \underline{4581} \\
 45822) \overline{) 181900} \\
 \underline{91644} \\
 458248) \overline{) 4025600} \\
 \underline{3665984} \\
 359616
 \end{array}
 \qquad
 \begin{array}{r}
 250 \cdot 0 \\
 229 \cdot 128 \\
 \hline
 20 \cdot 872, \text{ versed sine as required.}
 \end{array}$$

PROBLEM III.

Given the radius of a circle, and the versed sine of a segment of that circle, to find the distance between the centre of the circle and the chord of the segment.

Let DBEF be the circle, O its centre, DE a chord, C the middle of the chord, and BC the versed sine; then, if BC be produced to F, BF will be a diameter, and OC will be the distance between the centre O, and the chord DE and OB the radius of the circle.

Now $OC = OB - BC$.



RULE.

From the diameter subtract the versed sine, and the remainder is the distance between the centre of the circle and the chord.

EXAMPLE.

Suppose the radius of a circle to be 250, and the versed sine $20 \cdot 872$; required the distance of the chord from the centre.

$$\begin{array}{r}
 250 \cdot 0 \\
 20 \cdot 872 \\
 \hline
 229 \cdot 128, \text{ the distance between the centre and the chord, as required.}
 \end{array}$$

In the segment of a circle, any line parallel to the versed sine contained between the arc and the chord is called an *ordinate*, and the distance of that ordinate from the versed sine is called the *abscissa*.

PROBLEM IV.

Given the radius of a circle, the distance from the centre to a chord of that circle, and the distance of an ordinate in the segment from the versed sine, to find the length of the chord.

Let DBEF be a circle, O its centre, and DE a chord of that circle, C the middle of the chord, and PM the ordinate.

Draw OQ parallel to DE, and produce MP to Q.

Again, let $r = OB = OM$, the radius of the circle,

$d = OC = QP$, the distance of the chord from the centre,

$x = CP = OQ$, the abscissa,

and $y = PM$, the ordinate; then will $QM = d + y$.

By *Geometry*, Theorem xxxv, $QM^2 = OM^2 - OQ^2$;

wherefore $(d + y)^2 = r^2 - x^2$;

therefore $d + y = \pm \sqrt{r^2 - x^2}$;

whence . . . $y = \pm \sqrt{r^2 - x^2} - d$,

from which we have the following

RULE.

From the square root of the differences of the squares of the radius and of the abscissa subtract the distance from the centre of the circle to the middle of the chord, and the remainder is the ordinate.

EXAMPLE.

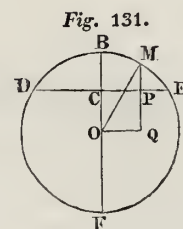
Given the radius of a circle 250 feet, the distance between the centre and a chord 229·128 feet, to find an ordinate of the segment at the distance of 10 feet from the versed sine.

$$\begin{array}{r}
 250 \\
 250 \\
 \hline
 12500 \\
 50 \\
 \hline
 62500 = r^2 \\
 100 = x^2 \\
 \hline
 62400 \sqrt{249 \cdot 799} \\
 4 \\
 44) 224 \\
 \hline
 176 \\
 489) 4800 \\
 \hline
 4401 \\
 4987) 39900 \\
 \hline
 34909 \\
 49949) 499100 \\
 \hline
 449541 \\
 499589) 4955900 \\
 \hline
 4496301 \\
 459599
 \end{array}$$

The square root of the differences of the squares of the radius and the abscissa being now found to be 249·799
subtract 229·128

and the remainder 20·671
is the ordinate required.

Upon the principles of these calculations the largest circles may be drawn, by calculating a sufficient number of ordinates.



EXAMPLE.

Let it be required to construct a segment of a circle of which the chord is 200 feet, and the radius of the arc 250 feet.

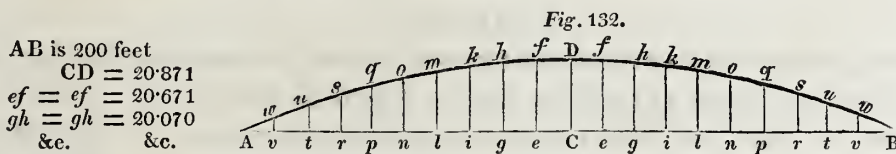
The method of doing this will be to find a sufficient number of points in the curve not exceeding the length of a board; say 12 feet. In this case we may calculate the ordinates for one half of the curve at about 10 feet distant from each other, in order that a twelve feet board may reach between any two adjacent points: this board must be curved on one of its edges to the arc of the circle required; then the curved edge, with its concavity towards the centre, being successively applied to every two adjacent points, and drawing the curve between them at every application on the surface, the entire arc will be described. The versed sine of the arc on the edge of the board will be found by Problem ii, and this arc being within reach, it may be described by Problem lvii, *Plane Geometry*.

Here follows the calculation of the ordinates at 10 feet distant from each other, the middle one being at the head of the column, and the others in succession towards one of the extremes.

<i>Feet.</i>	
20·871 versed sine	
20·671 first ord.	
20·070 second ord.	
19·065 third ord.	
17·651 fourth ord.	
15·820 fifth ord.	
13·565 sixth ord.	
10·873 seventh ord.	
7·726 eighth ord.	
4·110 ninth ord.	

In the first place, the versed sine of the entire arc must be found, by Problem II, the operation being performed in *Ex. 3*, p. 63; the result, 20·871 feet, is the versed sine, which subtracted from the radius gives 229·128, the distance between the centre and the chord; and having this distance between the centre of the circle and the middle of the chord, the radius of the circle and the distance of an ordinate in the segment from the middle of the chord, any ordinate is found by Problem iv. The calculation of the first ordinate at 10 feet distant from the middle of the chord is exhibited in *Ex. p. 64*.

Here follows the segment drawn according to this calculation.



The following Table exhibits the versed sines for the segments of circles which have chords of 10 feet each, from a radius of 10 feet to 110, increasing successively by unity; and from 110 to 132 feet, increasing successively by 2 feet at a time; and from 200 to

300 feet, increasing successively by 10 feet. To make this table useful, it would be better to calculate the ordinates at 9 feet distant.

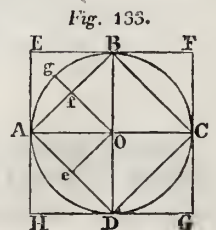
Rad.	Versed sine.		Rad.	Versed sine.		Rad.	Versed sine.		Rad.	Versed sine.				
ft.	ft.	inches.	ft.	inches.	ft.	ft.	inches.	ft.	ft.	inches.	ft.	inches.		
10	1	4.07800	43	3.50028	76	1.97592	109	1.37688	In the following the difference of the radii is 2 feet.					
11	1	2.54580	44	3.42024	77	1.95012								
12	1	1.09548	45	3.34380	78	1.92516								
13	1	0.00000	46	3.27060	79	1.90068								
14	0	11.07972	47	3.20064	80	1.87692								
15	0	10.29444	48	3.13356	81	1.85364	110	1.36461	In the following the difference of the radii is 10 feet.					
16	0	9.61584	49	3.06924	82	1.83108	112	1.34004						
17	0	9.02316	50	3.00756	83	1.80888	114	1.31652						
18	0	8.50080	51	2.94828	84	1.78740	116	1.29372						
19	0	8.03640	52	2.89140	85	1.76628	118	1.27176						
20	0	7.621	53	2.83656	86	1.74576	120	1.25064	In the following the difference of the radii is 10 feet.					
21	0	7.24716	54	2.78376	87	1.72632	122	1.23012						
22	0	6.90864	55	2.73300	88	1.70604	124	1.21020						
23	0	6.60072	56	2.68404	89	1.68684	126	1.19100						
24	0	6.31944	57	2.63676	90	1.66706	128	1.17240						
25	0	6.06132	58	2.59104	91	1.632	130	1.154	In the following the difference of the radii is 10 feet.					
26	0	5.82355	59	2.54700	92	1.630	132	1.136						
27	0	5.60412	60	2.50440	93	1.613	In the following the difference of the radii is 10 feet.							
28	0	5.40060	61	2.46324	94	1.59696								
29	0	5.21148	62	2.42340	95	1.58376								
30	0	5.03532	63	2.38476	96	1.56360	In the following the difference of the radii is 10 feet.							
31	0	4.87068	64	2.34744	97	1.54752								
32	0	4.71648	65	2.31120	98	1.53168	200	0.75000				In the following the difference of the radii is 10 feet.		
33	0	4.57200	66	2.27604	99	1.51620	210	0.71448						
34	0	4.43592	67	2.24196	100	1.50096	220	0.68196						
35	0	4.30788	68	2.20896	101	1.48608	230	0.67836						
36	0	4.18704	69	2.17680	102	1.47156	240	0.65016						
37	0	4.07280	70	2.14560	103	1.45728	250	0.60000	In the following the difference of the radii is 10 feet.					
38	0	3.96468	71	2.11536	104	1.44324	260	0.56784						
39	0	3.84220	72	2.08596	105	1.42944	270	0.55572						
40	0	3.76488	73	2.05728	106	1.41600	280							
41	0	3.67236	74	2.02944	107	1.40268	290							
42	0	3.58448	75	2.00232	108	1.38972	300	0.49890						

PROBLEM V.

To construct the plan of a circus or semicircular crescent, or quadrant, to a given radius.

Let ABCD be a circle, O its centre, and let ABCD be a square inscribed in the circle, and EFGH a square circumscribing the circle.

Now, the inscribed square ABCD is half the circumscribing square EFGH, and therefore OeAf, which is a fourth part of the inscribed square, is the half of OAEB, the square of the radius; therefore the square of the half chord Af is half the square of the radius. Whence the radius and the half chord being given,



the versed sine fg will be found, by Problem ii, p. 61, and the segment AgB may be constructed by ordinates, by Problem iv, p. 64.

EXAMPLE.

Let it be required to draw the line of front for a circus of 300 feet radius.

Here, by availing ourselves of the construction of the figure and its properties, the difference of the squares of the radius and that of the half chord is half the square of the radius, or $\frac{1}{2}(300)^2 = 45000$ feet.

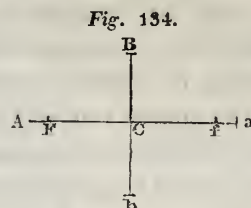
The square root of 45000 is 212.132 feet; this subtracted from the radius 300 feet, gives 87.868 feet for the versed sine of each quadrant; and 212.132 is the distance of each chord from the centre of the circle; and lastly, having the radius of the circle, and the distance of each chord from the centre, we may find as many ordinates in each of the four segments, by Problem iv, p. 64, as we please.

CURVES, ELLIPSE.

PROBLEM I.

The two axes Aa and Bb being given in position and magnitude, to find the focii.

From the extremity B of the axis minor, with the distance CA or Ca of the semi-axis major, describe an arc meeting the axis major Aa in F, f; and the points F, f are the two focii.



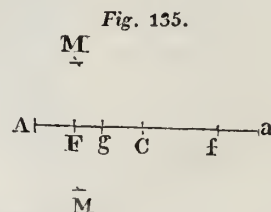
DEMONSTRATION.

For the distance of the focii from the centre is the leg of a right angled triangle, of which the other leg is the semi-axis minor, and the hypotenuse the semi-axis major; therefore, the distance of either focus from the semi-axis minor is equal to the semi-axis major, and consequently the curve is an ellipse.—See *Appendix*, Curve Lines, Prop. ii.

PROBLEM II.

Given the axis major Aa and the two focii F, f, to find any point in the curve of an ellipse.

Between Ff take any point g; from F with the radius Ag describe an arc at M, and from f, with the radius ag, describe another arc, meeting the former at M, and the point M is in the curve.



DEMONSTRATION.

The reason of the method for finding any point M in the curve, is because the sum of the two lines drawn from the focii to any point in the curve is equal to the axis major.—See Curve Lines *Appendix*, Proposition i, page 2.

SCHOLIUM.

It is evident that not only the same radii might have been employed in finding the point M' on the other side of the axis major, but that they might have been used in finding two other points at the same distance from the centre; so that by the two distances Ag, ga, four points in the curve might have been found.

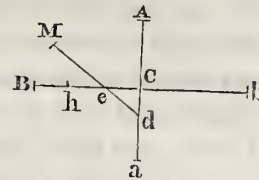
PROBLEM III.

Given the two axes Aa, Bb, of an ellipse, in position and magnitude, to find any point in the curve.

METHOD 1.

Make Ch equal to CA. In Ca take any point d, so that Cd may be less than Bh. From d, with the radius Bh, describe an arc meeting BC in e. Produce de to M, and make eM equal to CA, or dM equal to BC, and the point M will be in the curve.

Fig. 136.

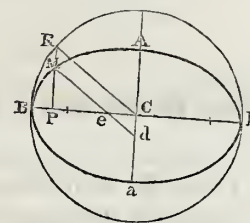


DEMONSTRATION.

For upon Bb describe the semicircle BRb, and through the point M draw PR parallel to CA meeting Bb in P.

Then, because de is equal to the difference of the two semi-axes, and eM equal to CA the lesser axis, dM is equal to BC the greater; and therefore dM is equal to CR.

Fig. 137.



Now, by similar triangles, PRC, PMe, . . . $PR \times Me = RC \times PN$;

and since . . . $RC = BC$,

and . . . $AC = Me$;

therefore, by eliminating RC, Me, there will result $PR \times AC = BC \times PM$; wherefore $PR : PM :: BC : AC$; and consequently (Prop. vi, Curve Lines, *Appendix*) the curve is an ellipse.

SCHOLIUM.

Upon this principle an instrument may be made to describe the curve by continued motion; for if dM be conceived as an inflexible line, and that the points d, e, M are fixed in this line, supposing the point e to be compelled to move in the line Bb, and the point d in Aa, and the point M to be moved from B to A; then the point M will describe the curve BAbA of the ellipse.

This instrument may be effected by making grooves in the plane of description instead of the lines Aa, Bb, of the position of the axis; and instead of the inflexible line dM, a rod with three projecting cylindric pins, which must have their diameters equal to the breadth of the grooves, so that, when the pins are inserted in the grooves, their axes may

be perpendicular to the plane of description, and their lower ends in a straight line, and at the same respective distances from each other that the points d , e , M are.

Such an instrument is called a *trammel*, and is generally made of two rectangular bars, with a groove in each, so fixed together, that when laid on the plane of description the grooves may be at right angles to each other, and the bottom of each groove parallel to the plane of description and their other two sides perpendicular to it, and that the upper surface from which the groove recedes may be a plane parallel to the plane of description. The rod may be contrived with two moveable pieces called *nuts*, with one of the cylindric pins fixed in each nut, and a pencil fixed at the remote extremity, so that the ends of the pins may be at the same distance from the under edge of the rod: but the pencil must be of sufficient length to reach the plane of description. Carpenters construct their trammels of wood; but the transverse piece is generally made entirely upon one side of the other, so as to describe a semi-ellipse at a time as they are mostly employed in the description of elliptic arches, or in less portions than the entire curve.

PROBLEM IV.

Given the two axes in position, the vertex A of the one, and another point M in the curve, to find the limits of the other axis.

From M , with the radius AC of the semi-axis given, describe an arc meeting the other unlimited axis at e , and let d be the point where Me either meets the axis Aa between the points M , e , as in No. 1, or, being produced, meets it beyond e , as in No. 2. Make CB , Cb each equal to dM , and Bb is the other axis, whether major or minor.

Fig. 138, No. 1.

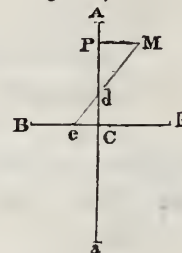
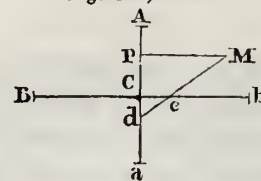


Fig. 138, No. 2.



Aa is the axis major when the point d falls between the centre and one extremity A ; but if it falls beyond C , it is the axis minor.

This is too evident to require a demonstration.

Cor. 1. Hence, since de is the difference of the semi-axis, any point in the curve may be found without finding the other axis.

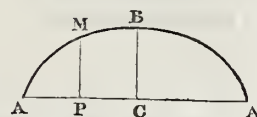
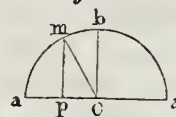
For taking the point d at pleasure, so that the distance Cd may be less than the given difference, from the point d as a centre, with a radius equal to the same difference, describe an arc meeting Bb at e , and produce de to M , and make eM equal to AC ; then M will be a point in the curve.

PROBLEM V.

Upon a given straight line AA as an axis, to find any point in the curve, the length of the other axes or semi-axis being known.

Draw the straight line AA equal in length to the other axis which is only given in length and not in position. Upon aa, as a diameter, describe the semicircle amba. Bisect AA in C and aa in c. Divide CA in P and ca in p, so that $CA : CP :: ca : cp$. Draw CB and PM perpendicular to AA, also cb and pm perpendicular to aa. Make CB equal to cb, and PM equal to pm, and M is a point in the curve answering to the axis AA and the semi-axis CB.

Fig. 139.



DEMONSTRATION.

Let $CA = a$, $CB = ca = cb = b$, $PM = pm = y$, $CP = x$, and $cp = v$.

By the right angled triangle cpm (Th. xxxv, *Geom.*), $v^2 = b^2 - y^2$

and by construction, $a : x :: b : v$ $\therefore b^2 x^2 = a^2 v^2$

wherefore, eliminating v from these two equations, there will result $a^2(b^2 - y^2)$, which is an equation to the ellipse.

It makes no difference to the demonstration whether it is the axis major or minor that is given in position.

PROBLEM VI.

To trace or draw the curve of an ellipse, or that of any other figure, through points given or found in the curve.

METHOD 1.

Join two adjacent points by an arc drawn by the eye; join one extremity of this arc to the next point, in the same manner; and the last extremity to the next point in succession by another arc, and so on till the two ends of the line meet, and form one continued line, or the required length; and the curve thus drawn will be the more accurate as the perception of the eye is quick in its judgment of the figure, and the hand that draws is steady. The greater the number of points and nearer the points are at equal distances, the easier the curve will be drawn.

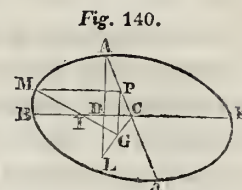
METHOD 2.

Put in pins in all the points, and bend a flexible elastic slip of wood or metal round the convex or outside of the pins; then, when the side of the slip comes in contact with every pin, draw a curve on the inner or concave side; and if the slip is not of sufficient length, repeat the operation as often as may be found necessary, till the curve forms one continued line, or an arc of the length intended.

PROBLEM VII.

Given two diameters in position and magnitude, to find any point in the curve.

From the vertex A of the semidiameter AC, draw AL perpendicular to the semiconjugate BC, cutting it in D. Make AL equal to BC, and join CL. From any point G in CL with a radius DL describe an arc meeting BC in I; join GI, and produce it to M. Make GM equal to LA; then will M be a point in the curve.



DEMONSTRATION.

For join PG. Let $GP = v$, and because $AL = BC = MG$ by construction, and $BC = b$ by notation, $BC = AL = MG = b$.

Because of the right angled triangle MPG, ... $v^2 = b^2 - y^2$

and of the similar triangles CAL, CPG, ... $b^2x^2 = a^2v^2$;

wherefore, eliminating v , there will arise $b^2x^2 = a^2(b^2 - y^2)$, which is the equation of the co-ordinates of the ellipse.

SCHOLIUM.

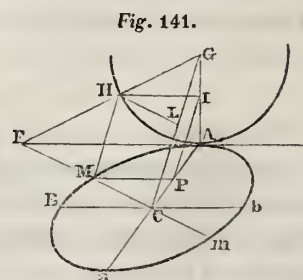
Upon this principle the curve may be described by the continued motion of a point. For suppose GIM to be an inflexible line, or the straight edge of a ruler, and G, I, M to be fixed points in it: then, suppose the point M to be moved, so that the point I may be always in the line Bb, and the point G in the line LC, the point M will describe the curve of the ellipse, or as much of it as may be found necessary.

A machine so constructed is called an *oblique trammel*.

PROBLEM VIII.

Given any two diameters Aa, Bb in position and magnitude, to find the points where a line drawn through the centre in a given position meets the curve.

Through the vertex A of the given diameter Aa, draw FA parallel to the other given diameter Bb, and draw AG perpendicular to AF. Make AG equal to BC. From G, with the radius GA, describe the arc AH. Draw FG meeting the arc AH in H. Join GC, and draw HM parallel to GC, meeting Fm in M. Make Cm equal to CM, and M, m are the points required.



DEMONSTRATION.

If Mm is a diameter, the point M is in the curve. To prove this, draw MP and HI parallel to BC , MP , meeting CA in P , and HI meeting AG in I ; and draw LH parallel to CM , meeting CG in L , and join PI .

Let $CF = m$, $CM = LH = n$, $GF = p$, $GH = q$, $CA = a$, $CP = x$, $HG = AG = CB = b$, $GI = v$, and $IH = PM = y$.

$$\begin{array}{l} \text{By similar triangles } \left\{ \begin{array}{ll} \text{PCM, ACF} & m^2x^2 = a^2n^2 \\ \text{IGH, AGF} & b^2q^2 = p^2v^2 \\ \text{LHG, CFG} & p^2n^2 = m^2q^2 \end{array} \right. \\ \text{and by the right angled triangle GIH} & v^2 = b^2 - y^2 \end{array}$$

Whence, eliminating m, n, p, q, v , we have $b^2x^2 = a^2(b^2 - y^2)$, which is the equation of the co-ordinates of an ellipse.

PLATE I.—Ellipse, exhibiting examples of the various methods of describing the curve by points and by continued motion.

Ex. 1.—In *fig. 1*, the axis Aa, Ab are given in position and magnitude, to describe the curve.

Find the focii F, f , by Prob. i; taking the points g, h, i, k , &c. at pleasure, between F, f , by Prob. ii, find the points in the curve, viz.— m, m from g, m', m' from h, m'', m'' from i , &c. and through all the points $m, m', m'',$ &c. draw the curve, by Prob. iii.

Ex. 2.—In *fig. 2*, the axes are given in position and magnitude, to describe the curve without the focii.

Here, on the edge of an ivory rule or thick slip of paper, mark the distance of the points m, o , equal to the semi-axis major, and the distance of the points m, n equal to the semi-axis minor. Place the point n in the axis major Aa , and the remote point o in the line Bb of the axis minor; then mark the plane of description at m , and m is a point in the curve. Having, in this manner, found a sufficient number of points, draw the curve by Problem vi.

Ex. 3.—*Fig. 3* exhibits the drawing or tracing the curve of an ellipse by means of the trammel described in Scholium, Prob. iii. Having set the distance mn from the pencil to the first sliding point equal to CB the semi-axis minor, and mo equal to the semi-axis major; then the sliding points being in the grooves move the end R , and the pencil at m will describe the curve.

Ex. 4.—*Fig. 4* and *5* exhibit the method of drawing the curve when the semi axis CA and an ordinate PM or Pm is given. Having drawn the unlimited axis Bb in its right angled position to AC, and having, on the edge of an ivory rule or slip of thick paper, made the distance *mo* equal to the given axis AC, place the point *m* or the edge of the

rule upon the given point m in the curve, and move the point Q so, that the point o in the edge of the rule may fall on the unlimited axis Bb . Mark the point n on the edge of the rule where it meets the line Aa of the given axis; then any point in the curve will be found, as in Example 2, *fig. 2*.

Ex. 5.—*Fig. 6* exhibits the method of describing a semi-ellipse upon a given axis Aa , according to the principle in Problem v. Draw any angle ACH , No. 3. Make CA equal to the semi-axis CA , No. 1, and CH equal to the other semi-axis which is only given in length. In CA take any number of points $P, P', P'', \&c.$ Join AH , and draw the lines $Pp, P'p', P''p'', \&c.$ parallel to AH , meeting CH in the points $p, p', p'', \&c.$ In the semi-axis Ca , No. 1, make $CP, CP', CP'', \&c.$ respectively equal to $CP, CP', CP'', \&c.$ No. 3. At any convenient place No. 2, draw the straight line GH . From any point c in GH , with the radius cH , No. 3, describe the semicircle GbH . Make $cp, cp', cp'', \&c.$ respectively equal to $Cp, Cp', Cp'', \&c.$ No. 3. In No. 1, draw the straight lines $CB, PM, P'M', \&c.$ perpendicular to Aa ; and in No. 2, draw $cb, pm, p'm', \&c.$ perpendicular to GH , meeting the circular arc at $b, m, m', \&c.$ In No. 1, make $CB, PM, P'M', \&c.$ respectively equal to $cb, pm, p'm', \&c.$, and the points $B, M, M', \&c.$ will be in the curve, by Prob. v. The curve itself is drawn by Prob. vi.

Fig. 7 exhibits the application of the oblique trammel; to the description of an ellipse of which, the two diameters Aa, Bb are given in position and magnitude.

From the vertex B of the minor or less diameter, draw BD perpendicular to the greater Aa , meeting it in E . Make BD equal to AC , and join CD : then the moving points of the trammel rod from the pencil or describing point being set to the distances BE and BD , the curve may be described as shown, the grooves of the trammel being in the lines Aa, CD .—See the principle, Prob. vii.

Fig. 8 is an example of the application of Prob. viii to finding the points where a line drawn through the centre in a given position meets the curve from the diameters Aa, Bb given in position and magnitude. Through B , draw FG parallel to Aa , and draw BE perpendicular to FG , and make BE equal to AC . From E , with the radius EB , describe the arc HBI . Produce mM to F , and join FE , meeting the arc HBI at H . Join CE , and draw MH parallel to CE ; make Cm equal to CM , and the points M, m are the vertices of the diameter Mm ; so that the position of three diameters being given, of which two are conjugate to each other, and of a given magnitude, the vertices of the third will be found.

In this manner we may find the extremities of as many unlimited diameters given in position as we please, and a curve being drawn through their vertices will form the ellipse.

CURVE LINES.

ELLIPSE.

PLATE 1.

Fig. 1.

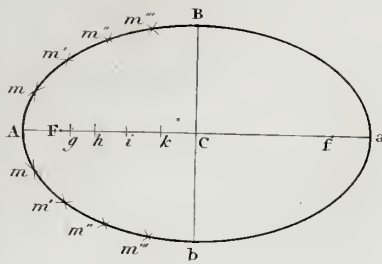


Fig. 2.

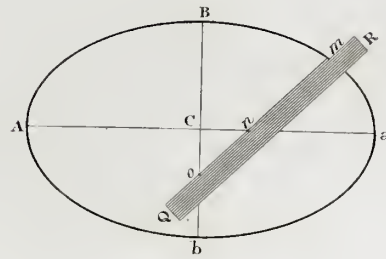


Fig. 3.

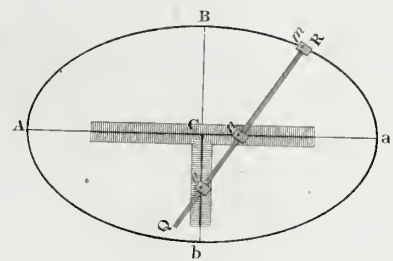


Fig. 4.

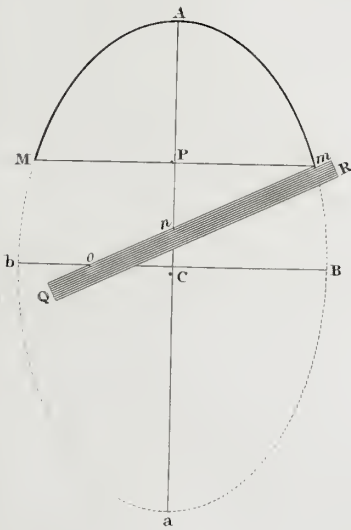


Fig. 5.

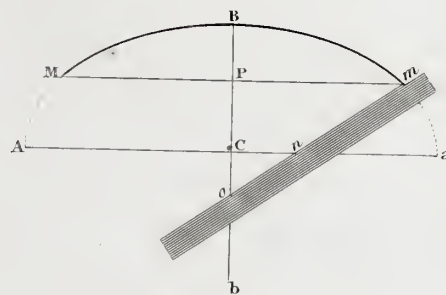


Fig. 6.

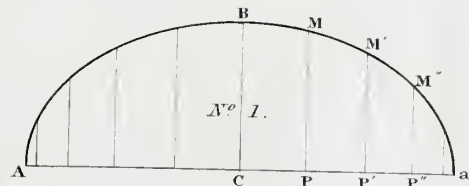
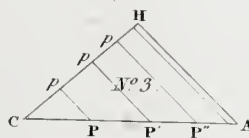
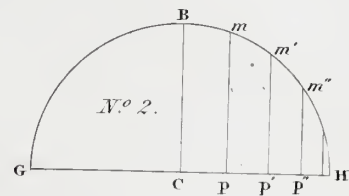


Fig. 7.

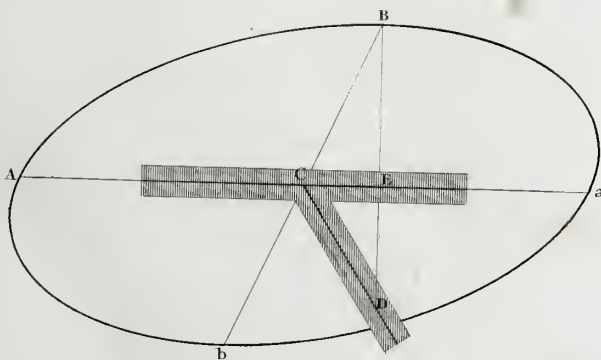
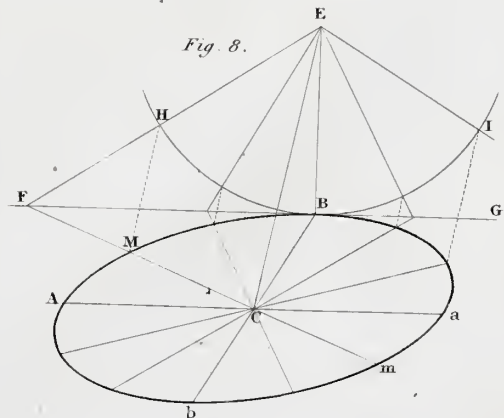


Fig. 8.



PROBLEM IX.

An ellipse being given, to describe a concentric ellipse through a given point in one of the semi-axes.

Join AB (*fig. 1, plate 3, Curve Lines*), the two extremities of the semi-axes CA, CB, and through the given point E in CA draw ED parallel to AB, meeting CB in D; then with the semi-axes CE, CD, by Prob. iii, find a sufficient number of points, and trace the curve through them, by Prob. vi.

PROBLEM X.

To circumscribe an ellipse about a rectangle so that the axis of the ellipse may have the same ratio as the sides of the rectangle.

Let GHIK, *fig. 2*, be the rectangle. Draw the straight lines Aa, Bb, bisecting the sides of the rectangle and each other at C. Let Aa bisect GH in E, and KI in e, and let Bb bisect GK in D, and HI in d. In Bb make DP equal to DK, and draw PK cutting Aa at N. Make CB, Cb each equal to NK, and CA, Ca each equal to PK; then describe an ellipse upon the axes Aa, Bb.

DEMONSTRATION.

Draw PQ parallel to Aa, meeting the side KI of the rectangle in Q.

Let CA = PK = a, CB = KN = b, KQ = DP = CE = p, and eK = CD = q.

By similar triangles QKP, eKN, ... $bp = aq$
therefore $a : b :: p : q$, or CA : CB :: CE : CD.

PROBLEM XI.

The curve ADGaHE of an ellipse being given, to find the centre.

Draw any two parallel lines DE, GH (*fig. 3*) to meet the curve in E, D, H, G. Bisect the lines DE and GH, and through the points of bisection draw Aa, meeting the curve in the points A, a; bisect Aa in C, which is the centre required.

PROBLEM XII.

*Given the ellipse ABab (*fig. 4*), to find the axes in position and magnitude.*

From the centre C, found by Prob. xi, describe a circle meeting the curve in D, E, F, G. Join DG. Bisect DG, and through the point of bisection and the centre draw Aa, meeting the curve in A, a; and through C draw Bb perpendicular to Aa, meeting the curve in Bb; then Aa, Bb are the two axes as required.

PROBLEM XIII.

An ellipse mAM (fig. 5) and a tangent Tt being given, to find the point of contact.

Draw Mm parallel to Tt, meeting the curve in M, m. Bisect Mm in P, and draw AC through P, and the centre C and A is the point of contact.

PROBLEM XIV.

An ellipse mAM (fig. 6) being given, to draw a tangent through a given point A in the curve without having any diameter or focii.

Draw CA; from C with any radius describe a circle within the curve of the ellipse. Draw LM and Km parallel to AC tangent to the circle, meeting the ellipse in M, m: join Mm, and through A draw Tt parallel to Mm, and Tt is the tangent required.

PROBLEM XV.

Given an ellipse ABab (fig. 7) and the two axes Aa, Bb, to draw a perpendicular through a given point M in the curve.

Find the focii F, f, by Prob. i. Through M draw FH and fG, and draw MN bisecting the angle HMG, and MN is perpendicular to the curve.

Upon this principle a trammel may be made to draw the joints of any elliptic arch, as exhibited in fig. 8.

PROBLEM XVI.

An ellipse AMa (fig. 9) and the focii F, f being given, to draw a tangent through a given point M in the curve.

Draw FM, fM, and produce FM to H. Draw QT bisecting the angle fMH, and QT will be a tangent to the curve at M.

PROBLEM XVII.

An ellipse ABab (fig. 10), and the axes Aa, Bb being given, to draw a tangent through a given point M in the curve without the focii.

Join AM, and bisect AM in d. Through d draw CQ, and draw AQ parallel to CB. Draw QR through M, and QR is the tangent required.

Or,

Join Ma, and bisect Ma in e. Through e draw CR, and draw aR parallel to CB. Draw QR through M, and QR is the tangent required.

CURVE LINES.

ELLIPSE.

PLATE III.

Fig. 1.

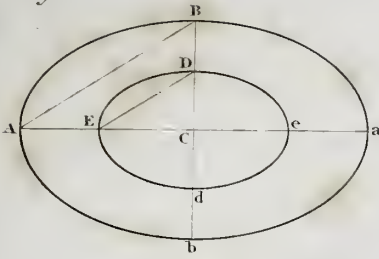


Fig. 2.

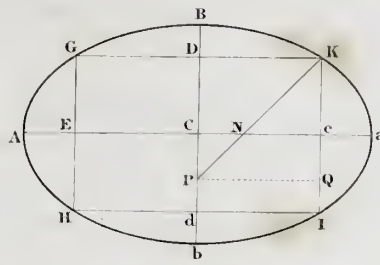


Fig. 3.

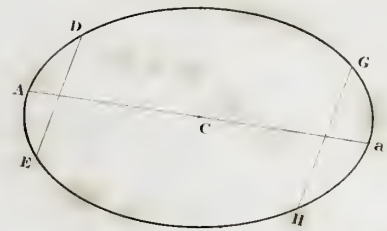


Fig. 4.

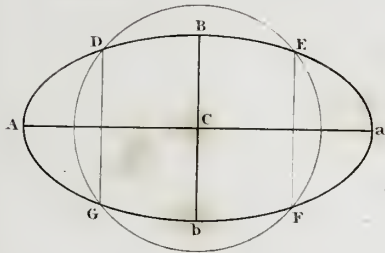


Fig. 5.

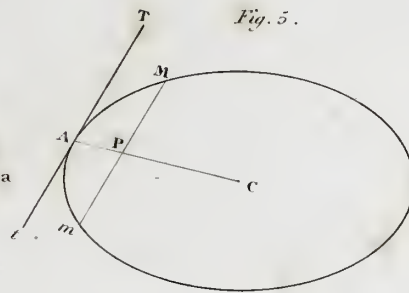


Fig. 6.

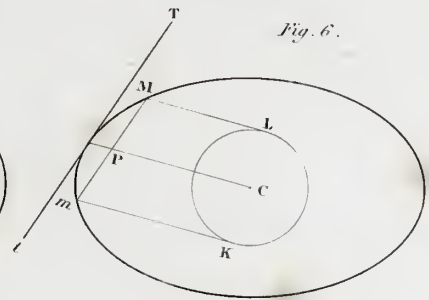


Fig. 7.

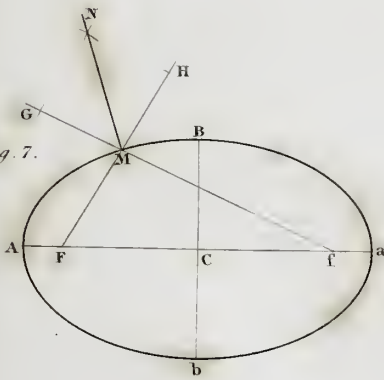


Fig. 8.

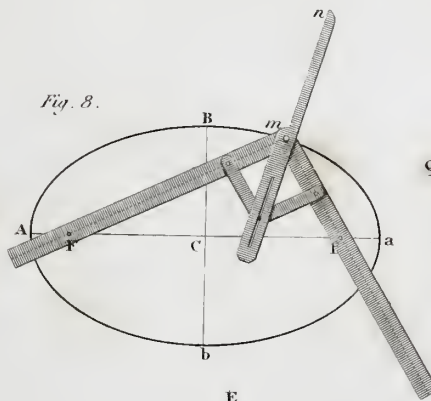


Fig. 9.

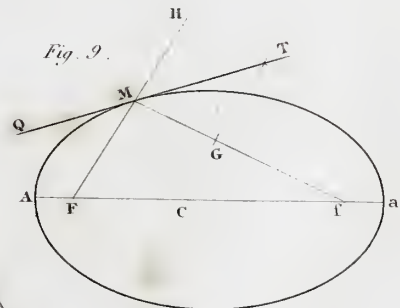


Fig. 10.

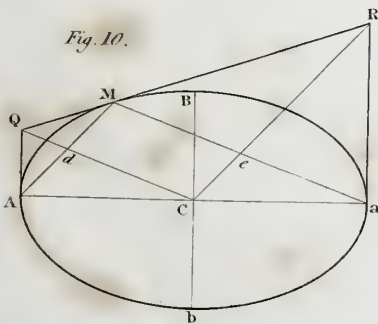


Fig. 11.

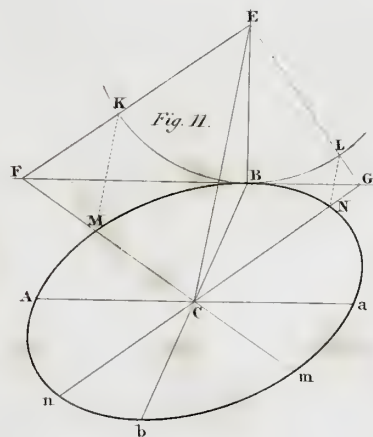
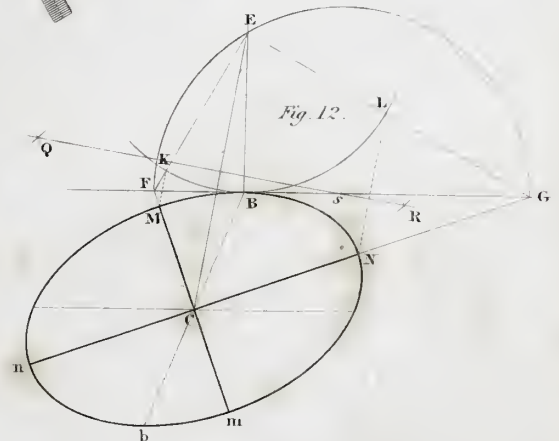
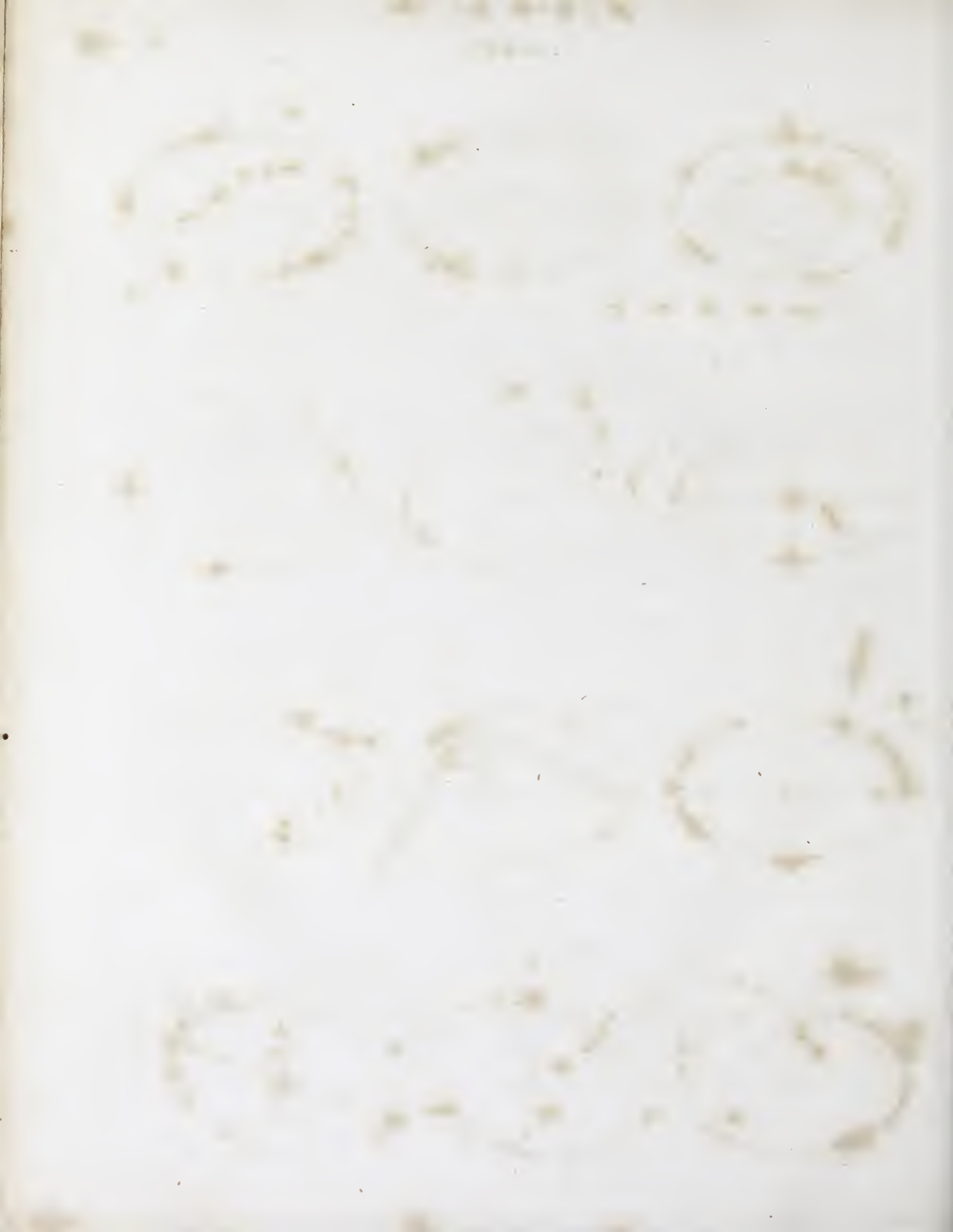


Fig. 12.





PROBLEM XVIII.

Given three straight lines passing through the centre in position, of which two are conjugate diameters, to find a conjugate diameter to the third straight line.

Let Aa, Bb (*fig. 11*) be the two conjugate diameters : it is required to find a conjugate diameter to the third straight line Fm.

Through B draw FG parallel to Aa. Draw BE perpendicular to FG, and make BE equal to CA. Join EF, and draw EG perpendicular to EF. Draw Gn through the centre C. From E, with the radius EB, describe the arc KBL, meeting EF at K, and EG at L. Join EC. Draw KM and LN parallel to EC, meeting Fn and Gn in M and N. Make Cm equal to CM, and Cn equal to CN ; then Mm and Nn will be two conjugates as well as Aa, Bb.

DEMONSTRATION.

For by Prop. xvii, *Appendix*, Curve Lines, Ellipse, $FB \times BG = AC^2$;
 but $AC^2 = BE^2$;
 therefore $FB \times BG = BE^2$; wherefore the
 points F, E, G are in a semicircle ; therefore the angle FEG is a right angle.

PROBLEM XIX.

*Given two conjugate diameters Aa, Bb (*fig. 12*), to find the two axes.*

Through B draw FG parallel to Aa. Make BE equal to CA, and join EC. Bisect EC by the perpendicular QR, meeting FG at s. From s, with the distance sE, describe the semicircle FEG. Join FE and GE ; also join FC and GC, and produce FC to m, and GC to n. From E, with the radius EB, describe the circle KBL, meeting FE and GE at K and L. Draw KM and LN parallel to EC, meeting Fm at M, and Gn at N. Make Cm equal to CM, and Cn equal to CN ; then Mm and Nn are the axes.

DEMONSTRATION.

Because Qs is perpendicular to EC, the distances sE, sF, sG, sC are all equal ; therefore the angle FCG is a right angle as well as FEG : but, because FEG is a right angle, Mm and Nn are conjugate diameters, by the preceding Problem ; but when two conjugate diameters are at right angles to each other, these two conjugate diameters are the axes.

CURVES, HYPERBOLA.

PROBLEM I.

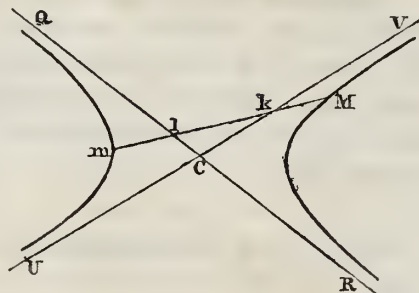
Given the asymptotes QR, UV, and a point M in one of the opposite curves, to find any point in the other opposite curve.

Draw Mm meeting UV in k, and QR in l. Make lm equal to kM, and m will be a point in the curve.

Hence, from the same point M we may find as many points in the curve contained within the angle QCU as we please.

In this manner, *Fig. 1, Plate 1, Curve Lines*, Hyperbola, is constructed; all the points being found from the same point a in the opposite curve.

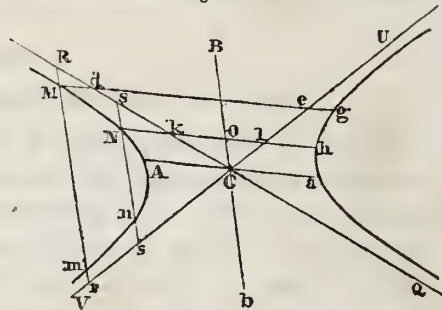
Fig. 142.



DEMONSTRATION.

Let M and N be two points in the same branch of the curve. Draw Mg, Nh parallel to each other, meeting QR in the two points d, k, and UV in the two points l, e, and the opposite curve in the two points h, g. Bisect kl in O. Through the two points O, C draw Bb, and through the two points M, N draw the straight lines Rr, Ss parallel to Bb, meeting QR in the two points R, S, and UV in the two points r, s, and the curve in the two points m, n.

Fig. 143.



By similar triangles	$\begin{cases} \text{RMd, SNk,} \\ \text{rMe, sNl,} \end{cases}$	$\begin{matrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{matrix}$	$\begin{matrix} \text{Nk} \times \text{RM} = \text{Md} \times \text{SN} \\ \text{Nl} \times \text{rM} = \text{Me} \times \text{sN} \\ \text{SN} \times \text{Ns} = \text{RM} \times \text{Mr} \end{matrix}$	
and by Prop. ix, <i>Appendix</i> , Curve Lines, Hyperbola				
wherefore, eliminating RM, SN, Mr, Ns,				$\text{Nk} \times \text{Nl} = \text{Md} \times \text{Me}$
In the same manner,				$\text{hl} \times \text{hk} = \text{ge} \times \text{gd}$
hence				$\text{Nk} \times \text{Nl} = \text{hl} \times \text{hk}$
and since kl is bisected in O	$\left\{ \begin{matrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{matrix} \right.$			$\begin{matrix} \text{Nk} \times \text{Nl} = \text{ON}^2 - \text{Ok}^2 \\ \text{Oh}^2 - \text{Ol}^2 = \text{hl} \times \text{hk} \\ \text{hk} \times \text{hl} = \text{Nk} \times \text{Nl} \end{matrix}$
and since it has been shown that				
therefore, eliminating Nk, Nl, hk, hl, and we shall have				$\text{Oh}^2 - \text{Ol}^2 = \text{ON}^2 - \text{Ok}^2$; and
since Ok = Ol, therefore Oh = ON.				

PROBLEM II.

Given the transverse axis Aa of an hyperbola, and any point M in the curve, to find the conjugate axis.

Bisect aA in C . Through the centre C , draw FG perpendicular to Aa , and through the vertex A draw HI parallel to FG , and draw Me parallel to Aa , meeting FG in e . From e , with the distance eM , describe an arc, cutting HI in k . Join ek , meeting CA in d , or, if necessary, produce ek to meet CA in d ; then Cd is the magnitude or length of the semiconjugate axis.

If in FG be made CB , Cb each equal to Cd ; then Bb is the conjugate axis in position and magnitude.

In No. 1, the conjugate axis is less than the transverse axis, and in No. 2 it is greater than the transverse axis.

If the point k coincide with A , both the axes are equal, and the axes belong to the equilateral hyperbola.

The reason of this method for finding the conjugate axis will be understood from the Demonstration given at the end of Problem v following.

Fig. 144, No. 1.

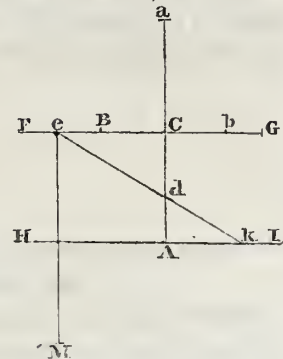
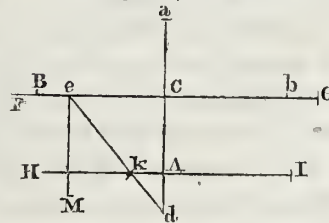


Fig. 144, No. 2.

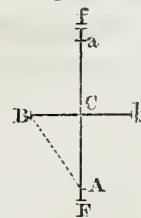


PROBLEM III.

Given the transverse Aa, and conjugate Bb, axes of an hyperbola, in position and magnitude, to find the focii.

In the line of the transverse axes, make CF , Cf , each equal to AB or Ab ; then F and f are the focii.

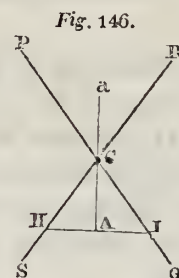
Fig. 145.



PROBLEM IV.

Given the transverse axis of an hyperbola in position, and the length of both axes, to find the asymptotes.

Through A, draw HI perpendicular to the transverse axis Aa, and make AH, AI, each equal to the semi-conjugate axis. Through the points C, I, draw PQ, and through the points C and H draw RS; then the straight lines PQ, RS, are the asymptotes.



PROBLEM V.

Given the transverse axis Aa of an hyperbola in position and magnitude, and the magnitude of the conjugate axis, to find any point in the curve.

Bisect Aa in C. Through the centre C draw FG perpendicular to Aa, and through A draw HI parallel to FG. In CA, make Cd equal to the semi-conjugate axis. Through any point e in FG, draw ed meeting HI in k, as in No. 2, or produce ed, if necessary, to meet HI in k. Through e, draw Mm parallel to Aa. Make eM, em, each equal to ek, and M, m are points in the opposite hyperbolas.

In this manner all the points in the opposite curves, in Fig. 2, Plate 1, Curve Lines, Hyperbola, and in the succeeding diagram here exhibited, are found; the opposite hyperbolas shown in Plate 1, Fig. 2, Curve Lines, being constructed according to No. 1, and those in the diagram here exhibited according to No. 2: in the diagram, Plate 1, Fig. 2, the transverse axis is greater than the conjugate; but in the following diagram, the conjugate axis is greater than the transverse.

When both the axes are equal, the points d and k both coincide with the vertex A, and the opposite hyperbolas are called

Fig. 147, No. 1.

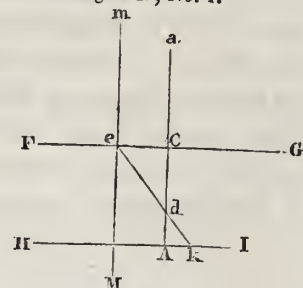


Fig. 147, No. 2.

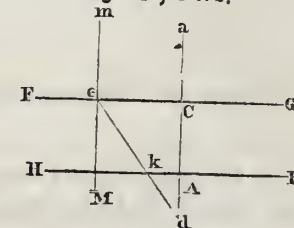
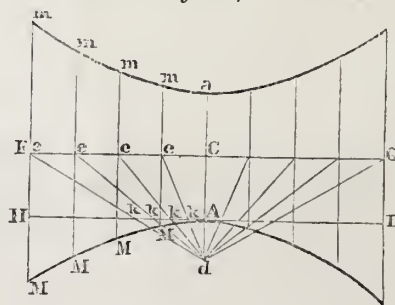


Fig. 147, No. 3.

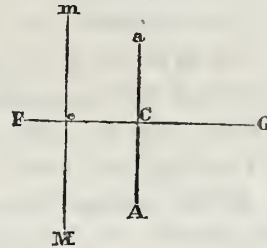


equilateral. In this case, when one of the axes is given, the other is also given. The construction to find any point in each opposite curve is simply as follows:—

Bisect the transverse axis Aa in C. Through the centre C, draw FG perpendicular to Aa. Through e, any point in CF, draw mM, and make em, eM each equal to eA, and M, m are each a point in every opposite curve.

The complete construction of equilateral opposite hyperbolas is exhibited in *Plate 1, Fig. 3, Curve Lines, Hyperbola.*

Fig. 147, No. 4.



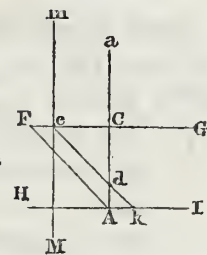
GENERAL DEMONSTRATION.

The construction being made as in *No. 1*, draw AF parallel to ke.

Now, let $CA = Ca = a$, $Cd = b$
and $AF = ek = eM = x$, $Ce = y$, and $de = v$.

By similar triangles Cde, CAF, ... $b^2x^2 = a^2v^2$
and by the right angled triangle dCe $v^2 = b^2 + y^2$
therefore, eliminating v , there will arise $b^2x^2 = a^2(b^2 + y^2)$, or, by transposition, $a^2y^2 = b^2(x^2 - a^2)$, which is the central equation of the hyperbola.—See *Appendix, Curve Lines, Hyperbola, Prop. iv, page 9.*

Fig. 148.



PROBLEM VI.

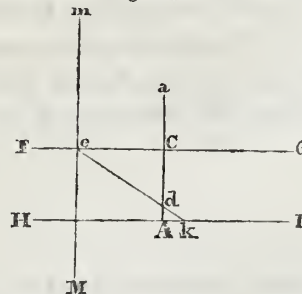
Given the transverse axis Aa of an hyperbola, and any point in one of the opposite curves, to find any other point in the curve in which the point is given, or in the opposite curve.

Find the conjugate axis or point d in the transverse axis by *Prob. ii*, and let the constructive lines FG and HI remain.

Through any point e in FG draw Mm parallel to Aa. Draw ed, meeting HI in k, and make eM equal to ek, and M is a point in the curve as required.

And by making em equal to eM, the point m is in the opposite curve.

Fig. 149.



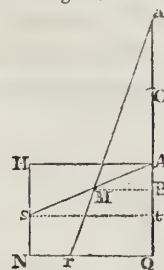
PROBLEM VII.

Given a diameter Aa , the abscissa CQ , and ordinate QN , to find any point in the curve.

Let M be the given point in the curve. Produce aA to Q . Draw NQ and HA perpendicular to AQ , and draw NH parallel to Aa . Divide NQ and NH each in the same ratio in r and s . Draw ra and sa meeting each other at M , and M is a point in the curve.

In this manner the opposite curves, *Fig. 4, Plate 1, Curve Lines, Hyperbola*, are described.

Fig. 150.



DEMONSTRATION.

Draw st and MP parallel to NQ , meeting AQ in t and P . Let $CA = Ca = a$, $CP = x$, $PM = y$, $CQ = z$, $QN = \gamma$, $At = Hs = v$, and $Qr = w$; then will $aP = CP + Ca = x + a$, $AP = CP - CA = x - a$, $aQ = CQ + Ca = z + a$, and $AQ = CQ - CA = z - a$.

By similar triangles aPM , aQr , . . . $(x + a)w = (z + a)y$,

and . . . APM , Ats , . . . $(x - a)\gamma = vy$,

and by construction $HN : Hs :: QN : Qr$. . . $v\gamma = (z - a)w$;

therefore, eliminating v and w , we have $(x^2 - a^2)\gamma^2 = (z^2 - a^2)y^2$, and therefore the curve is an hyperbola.

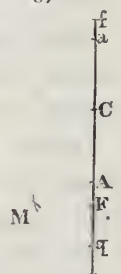
PROBLEM VIII.

To find a point in the curve by another method.

Find the conjugate axis by Prob. ii, and the foci by Prob. iii; then the transverse axis Aa , and the foci F, f being now given, any point in the curve will be found by the following method:—

In fF produced, take any point q . From F , with the distance Aq , describe an arc at M , and from f , with the distance aq , describe another arc meeting the former arc at M ; then M is a point in the curve.

Fig. 151.

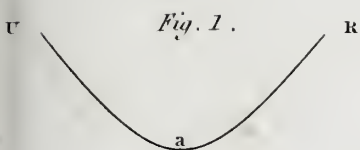


DEMONSTRATION.

This is evident from *Def. 7, Curve Lines, Appendix, p. 1*.

In *fig. 5, Curve Lines, Hyperbola*, the preceding points are found by Prob. viii immediately.

HYPERBOLA.



k

C

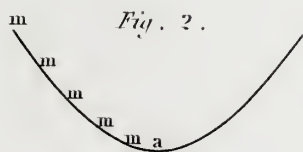
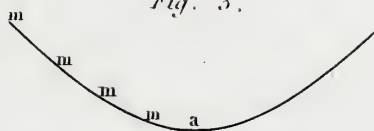
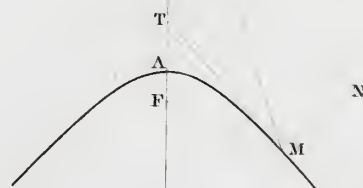
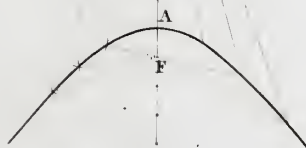
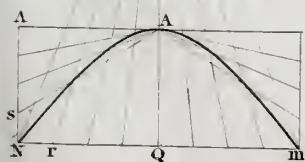
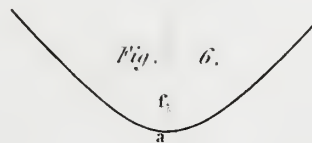
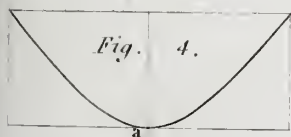
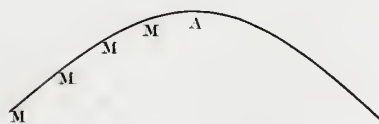
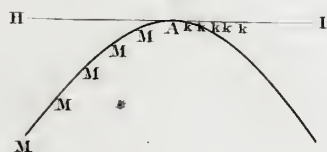


Fig. 3.



F e e e e C G

F e e e e C G



Engraved by C. Armstrong.

London Published by John Day & Co Octo. 20. 1822.



PROBLEM IX.

Given either curves of an hyperbola and the focii, to draw a tangent to the curve from any point M.

Fig. 6, Plate 1, Curve Lines, Hyperbola. Join FM, fM, and bisect the angle fMF, and the bisecting line MT is the tangent required.

PROBLEM X.

Given either opposite curve of an hyperbola and the focii, to draw a straight line from any point M in the curve perpendicular to that curve.

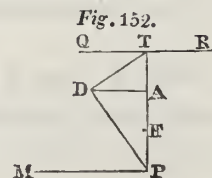
Fig. 6, Plate 1, Curve Lines, Hyperbola. Find the tangent MT by the immediately preceding Problem, and draw MN perpendicular to MT; then MN is perpendicular to the curve from M, as required.

OF THE PARABOLA.

PROBLEM I.

In a parabola are given in position the axes and its vertex, and any other point in the curve, to find the directrix and focus.

Through the vertex A , draw AD perpendicular to the line TP of the axis, and from the given point M in the curve draw MP parallel to DA . Make AD equal to the half of PM . Join PD , and draw DT perpendicular to DP . Through T draw QR parallel to AD . In AP , make AF equal to AT ; then QR is the directrix, and F the focus.



DEMONSTRATION.

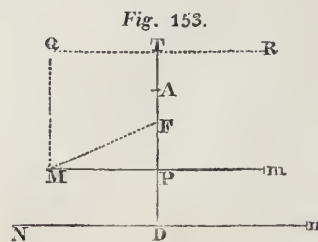
This will be evident by considering the *Coroll.* to *Def. 7*, Curve Lines, *Appendix*.—See the definition of directrix in page 11 of the same.

PROBLEM II.

METHOD 1.

In a parabola are given the vertex A , the position TD of the axis, and a point N in the curve, to find a double ordinate.

Draw Nn perpendicular to TD , meeting TD in D . Find the focus F and the point T , through which the directrix passes. Through any point P in AD or AD produced, draw Mm parallel to Nn . From the focus F , with the distance TP , describe two arcs meeting Mm at M, m , and the points M, m are in the curve, and consequently Mm is a double ordinate.

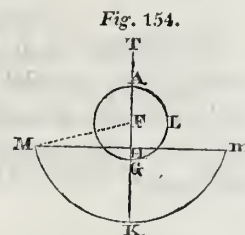


DEMONSTRATION.

Draw the directrix QR , and join MF . Now TP is equal to QM ; but by *Coroll.* to *Def. 7*, Curve Lines, *Appendix*, the point M is in the curve, whence QM is equal to MF .

METHOD 2.

From F, with the radius FA, describe a circle AGL, and from F with any radius describe another circle MKm. In AK, make AH equal to GK, and draw Mm perpendicular to AK, the points M, m are in the curve, and Mm is a double ordinate.



DEMONSTRATION.

Produce FA to T, and make AT equal to AF. Then T will be the point in the axis through which the directrix passes. We have now only to prove that FM or FK is equal to TH.

Now, by construction, $GK = AH$,

and by the parabola, $FG = AT$;

therefore, by addition, $FG + GK = HA + AT$.

Now $FG + GK = FK$, and $HA + AT = HT$; therefore $FM = FK = HT$, as was to be shown.

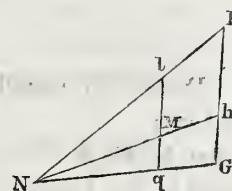
PROBLEM III.

Given a tangent NR, a double ordinate NG from the point of contact, and the position of a diameter, to find any point in the curve of the parabola.

METHOD 1.

Draw GR and ql parallel to the diameter, meeting NR in l. Make Rh equal to lq, and draw hN meeting ql in M; then M is a point in the curve.

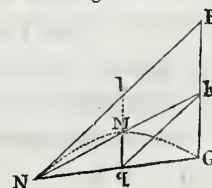
Fig. 155.



METHOD 2.

Draw any line Nh between NR and NG, and draw GR parallel to the diameter, meeting Nh in h. Draw qh parallel to NR, meeting NG in q, and draw qM parallel to GR, and M is a point in the curve.

Fig. 156.



DEMONSTRATION OF METHOD 1.

Since (Prop. x, p. 13, Curve Lines, *Appendix*) $Nq : NG :: MI : ql$, $Nq \cdot ql = NG \cdot MI$ and since by parallel lines, $MI : ql :: hR : GR$, ... $MI \cdot GR = ql \cdot hR$ and by similar triangles Nql , NGR $ql \cdot NG = Nq \cdot GR$ therefore, eliminating the common quantities, will be found $ql = hR$; therefore M is in the curve.

DEMONSTRATION OF METHOD 2.

Because ql has been shown to be equal hR , and since ql and hR are parallel lines, the straight lines NR and qh will also be parallel.

PROBLEM IV.

In a parabola are given an ordinate DN, and the abscissa AD, to find any other point in the curve.

METHOD 1.

Draw Mq parallel to AD , meeting DN in q , and find the point l , so that $DN : Dq :: AD : Al$, and draw Nl meeting MH at M , then M is a point in the curve.

Fig. 157, No. 1.

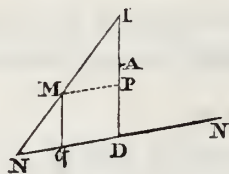
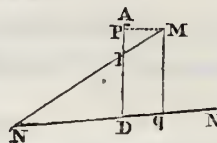


Fig. 157, No. 2.



DEMONSTRATION.

Draw PM parallel to ND , and let $AD = a$, $DN = b$, $AP = x$, $PM = Dq = y$, and $Pl = v$.

then in *No. 1* . . . $AI = Pl - AP = v - x$, and in *No. 2* $= Pl + AP = v + x$,

and in *No. 1* and *2* . $PD = AD - AP = a - x$,

and in *No. 1* . . . $Nq = ND - qD = b - y$, and in *No. 2* $= ND + Dq = b + y$.

Now by construction $Al : AD :: Dq : DN$; $\therefore b(v - x) = ay$,

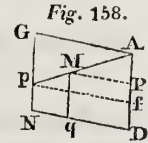
and by similar triangles NqM , MPl , ... $v(b - y) = y(a - x)$

By finding the value of v in each of these equations, and by putting these values equal to each other, there will result $b^2x = ay^2$, which is the equation of the parabola.

The demonstration is the same for the second figure, except that $Al = v + x$, and $Nq = b + y$.

METHOD 2.

Draw AG parallel to DN, and NG parallel to DA. In DN, take any point q, and draw qM parallel to DA. Divide GN in p in the same ratio that DN is divided in q, and join pA meeting qM at M, the point M is in the curve.



DEMONSTRATION.

Draw MP and pf parallel to ND, meeting AD in P and f. Let $AD = GN = a$, $DN = fp = b$, $AP = x$, $Dq = PM = y$, $Af = Gp = p$.

By similar triangles APM, Afp, ... $bx = py$,

and by construction $GN : Gp :: DN : Dq$, ... $bp = ay$;

therefore, eliminating p , there will be found $b^2x = ay^2$, which is the equation of the parabola.

Description of the Diagrams, Plate 1, Curve Lines, Parabola.

Fig. 1 shows the method of finding the focus, the axis and ordinate being given. The principle is, that the distance of the focus from the vertex of the curve is a third proportional to Ap, Pq, or to the abscissa and to half the ordinate. This is the same in principle as Prob. i.

Fig. 2 shows the description of a parabola by means of points found as in Prob. ii, viz.: any point M is thus found by drawing a perpendicular through M to the axis, and describing an arc or two arcs from the focus, with the distance between the perpendicular and directrix, meeting the perpendicular in M, m. In this case the directrix QR and the lines QM need not be drawn; for the distance TP being applied as a radius from the focus, the points M, m will be found as before.

Fig. 3 depends upon the same principle as shown in Prob. ii, Method 2.

Fig. 4 depends upon the principle shown in Prob. iii, where it is evident that the points q, h divide the lines GN and GR in the same ratio. Hence we may find as many points in the curve as we please by dividing the lines GR and GN, each into the same number of equal parts.

Figs. 5 & 6 show the application of Prob. iv, *fig. 5*, as described in No. 2, and *fig. 6* as in No. 1. Indeed, the method here applied is evident from Prob. iii.

Fig. 7 is described upon the principle shown in Prob. iii, Method 2.

Fig. 8 and *9* are described upon the principle demonstrated in the *Appendix*, Curve Lines, Prop. ii, p. 13, viz., by dividing each of the lines CD, DE in the same proportion, and drawing straight lines through the corresponding points of section.

Fig. 10 is described upon this principle, viz., that the ordinates are as the squares of the abscissas.

Fig. 11 is described by the principle shown in Prob. iv, Method 2.

PROBLEM V.

Given the curve of a parabola and a diameter AP, and the vertex A, to find a double ordinate.

Produce PA to O, and through O draw JK perpendicular to OP. Take J at any distance from O, and make OK equal to OJ. Parallel to OP draw JM, meeting the curve in M, and Km meeting the curve in m, and join M, m; then Mm is the double ordinate required.

This is so evident, as not to require any formal demonstration.

PROBLEM VI.

Given the parabola (fig. 13), a point M in the curve, the axis AD, and the focus F, to draw a tangent through the point M.

Join FM, and draw MG parallel to DA. Bisect the angle FMG, and the bisecting line MT is a tangent to the curve.

DEMONSTRATION.

For the line bisecting the angle made by the radius vector, and a line from that point of the curve where the radius vector meets it perpendicular to the directrix, is a tangent to the curve, *Appendix*, Prop. ii, Curve Lines, p. 12.

PROBLEM VII.

Given the parabola (fig. 14), a point M in the curve, and the axis AD, to draw a tangent to the curve through the point M.

Produce PA to T, and draw MP, meeting the axis AD in P. Make AT equal to AP, and draw TM, which is the tangent required.

DEMONSTRATION.

Because the subtangent of the axis is double the abscissa, *Appendix*, Curve Lines, Prop. iii, p. 12.

PROBLEM VIII.

Given the parabola (fig. 15), a point M in the curve, and a diameter AP, to draw a tangent through a given point M in the curve.

Find the ordinate pm, by Prob. ii, and draw the ordinate PM parallel to pm. Produce AP to T, and make AT equal to AP, and draw MT, which is the tangent required.

CURVE LINES.

PLATE 1.

PARABOLA.

Fig. 2.

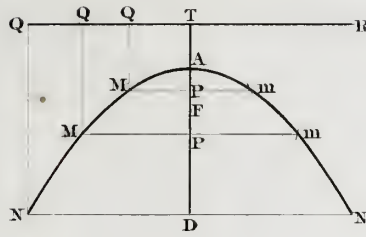


Fig. 3.

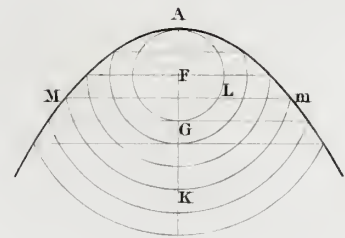


Fig. 1.

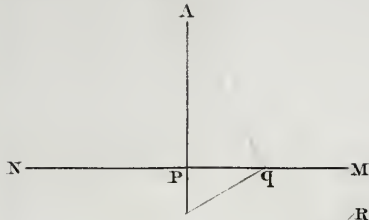


Fig. 4.

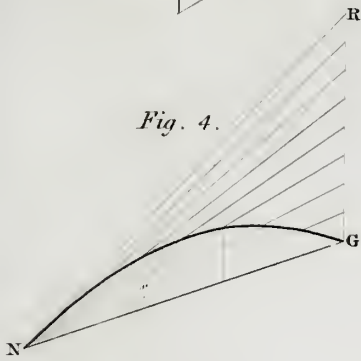


Fig. 5.

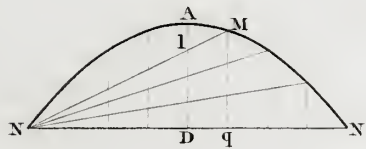


Fig. 6.

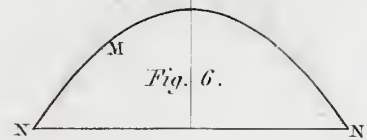


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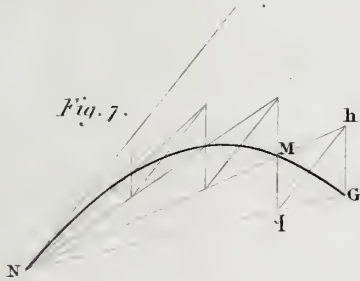


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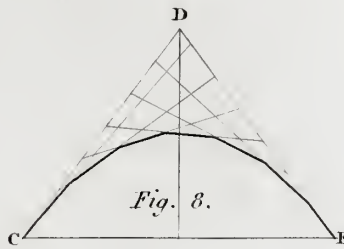


Fig. 9.

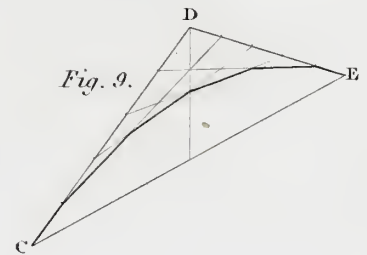


Fig. 10.

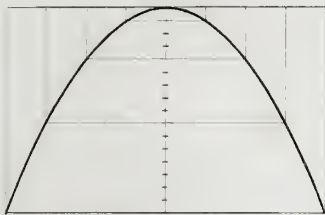


Fig. 11.

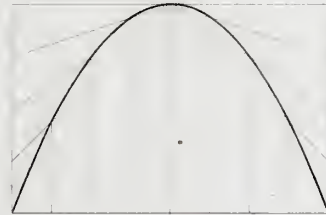


Fig. 12.

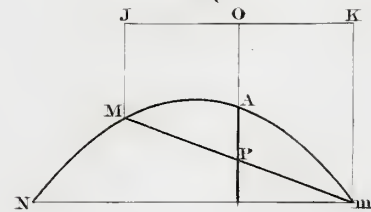


Fig. 13.

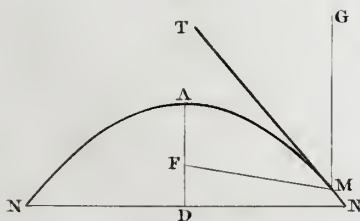


Fig. 14.

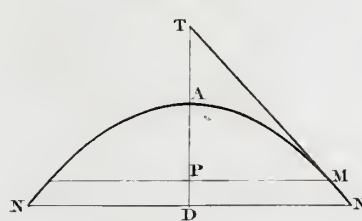
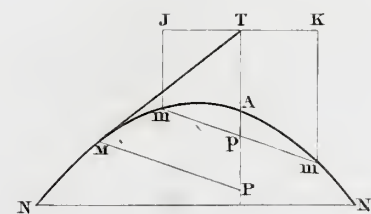


Fig. 15.



ELLIPSE, HYPERBOLA & PARABOLA.

Fig. 1.

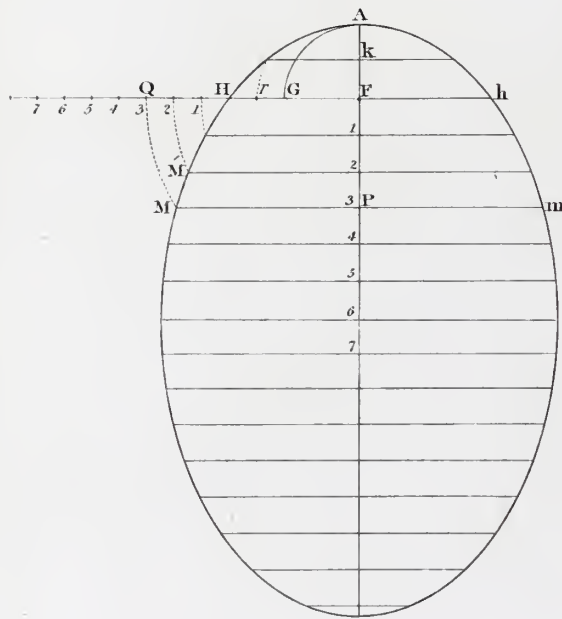


Fig. 2.

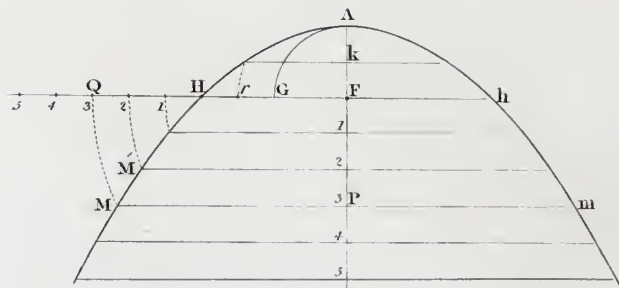


Fig. 3.

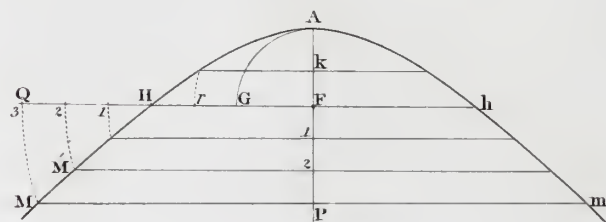


Fig. 4.

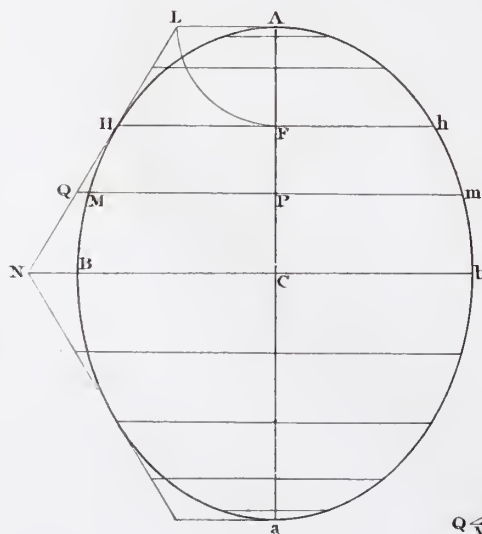


Fig. 5.

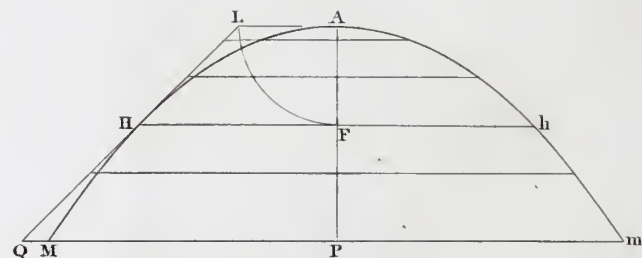
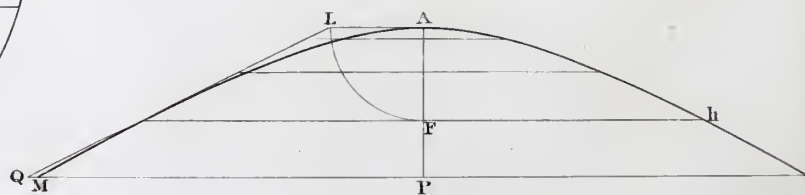


Fig. 6.



Engraved by W. Lowry.

PROBLEM IX.

Given the vertex, the focus, and the latus rectum in position and magnitude, to find any point in the curve of an ellipse, hyperbola, or parabola.

METHOD 1.

In *Plate 3, Curve Lines, fig. 1, 2, 3*, let A be the vertex, F the focus, and FH the half of the latus rectum.

From F, with the radius FA, describe an arc AG, meeting FH in G. Make FP to HQ as AF is to GH. Draw Mm parallel to FH, and from F, with the radius FQ, describe the arc QM, and the point M will be in the curve.

If with the same distance and from the same point another arc be described meeting Mm at m, m will be another point in the curve.

Or, divide AF into any number of equal parts, as two, and run any number of these parts from F in the line FP; also divide GH into the same number of equal parts, and run as many of these parts from H in the line HQ; then proceed with any two corresponding points 2, 2, as has been done with the points P and Q, and we shall have one point M' at the extremity of an ordinate, or one in the extremity of each equal and opposite ordinate.

The curve is that of an ellipse when GH is less than AF, or AF greater than the half of FH, as in *fig. 1*. The curve is that of a parabola when GH is equal to AF, or AF the half of FH, as in *fig. 2*; and the curve is that of an hyperbola when GH is greater than AF, or when A is less than the half of FH, as in *fig. 3*. This properly depends upon the principle of the directrix, which has not been demonstrated in the *Appendix*.

METHOD 2.

Figs. 4, 5, and 6. Draw AL parallel to FH. From A, with the distance AF, describe an arc AL. Through the two points L, H, draw LQ, and produce LQ at pleasure. Draw Qm parallel to FH, meeting AF produced in P. From F, with the distance PQ, describe an arc meeting Qm at M; then M is a point in the curve.

If from the same point F, with the same distance PQ, another arc be described meeting Qm in m, m will be another point in the curve.

In the ellipse, *fig. 4*, if the two axes Aa, Bb be given in position and magnitude, the curve may be described by the same method.

For find the focus F by Prob. i, Ellipse, and draw AL parallel to FH, and produce CB to N. Make CN equal to CA, and AL equal to AF, and join LN; then proceed as in Method 2.

This method depends also upon the property of the directrix, as is the case with the first.

With regard to CM being made equal to CA, and AL to AF, see *Emerson's Conic Sections, Prop. xxxviii, page 23, Ellipse*.

PROBLEM X.

Given the abscissa, an ordinate, and the axis of a curve of the second order of lines, to find any point or points in that curve.

Let Aa be the axis (*fig. 1, 2, 3, No. 1, 2, 3, Ellipse, Parabola, and Hyperbola, Plate 1*), PM the ordinate either from some point in the axis, or from some point beyond it. Draw AD parallel to PM, and MD parallel to PA. Divide MP in h, and MD in g, each in the same proportion. Draw ah and gA, meeting each other at m, and m is a point in the curve.

No. 2 in each *fig.* shows the method of describing the curve by finding a sufficient number of points.

In the parabola, *fig. 3*, the axis Aa is of infinite length; therefore only the point A is given; and since the axis is of infinite length, the line hm must be drawn parallel to the abscissa.

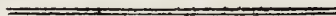
PROBLEM XI.

Given the vertex, the ordinate, and a tangent at the extremity of that ordinate of an ellipse, parabola, or hyperbola, to find the centre, and to determine the species of the curve.

Figs. 1, 2, 3, Plate 2, Ellipse, Hyperbola, and Parabola. Through the vertex A, draw AT parallel to the ordinate PM, meeting the tangent MT in T. Join AM, and bisect AM in d. Through the point of concurrence T, and the point of bisection d, draw TC meeting AP in C; then C is the centre.

If the centre fall on the same side of the line AT on which the ordinate lies, the curve is an ellipse; and if the centre fall on the contrary side of the line AT, the curve is an hyperbola; but if the line dT happen to be parallel to the abscissa AP, the curve is a parabola.

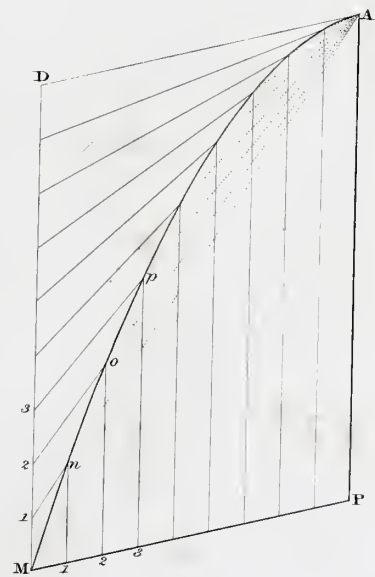
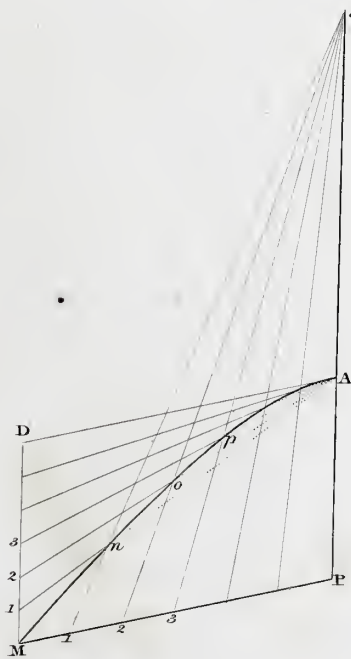
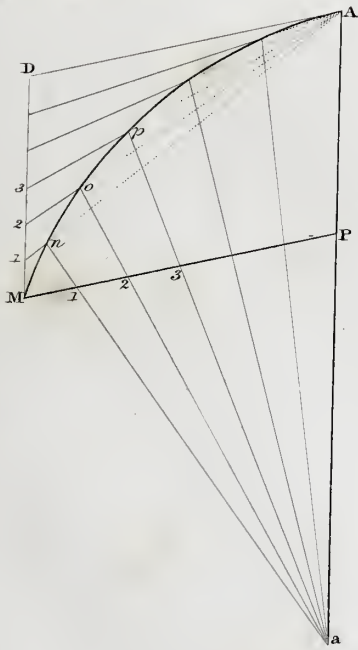
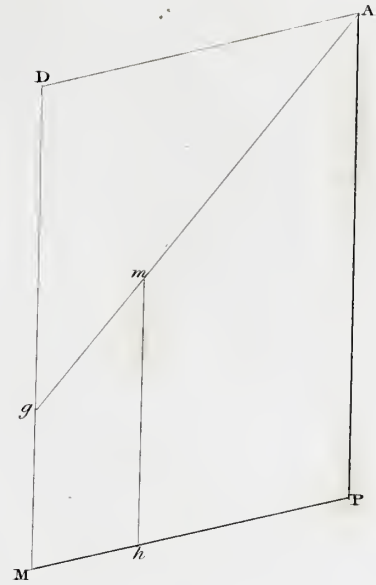
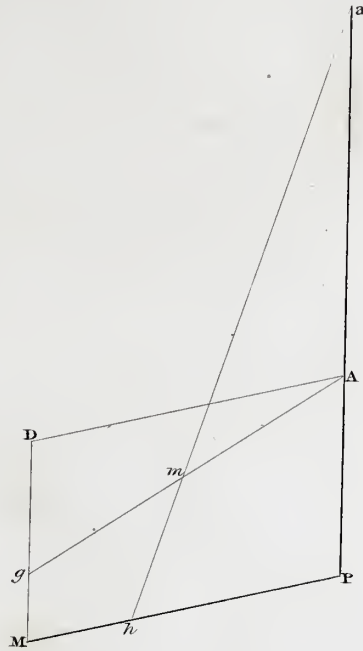
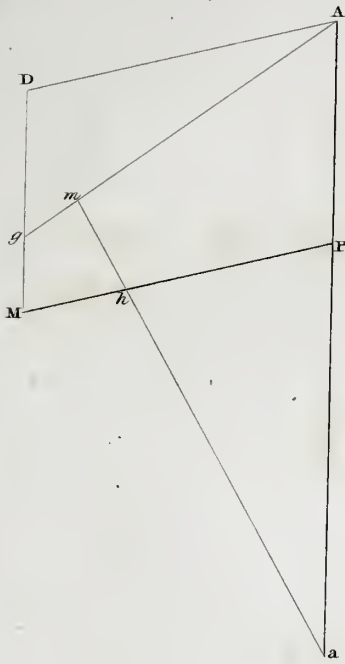
Hence, since the diameter is found by the above method, we may describe the curve by finding a sufficient number of points, as in No. 2 of each *fig.* by Prob. x; for then a diameter and double ordinate will be given to find these points.



CURVE LINES.

ELLIPSE HYPERBOLA & PARABOLA.

PLATE 1.



Introduced by E. Nicholson.

London, Published by John Day & Co Feb. 20 1823.



C U R V E L I N E S.

PLATE II.

Fig. 1.

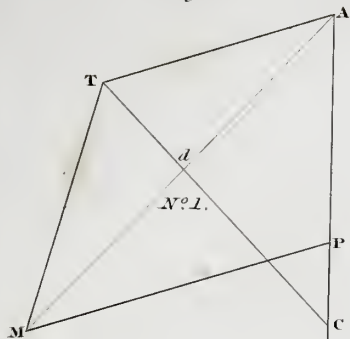


Fig. 2.

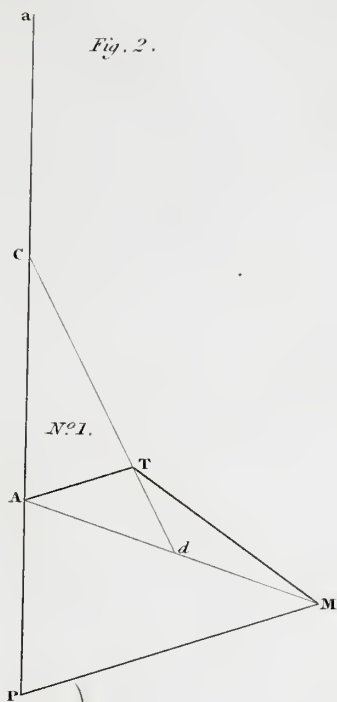


Fig. 3.

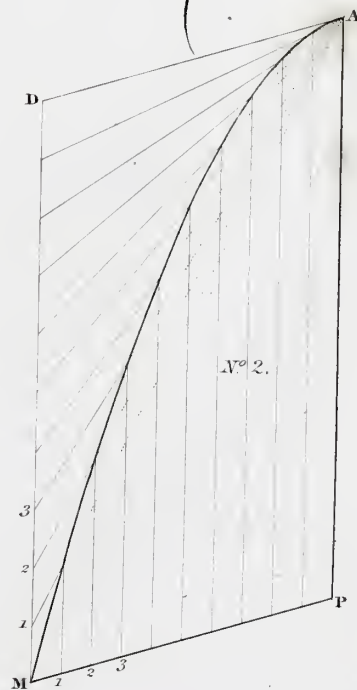
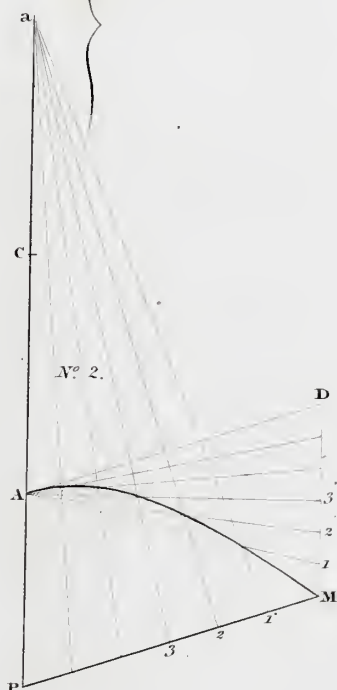
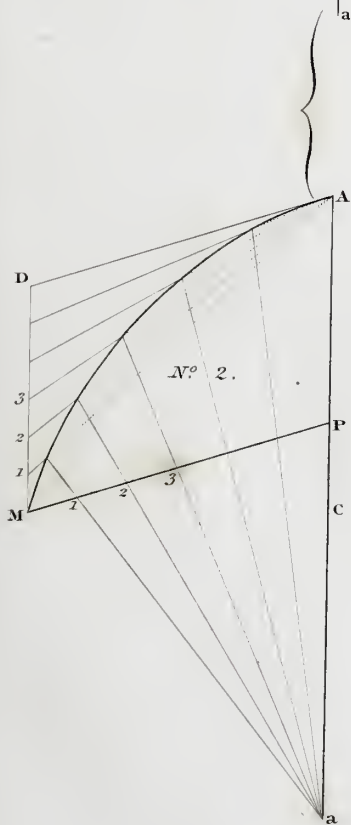
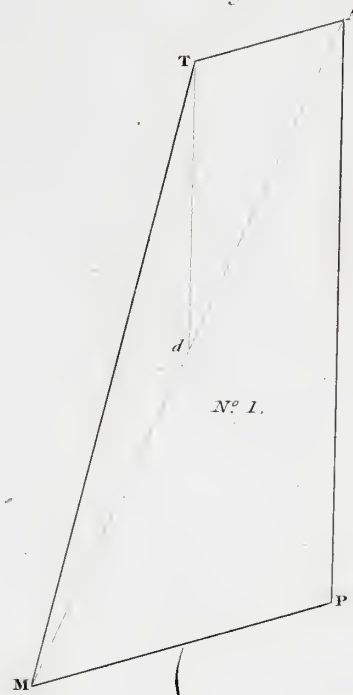




Fig. 1.

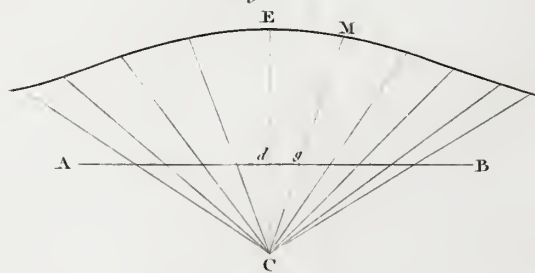


Fig. 2.

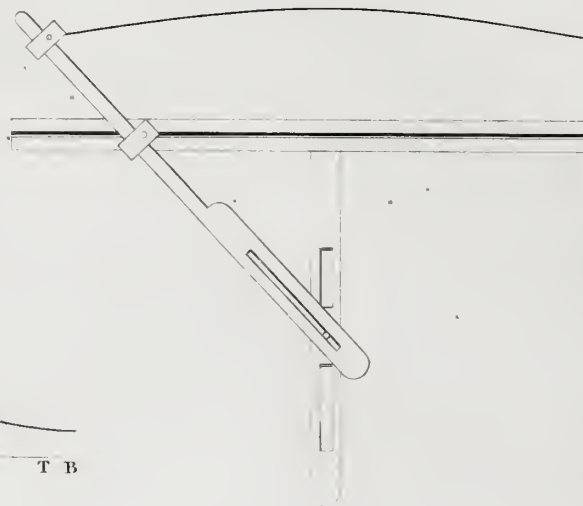


Fig. 3.

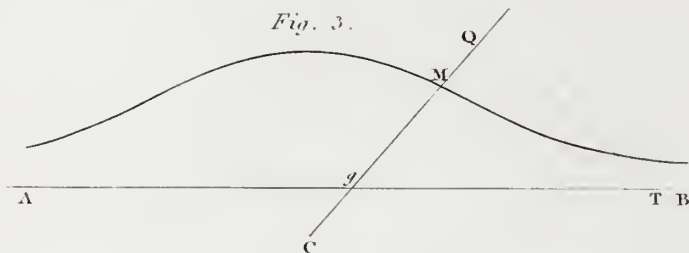
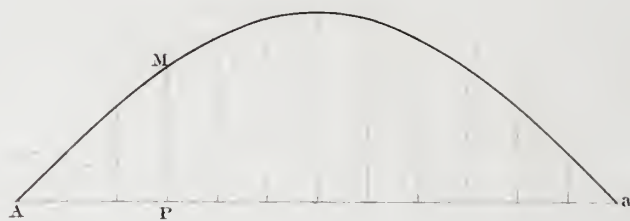


Fig. 4.



a.

Fig. 5.

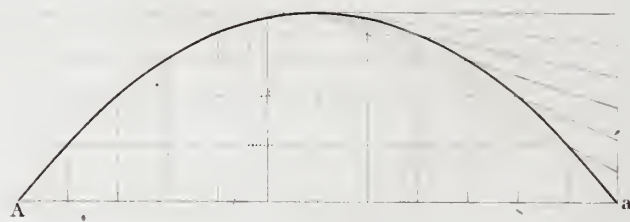
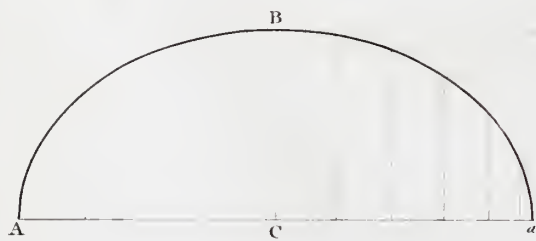
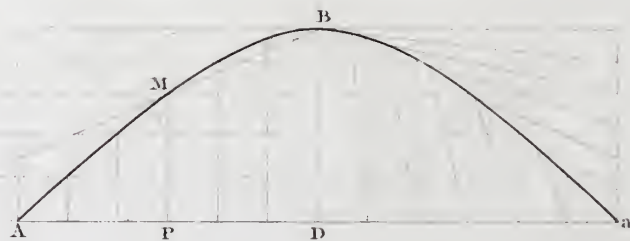


Fig. 6.



CURVE LINES OF THE HIGHER ORDERS.

THE CONCHOID OF NICHOMEDES is a curve of such a nature, that if in *fig. 1, Plate 1*, Curve Lines, AB be a straight line, C a point out of it, and CE a line perpendicular to AB, cutting AB in d; and if dE be a given distance, and if any number of straight lines CM be drawn, all the distances gM between the straight line AB and the curve will be equal to each other.

DEFINITION.

The fixed point C is called the *Pole*.

Hence the curve may be described by a trammel, as in *fig. 2*.

PROBLEM I.

To draw a tangent to the conchoid through any point M in the curve, fig. 3.

Produce CM to Q, and make MQ equal to Cg. Draw QT perpendicular to CQ, meeting AB in T, and join TM; then TM is the tangent required.—See *Newton's Fluxions*, Ex. 2, p. 64, and also p. 72, 8vo edit.

THE SINIC CURVE.

The Sinic Curve, or figure of the sines, is a curve of such a nature, that the abscissa AP, *fig. 4*, is equal to the arc ap of a circle, and the ordinate PM is equal to the sine pq of that arc. The figure may therefore be easily described by taking equal parts in the arc, and repeating one of them in the line AB as often as necessary, for any part or the whole of the semi-circumference.

Figs. 5 and 6 are described from an elliptic curve in the same manner.

The reader may here observe the very coincidence near of these curves with those of the first order. When the axis major is to that of the axis minor in the ratio of 4 to 3, as in *fig. 5*, the curve is nearly that of a parabola. When the generating ellipse stands with its axis major parallel to the ordinates, the figure will approach to an hyperbola. The axis Ba of the nearest hyperbola may be found by Prob. xi, p. 90, supposing BD the abscissa, DA the ordinate, and M a point in the curve given.

SPIRALS.

DEFINITIONS.

A Spiral is a curve making any given number of revolutions round a fixed point without meeting itself.

The fixed point is called the *centre of the spiral*.

A line drawn from the centre of the spiral to the curve, is called an *ordinate*.

SPIRAL OF ARCHIMEDES.

DEFINITION.

If the arc passed over by the radii be always in a given ratio to the difference of the ordinates, the spiral is called the *spiral of Archimedes*.

Therefore to draw the spiral of Archimedes, we need only to draw lines forming equal angles round the centre, and fix upon one of these lines as the greatest ordinate; which being determined, divide it into as many equal parts as the number of revolutions intended, and subdivide each part into as many smaller equal parts as the number of angles: make the second or next ordinate one part less; the third two parts less; the fourth three parts less, &c. than the first, and draw the curve through these points.—See *fig. 1, Plate 2, Plane Geometry, Curve Lines*.

PROBLEM II.

To draw a tangent through any point M in the curve, Fig. 1, Plane Geometry, Curve Lines, Plate 2.

From the centre C, with the distance CM, describe a circle. Draw CT perpendicular to CM, and make CT equal to the circumference of the circle: then TM being drawn, is the tangent.

LOGARITHMIC SPIRAL.

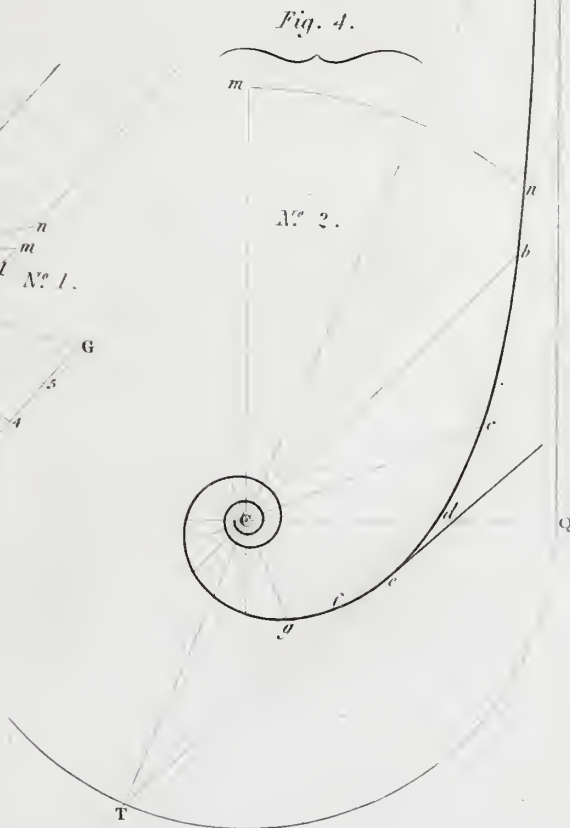
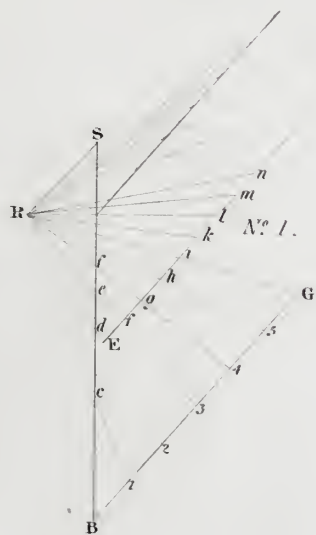
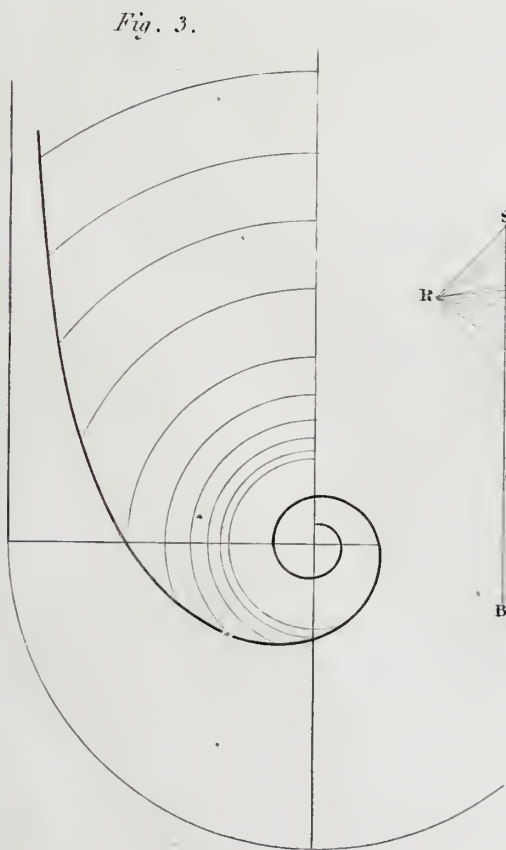
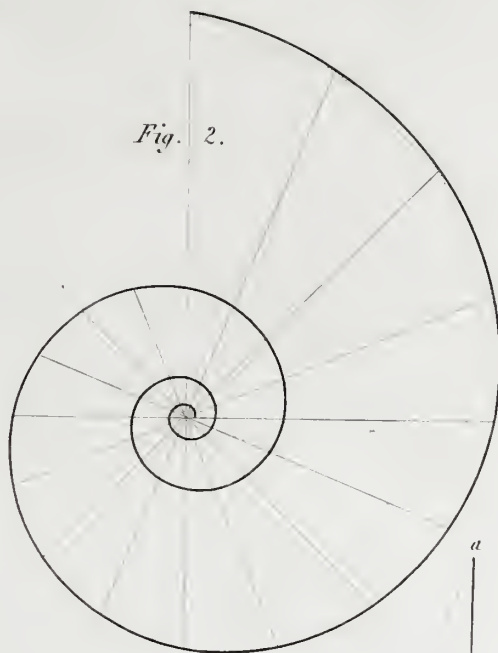
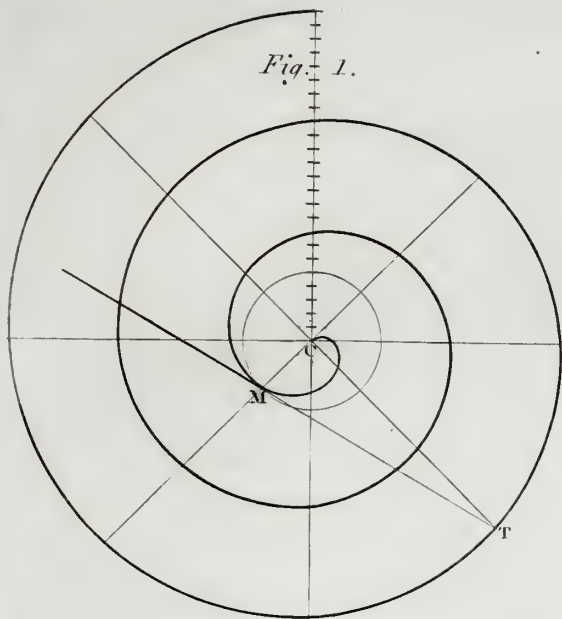
If the ordinates form equal angles at the centre, and all the succeeding ordinates and chords form equal angles at the curve, the spiral is called the *logarithmic* or *proportional spiral*, as in *fig. 2, Spirals*.

Hence the description of the curve is evident by similar triangles.

PLANE GEOMETRY.

PLATE 2.

CURVE LINES.





HYPERBOLIC SPIRAL.

DEFINITIONS.

If from any point in a straight line, arcs of circles of equal length be described on the same side of it and terminate in it, the curve passing through all the other extremities is called the *hyperbolic spiral*.—See *fig. 3*, Spirals.

The straight line in which the arcs terminate, is called the *axis*.—See *fig. 3*.

A straight line drawn on the other side of the curve, opposite and parallel to the axis, is called the *asymptote*.

PROBLEM III.

To describe the hyperbolic spiral, the axis, the centre, and the ordinate next to the axis, being given, fig. 4.

Let BS be equal to the given ordinate. Draw BG at any angle with BS and SR parallel to BG. In the straight line BG, make B1 equal to any distance, and make B2, B3, B4, &c. equal to twice, three times, four times, &c. that distance, and make SR equal to B2. Draw 1R, 2R, 3R, &c. meeting BS in the points C, D, E, &c. Having drawn the ordinates Ca, Cb, Cc at equal angles, make the ordinate Ca equal to twice SB, the ordinate Cb equal to Sc, the ordinate Cd equal to Sd, &c.; then the curve passing through all the points a, b, c, d, &c. is the hyperbolic spiral.

Note in the subsidiary diagram, No. 1, the line BG might require to be extended more than the space would allow. In this case we might draw any lines parallel to BG, and repeat one of the equal parts intercepted by any two adjacent lines drawn from R to the divisions in BG; thus, making ik, kl, lm, &c. each equal to ik, or hg, or gf, &c.; and when this becomes inconvenient, take another parallel line nearer to S.

PROBLEM IV.

To draw a tangent to any point e in the curve, fig. 4.

Describe the arc mn between the axis and the curve. Draw CQ perpendicular to the axis Cm, and make CQ equal to the arc mn. From C, with the radius CQ, describe the arc QT, and draw CT perpendicular to Ce; then Te being drawn is the tangent required.

OF PLANES.

DEFINITIONS.

1. A straight line is perpendicular or at right angles to a plane, when it makes right angles with every straight line meeting it in that plane. Reciprocally, a plane is perpendicular to a straight line.

2. A straight line is parallel to a plane, when both the straight line and the plane are extended indefinitely without meeting. Reciprocally, the plane is parallel to the straight line.

3. Two planes are parallel to each other when both are produced indefinitely without meeting.

Axiom.—The common section of two planes is a straight line.

For any two points in the common is in both planes; but a plane is that in which any two points being taken, the straight line between them is wholly in that plane; therefore the straight line must be in both planes; and since all straight lines coincide through the same two points, the common section is a straight line.

4. The angle contained by two straight lines, drawn each from any point in the common section of two planes perpendicular to that common section in each plane, is called the *angle* or *inclination* of these two planes.

5. If this angle is a right angle, the planes are perpendicular.

THEOREMS.

1. A plane which passes through two straight lines can have only one position.

2. If two planes cut one another, their common section is a straight line.

3. If a straight line be perpendicular to two straight lines at the point of their intersection, it is perpendicular to the plane in which these lines are.

4. If a straight line be perpendicular to a plane, every straight line parallel to that straight line is perpendicular to the plane.

5. Two planes perpendicular to the same straight line are parallel to each other.

6. The intersections of two parallel planes with a third plane are parallel.

7. A straight line perpendicular to one of two parallel planes is also perpendicular to the other.

8. Parallel straight lines intercepted between two parallel planes are equal.

9. If two straight lines meeting one another be each parallel to each of two others that meet one another, though not in the same plane with the first two, the first two and the last two shall contain equal angles, and the plane passing through the first two shall be parallel to the plane passing through the last two.

10. If three straight lines not situate in the same plane are equal and parallel to each other, the triangles formed by joining the extremities of these lines are equal, and their planes parallel.

11. If two straight lines be cut by three parallel planes, the middle plane will divide the finite lengths of the lines terminated by the other two planes in the same ratio.

12. If a straight line be perpendicular to a plane, every other plane which passes along that straight line shall be perpendicular to the first plane.

13. If two planes be perpendicular to each other, and a straight line be drawn in one of them perpendicular to their common section, the straight line shall also be perpendicular to the other plane.

14. If two planes be perpendicular to a third, their common section is perpendicular to the third.

SOLID ANGLES.

DEFINITION.

A **SOLID ANGLE** is the surface contained by the meeting of the angular points of more than two plane angles which are not in the same plane.

THEOREMS.

1. If a solid angle be formed by three plane angles, the sum of any two of them is greater than the third.
2. The sum of all the plane angles which form any solid angle is less than four right angles.
3. If two solid angles be each composed by three plane angles, of which the plane angles of the one solid angle are respectively equal to the angles of the other solid angle, any two planes of the one solid angle shall have the same inclination as the corresponding planes of the other.

DEFINITIONS.

1. A solid is that which has length, breadth, and thickness.
2. A prism is a solid contained by parallelograms terminating at each end in two parallel plane figures.
3. The parallel plane figures are called the *bases* of the prism, and the parallelograms are called the *sides*, which taken together constitute the *convex surface*.
4. The altitude of a prism is the distance between its bases.
5. A prism is right when the lateral edges are each perpendicular to the planes of its bases; then each of them is equal to the altitude of the prism.
6. A prism is oblique when the lateral edges are not perpendicular to the bases.
7. A prism is triangular, quadrangular, pentagonal, hexagonal, &c. according as the base is a triangle, a quadrilateral, a pentagon, a hexagon, &c.
8. A prism which has a parallelogram for one of its bases, is called a *parallelopiped*.
9. A parallelopiped is rectangular when all its faces are rectangles.
10. When the rectangles are squares, the parallelopiped is called a *cube*.
11. A pyramid is a solid formed by more than two plane triangles, of which all the angular points, one of each triangle, terminate in the same point, and the other angular points terminate in a plane figure.

12. The triangles which unite in a point and terminate in the plane figure, are called the *sides of the pyramid*; the point where they meet, is called the *vertex*; and the plane figure in which the sides terminate, is called the *base*.

13. The altitude of a pyramid is the perpendicular drawn from its vertex to the plane of its base.

14. A pyramid is triangular, quadrangular, &c. according as the base is a triangle, a quadrangle, &c.

15. Two solids are similar when they are contained by the same number of similar planes, similarly situated, having like inclinations to each other.

A CYLINDER.

DEFINITIONS.

1. A solid generated by the revolution of a rectangle about one of its sides which remains fixed, is called a *cylinder*.

2. The fixed straight line of the rectangle is called the *axis* of the cylinder.

3. The two circles described by the opposite sides of the rectangle perpendicular to the axis, are called the *ends* or *bases* of the cylinder.

4. The surface generated by the side of the rectangle which is parallel to the axis, is called the *convex surface*.

5. If a cylinder be cut by a plane parallel to the axis, the least portion is called the *archoid* of a cylinder.

6. The portion of a cylinder contained between two planes passing along the axis, is called a *sectroid* of a cylinder.

7. If the two planes of a sectroid of a cylinder be at a right angle with each other, the sectroid of the cylinder is called a *quadrantal sectroid* of a cylinder.

A CYLINDROID.

DEFINITIONS.

1. Two equal and similar ellipses are similarly situated when they are parallel to each other, and when the two axes major are in one plane, and the two axes minor in another plane.

2. A solid, of which two sides are equal and similar ellipses and similarly situated, and of which a straight line perpendicular to these two sides will entirely coincide with the intermediate surface, is called a *cylindroid*.

3. The two equal and similar ellipses are called the *ends* or *bases* of the cylindroid.

4. The intermediate surface is called the *curved surface*.

5. A straight line passing through the centres of the two bases, is called the *axis* of the cylindroid.

6. If a cylindroid be cut by a plane along the axis or parallel to it, and through the axis of one of the ends or parallel to it, the half or the least portion of the cylindroid is called the *archoid* of a cylindroid.

CYLINOID.

DEFINITIONS.

1. A solid consisting either of the segment of a cylinder or cylindroid, or compounded partly of a prism, and partly of a cylinder or cylindroid, such that all the rectangular sides may be parallel to the axis of the cylinder or cylindroid, and that two of the surfaces of the solid may be perpendicular to that axis, is called a *cylinoid*.

2. The axis of the cylinder or cylindroid is called the *axis* of the cylinoid.

3. Each of the two surfaces which are perpendicular to the axis of the cylinoid, is called an *end* of the solid.

4. If a cylinoid be cut by a plane obliquely to the axis, but perpendicular to one of the plane sides which joins the ends, each of the parts is called the *ungula* of a cylinoid.

5. The plane of the cylinoid which joins the two bases, and which is perpendicular to the oblique section to the axis, is called the *determinating plane*.

6. The end of the ungula which is perpendicular to the axis, is called the *right end* of the ungula.

7. The end of the ungula which is oblique to the axis, is called the *oblique end* of the ungula.

STEREOTOMY.

DEFINITION.

STEREOTOMY is that branch of Geometry which treats of the sections of solids.

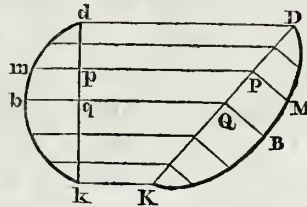
PROBLEM I.

Given the right end of the ungula of a cylinoid, and the determinant of the oblique end, to find the oblique end.

GENERAL RULE.

Let kKd be the determinant of the oblique end, KD being the line of position, and let kd be perpendicular to kK or dD ; then on kd describe $kbmd$, the right end of the prism. Draw mP parallel to dD or kK , meeting dk in p , and DK in P , and draw PM perpendicular to KD . Make PM equal to pm . In the same manner will be found the point B , making QB equal to qb : a sufficient number of points being found through all these points, draw a curve, and we shall have the oblique end $AKMD$.

Fig. 159.



SCHOLIUM.

Upon this principle, the angle ribs of groins and the angle brackets of coves are to be described, the right end being the given rib or bracket, and the oblique end the angle rib or bracket to be found.

Thus let the determinant be $AacC$, and the given rib or bracket be $amnc$, the angle rib or angle bracket will be $AMNC$.

Fig. 160, No. 1.

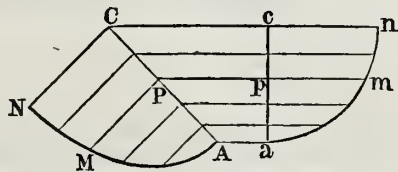
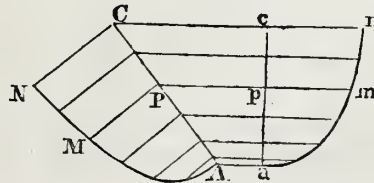
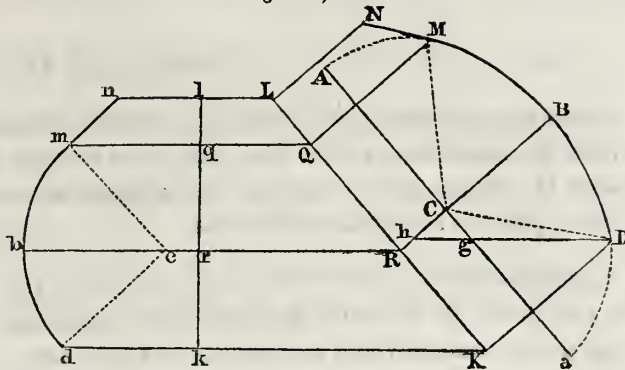


Fig. 160, No. 2.



Through the centre c of the arc mbd , draw the straight line bR parallel to IL or kK , meeting kl in r , and KL in R . Draw RB and KD perpendicular to KL , and make RB equal to rb , and KD equal to kd ; also make BC equal to bc : then here are given a semi-axis CB , and a point D in the curve, to describe the ellipse or portion MBD required; which being done*, join CD , and draw MN parallel to CD , and draw LN perpendicular to KL .

Fig. 160, No. 6.



PROBLEM II.

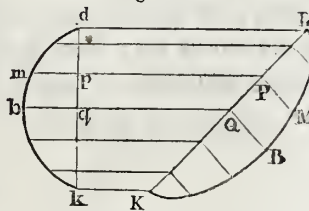
Given the oblique end, and the determining plane, to find the right end or base.

GENERAL METHOD.

Let $kKdD$ be the determinator, and $KBMD$ be the oblique section. Draw PM perpendicular to KD the line of position, meeting KD in P , and draw Pm parallel to kK or dD , meeting dk in p . Make pm equal PM , and the point m will be one of the points in the base. In the same manner we may find as many points as we please. A sufficient number of points being thus found, the curve may be traced.

Or, if the oblique end be an ellipse, or any given portion of an ellipse, or of a circle, the base or right end will also be the same portion of an ellipse or circle, and may therefore be described by a trammel.

Fig. 161.



* Or through C , draw Aa parallel to LK , and from D as a centre, with the distance BC , describe an arc meeting Aa in g . Draw Dg to meet BC prolonged in h . Make CA and Ca each equal to Dh ; then the ellipse or portion of it may be described by Prob. iii, p. 69.

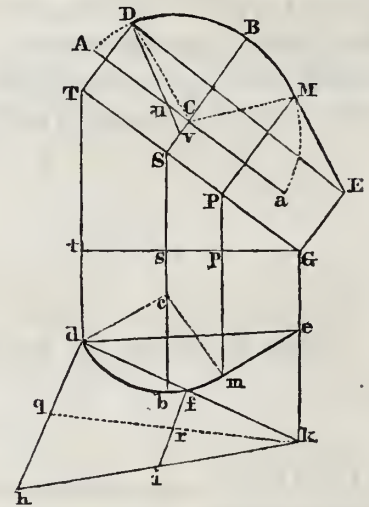
PROBLEM III.

The base or right end of the ungula of a cylinoid being given, and the length of two straight lines perpendicular to the base contained between two points in the outline of the base and the oblique end, to describe that oblique end so that it may pass through another given point in the outline of the base.

Let $dbfine$ be the outline of the base, and let d, f be the two points on which the perpendiculars stand, and e the point through which the oblique end is to pass.

Through the point f draw dk , and draw dh and fi each perpendicular to dk . Make dh equal to the height of the straight line upon the point d and fi equal to the height of the straight line upon the point f . Through i draw hk , and through e draw kG ; and through any point G in kG , draw Gt perpendicular to kG . Draw dT parallel to kG , meeting Gt in t . Make tT equal to dh , and join TG . Then if $t dbfineG$ be considered the end of the ungula of a prism, and tTG the determining plane, the oblique end $TDBMEG$ may be found by Prob. i, p. 99, by the General Method. Or if dbm be an arc equal to the fourth part of the whole circle, and me perpendicular to the radius cm ; then, in the oblique end, the curve DBM will be a portion of the semi-ellipse $ADBMa$, of which the centre is C , and ME parallel to CD will be a tangent to the curve at the point M .

Fig. 162.



A CONE.

DEFINITIONS.

1. A cone is a solid generated by the revolution of a right angled triangle about one of the sides containing the right angle.
2. The fixed side of the triangle, about which the other perpendicular side and the hypotenuse revolves, is called the axis of the cone.
3. The circle described by the revolving side of the triangle which is perpendicular to the axis, is called the *base* of the cone.

4. The other extremity of the axis which does not terminate in the base, is called the *vertex* of the cone.

5. The surface of the cone generated by the hypotenuse of the revolving triangle, is called the *convex surface*.

6. The portion of a cone contained between two planes perpendicular to the axis, is called the *frustum* of a cone.

7. The portion of a cone contained between the base and a plane, cutting the axis obliquely, is called the *ungula* of a cone.

8. The portion of the ungula of a cone contained by a plane which is perpendicular to the base and the oblique plane to the axis, and which also passes through the base, is the *semi-ungula* of a cone.

SECTIONS OF SOLIDS:—CONIC SECTIONS.

PROBLEM IV.

Given the triangular section DEF of a right cone through its axis, and the line of position of any other section of the cone perpendicular to the plane of the triangular section, to find that other section of the cone.

CASE 1.

Let GH the line of position meeting both sides DE, DF of the cone, or DE, DF produced; then this position of the cutting plane will cause the section to be an ellipse; therefore proceed as follows:—

Bisect GH in i, and through i draw kl parallel to EF, the base of the triangular plane, meeting DE in k, and DF in l.

In No. 3, draw the straight lines Aa, Bb at right angles, meeting each other in C. Make CA, Ca, each equal to iG or iH, No. 1, and make CB, Cb, each equal to a mean proportional between ik and il, No. 1; then by Prob. iii, p. 69, on the two axes Aa, Bb, describe the ellipse ABab, which will be the section of the cone for this position of the line GH, No. 1.

In order to refresh the reader's memory, the method of finding the mean proportional is exhibited at No. 2 in this and in the following figure.—See Prob. xl, p. 43.

Fig. 163, No. 1.

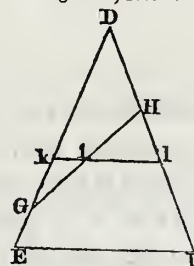


Fig. 163, No. 2.

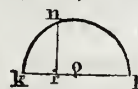
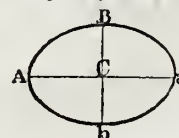


Fig. 163, No. 3.



CASE 2.

Let the straight line GH , No. 1, meet the section DEF , which passes through the axis of the cone in G , and the section Def of the opposite cone in the same plane in H .

Bisect GH in i , and draw il perpendicular to the axis of the cone, meeting Ff in k , and Ee in l . Find a mean proportional between ik and il , and let in, No. 2, be the mean proportional. Draw Aa , No. 3. From C in Aa make CA , Ca , each equal iG or iH , No. 1. Now having the transverse axis Aa , and the semi-conjugate in, the opposite hyperbolas may be described.

Fig. 164, No. 1.

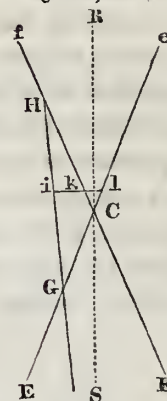
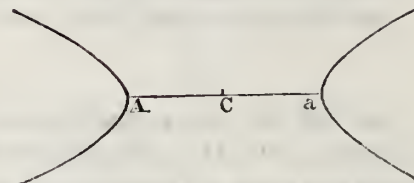


Fig. 164, No. 2.



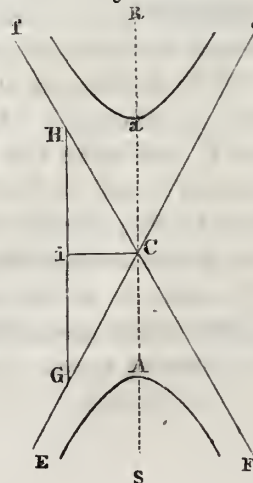
Fig. 164, No. 3.



SCHOLIUM.

If the line GH be parallel to the axis RS , then the points I , k will coincide with C , and Ci will then be the semi-conjugate axis; and if GA and Ha be drawn perpendicular to GH , meeting RS in A , a , then Aa will be the transverse axis. And because AG and aH are each equal to Ci , and that Aa is bisected in C , and the two straight lines Ee , Ff pass through the points G , C , H , the two straight lines Ee , Ff are asymptotes to an hyperbola, of which the transverse axis is Aa , and the semi-conjugate Ci ; therefore the curve may be described by Prob. i, p. 78.

Fig. 165.



CASE 3.

If the line GK be parallel to one side CE of the cone, and CE and CF equal to each other, the line GK meeting EF in K ; then if AB , No. 3, be made equal to GK , and Dd be drawn perpendicular to AB , and BD , Bd be made each equal to kn , No. 2, a mean proportional between EK , KF , No. 1; and, lastly, if a parabola be described upon the abscissa AB , and upon the semi-ordinate BD or Bd , this curve will be the section of the cone through GK , No. 1.

Fig. 166, No. 1.

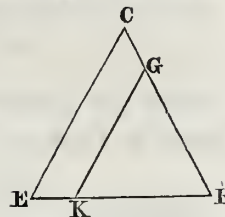


Fig. 166, No. 2.

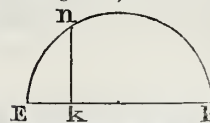
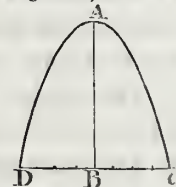


Fig. 166, No. 3.



CONOID.

DEFINITIONS.

1. If the portion of a conic section contained by the axis, an ordinate, and the part of the curve between them, be revolved round the axis which remains fixed, the solid generated is called a *conoid*.
2. The axis of the generating figure is called the *axis* of the conoid.
3. The surface of the solid generated by the curve, is called the *curved surface* of the conoid.
4. The side of the solid generated by the ordinate, is called the *base* of the conoid.
5. If the conoid be cut by a plane parallel to its base, the part of the solid contained between the base and the cutting plane is called the *zone* of a conoid.
6. If the curve of the generating figure be an arc of an ellipse, the solid is called an *elliptic conoid*.
7. If the curve of the generating figure be an arc of an hyperbola, the solid is called an *hyperbolic conoid*.
8. If the curve of the generating figure be an arc of a parabola, the solid is called a *parabolic conoid*.

9. A portion of a conoid contained between two planes which meet the axis, and which are perpendicular to the base, is called the *sectroid* of a conoid.

These Definitions also include some of those belonging to the sphere and the cone.

GENERAL PROPERTY.

Every two parallel sections of a conoid are similar figures. Hence, if any section of a conoid be given, any section parallel to that section may be found.

C U N E O I D.

DEFINITIONS.

1. A cuneoid is a solid terminating in a straight line at one end, and in a plane parallel to that straight line at the other, in such a manner that another straight line perpendicular to the first from any point in it may coincide with the intermediate surface.

2. The straight line in which the solid terminates, is called the *directing edge* of the solid.

3. The plane figure is called the *base* or *end* of the solid.

4. If the base be a circle or ellipse, having either axis parallel to the directing edge, the solid is called a *cuniconoid*.

Coroll. 1. Every section of a cuniconoid made by a plane perpendicular to the directing edge, is a triangle perpendicular also to the base.

Coroll. 2. Every section of a cuniconoid passing through the directing edge is a rectangle.

DEFINITIONS *continued*.

5. The straight line passing through the centre of the base perpendicular to the directing edge, is called the *axis*.

6. The triangular section which passes along the axis, is called the *principal triangle*.

7. The rectangular section which passes along the axis, is called the *principal rectangle*.

8. If a semi-cuniconoid having the principal triangular section for a side be cut by a plane perpendicular to that side, the portion of the solid which contains the base is called the *ungula* of that semi-cuniconoid.

9. The side of the ungula which is oblique to the axis and to the principal triangle, is called the *oblique section*.

10. The part of the principal triangle contained between the base and the oblique end, is called the *determinating side*.

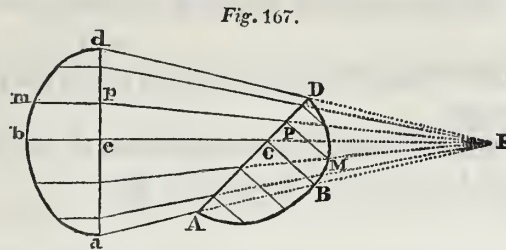
PROBLEM V.

Given the determining side of the ungula of a semi-cuniconoid in the plane of the principal triangle and the base of the solid, to find the oblique end.

Let $aADd$ be the determining side of the solid, $abmd$ the base, and AD the base line of the oblique end.

Draw pm perpendicular to ad , meeting ad in p . Prolong dD , aA to meet in E . Draw pE , meeting AD in P , and draw PM perpendicular to AD . Make PM equal to pm , and M will be a point in the curve;

in this manner we may find as many points as will be necessary to describe the curve $ABMD$.



PROBLEM VI.

Given the oblique end $ABMD$ of the ungula of a semi-cuniconoid and the determining side $dDAa$ in the plane of the principal triangle, to find the base of the solid.

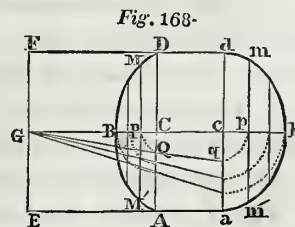
In AD take any point P , and draw PM perpendicular to AD . Through P draw Ep , meeting ad in p . Draw pm perpendicular to ad , and make pm equal to PM , and m will be a point in the curve of the base. In the same manner, as many points may be found as will be necessary to draw the curve $abmd$ of the base.

PROBLEM VII.

Given the determining side of a semi-cuniconoid upon the plane of the principal rectangle and the base of the solid, to find the section of the solid parallel to the base through a given straight line in the plane of the principal rectangle.

Let $aEFd$ be the plane of the principal rectangle, and $am'bmd$ the base of the solid, and let AD parallel to ad be the given line of section.

Bisect ad in c , and through c draw Gb parallel to Fd or Ea , meeting EF in G . In cb take cp any convenient distance from c , and through p draw mm' parallel to ad . In ca or cd make cq equal to cp , and draw Gq meeting AD in Q . In CG , make CP equal to CQ , and through P draw MM' parallel to AD . Make PM equal to pm , and PM' equal to pm' , and the points M , M' will be in the curve.



In the same manner, as many points may be obtained as will be found necessary to trace the curve AM'BMD.

If the base am'bmd be an ellipse, the curve AM'BMD will be an ellipse also; and therefore if CB, the semi-conjugate axis, be found, the curve AM'BMD may be described by a trammel.

That the curve is an ellipse may be shown thus:—

$$CB : CP :: cb : cp$$

and CD and PM are each equal to cd, pm; therefore, by Prob. v, p. 71, the curve is an ellipse.

This is applied to the description of the ribs for a hollow in the ceiling under the gallery of a Church, as in Waterloo Place.

DEFINITIONS.

1. If the ends of a prism be a conic section contained by a portion of the curve next to a vertex and double ordinate to the axis, and not exceeding the half of the entire figure, when the curve is an ellipse, the prism is called an *archoid*.

2. An archoid is denominated a *circular*, an *elliptic*, *hyperbolic*, or *parabolic* archoid, accordingly as the ends are portions of a circle, an ellipse, hyperbola, or parabola; the circular or elliptic archoid is also denominated a *cylindric* or *cylindroidic* archoid.

3. The side of an archoid, which is generated by a double ordinate, is called the *base* of that archoid.

4. The section of an archoid made by a plane passing through the axis of the ends, is called the *axal section*.

5. The two edges of the base which is generated by the extremities of the double ordinate, are called the *straight edges* of the curved surface.

6. If an archoid be cut by a plane perpendicular to the base, and parallel to one of the straight edges of the curve surface, each portion is denominated a *segment* of that archoid.

7. The side of the segment of an archoid which was separated from the others in dividing it into two segments, is called the *perpendicular side*, and the other plane side the *base*.

8. If two less segments of an archoid having their ends equal and similar be joined by their perpendicular sides, so that their bases may be in one plane, the solid made of the two is called a *pointed* archoid.

9. The plane side of a pointed conic prism which terminates the curved surfaces, is called the *base* of that pointed conic archoid.

10. The line in which the two curved surfaces meet, is called the *ridge line*.

Fig. 1, Plate 1, Architectural Solids, is an archoid.

A S P H E R E.

DEFINITIONS.

1. A SOLID generated by the revolution of a semicircle about its containing diameter which remains fixed, is called a *sphere*.
2. The diameter of the semicircle which remains fixed, is called the *axis* of the sphere.
3. The centre of the semicircle is called the *centre* of the sphere.
4. The surface generated by the semicircular arc, is called the *curved surface* of the sphere.
5. The portion of a sphere contained by a part of the curved surface and a plane, is called the *frustum* of a sphere.
6. The portion of a sphere contained between two parallel planes, is called a *zone* of the sphere.
7. The plane of the segment of a sphere is called the *base*.
8. If the base pass through the centre of a sphere, the segment is called a *hemisphere*.
9. The portion of a sphere contained between two planes passing through the centre, is called the *sectroid* of a sphere.
10. The portion of a sphere contained between any two planes of which their intersection does not pass through the axis, is called an *imperfect sectrum* of a sphere.

GENERAL PROPERTY.

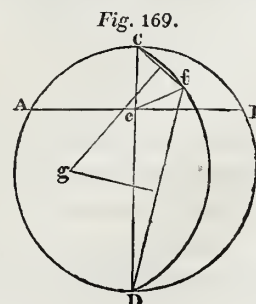
Every section of a sphere is a circle.

PROBLEM VIII.

Given the line of common section of the two circular segments of the sectern of a sphere, the inclination of their planes, and the versed sine of each segment, to find the radius of the great circle perpendicular to the line of common section.

Let AB be the line of common section. Bisect AB by a perpendicular CD, meeting AB in e.

Draw the line ef, making the angle Cef equal to the angle of inclination of the two planes. Make eC equal to the versed sine of the one segment, and ef equal to the versed sine of the other. Through the three points A, C, B, describe the circumference ACBD of a circle; also find the centre g of a circle that will pass through the three points D, f, C; then will gC or gD be the radius of the sphere required.



SOLIDS OF REVOLUTION.

DEFINITIONS.

1. If a plane figure which has one straight edge be revolved about that straight edge, so that any point in the figure may just describe a circle, the solid formed is called a *solid of revolution*.

2. The fixed straight line is called the *axis* of the solid.

3. If a solid have a side which is a plane figure perpendicular to the axis, this perpendicular side is called the *base* of the solid.

4. The section of the solid passing through the axis is called the *axal section*; and each part of the axal section on each side of the axis, is called a *radial* or *radial section*.

5. A portion of the solid comprehended between two planes which meet together in the axis, is called a *sectroid* of a solid of revolution.

7. If the solid be cut by any plane inclined or parallel to the axis, and if this inclined plane be cut perpendicularly by another plane passing through the axis, the line of section in the oblique or parallel section is called the *axis* of that oblique section, or of that parallel section.

8. A straight line drawn in the plane of the oblique section or parallel section perpendicular to the axis of this section, and terminated by the outline of the section, is called a *double ordinate*.

9. The axal section perpendicular to the oblique or parallel section, is called the *determinating section*.

GENERAL PROPERTIES.

All the sections of a solid of revolution parallel to the base are circles.

An ordinate of any oblique section is a mean proportional between the two segments of the straight line which passes through the intersection of the double ordinate perpendicularly to the axis of the solid, and which is terminated by the surface of that solid.

PROBLEM IX.

To describe any oblique section of a solid of revolution, the position of the cutting plane being given in the determining section.

Let VWX be the determining section, sn the axis or line of the oblique section, VZ the axis.

Through h , any convenient point in sn , draw fg perpendicular to VZ , meeting the opposite sides of the determining section in f and g . Find hi in No. 2, a mean proportional between fh and hg . In No. 3, make Aa equal to sn , AP equal to sh , and Pa equal to hn . Through P draw MM' perpendicular to Aa . Make PM and PM' each equal to the mean proportional hi , and the points M , M' will be in the curve; and thus the curve may be drawn through a sufficient number of points found in the same manner as the points M , M' .

Fig. 170, No. 1.

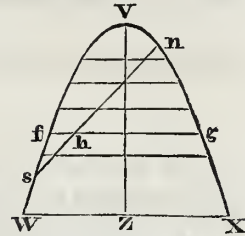
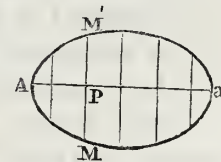


Fig. 170, No. 2.



Fig. 170, No. 3.



PROBLEM X.

To find the section of a solid of revolution by a plane parallel to the axis; given the generating figure, and the position of the line of section on the base of the solid.

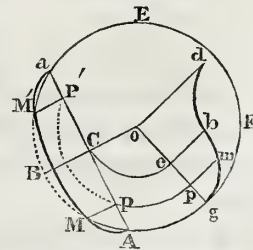
Let $AaEFg$ be the base of the solid, Aa the line of section, $ogmbd$ the generating figure, and og the line in which it meets the base, od being the altitude of the figure.

Draw oB perpendicular to Aa , meeting Aa in C . From the centre o with the radius oC , describe an arc Cc , meeting og in c , and draw cb perpendicular to og . Make CB equal to cb , and B is the summit or vertex of the curve.

In like manner from o with any convenient radius greater than oC , but less than og , describe the arc pPP' , meeting og in p , and AC in P , p . Draw pm perpendicular to og , and PM , $P'M'$ perpendicular to CA . Make PM , $P'M'$ each equal to pm , and the points M , M' will be in the curve.

A sufficient number of points being found in this manner, the curve $AMBM'a$ may be traced through them.

Fig. 171.



SCHOLIUM.

A solid of revolution comprehends that of a sphere, a cylinder, a cone, and the three conoids; the section may therefore be described by the rules peculiar to each of these solids.

E L L I P S O I D.

DEFINITIONS.

1. A SOLID, of which all its sections are similar ellipses, and are similarly situated and perpendicular to a straight line passing through their centres, so that every ellipse may have one of its axes in a section of the solid which is also an ellipse, is called an *ellipsoid*.
2. The section of the solid in which all the axes of the ellipses are situated, is called the *determinating ellipse*.
3. The centre of the determining ellipse is called the *centre* of the ellipsoid.
4. The line passing through the centre perpendicularly to the determining ellipse, and terminated by the surface of the solid, is called the *axis* of the solid.
5. A portion of the solid cut off by the determining ellipse, is called a *hemi-ellipsoid*.
6. In a hemi-ellipsoid, the determining ellipse is called the *base* of the solid.
7. The portion of an ellipsoid or of a hemi-ellipsoid contained between planes meeting each other in the axis of the solid, is called a *sectroid* of an ellipsoid, or *sectroid* of a hemi-ellipsoid.
8. The portion of the base of the solid which remains with the sectroid, is called the *base* of the sectroid.
9. The two surfaces which meet the axis, are called the *radial sides*.

GENERAL PROPERTIES OF THE ELLIPSOID.

1. Every section of the solid is an ellipse.
2. Every section of the solid perpendicular to the determining ellipse has one of its axes in the plane of the determining ellipse, and has the other perpendicular to that plane.
3. Any two sections of the solid cut by parallel planes, are similar ellipses and similarly situated.

Hence, if one of the parallel sections be given, the other may be found by having one of its axes given.

4. If the sectroid of a hemi-ellipsoid be cut by a plane parallel to the axis of the solid, and parallel to the chord which joins the two ends of the curve in the base of the sectroid, the points in which the cutting plane meet the curves of the radial sides are equally dist-

ant from the base.—See this principle applied in *Plate 5*, plan and elevation, *figs. 3* and *4*, which is of the greatest use in the construction of niches, domes, &c. where the ellipsoid, solid, or the hemi-ellipsoid, are employed.

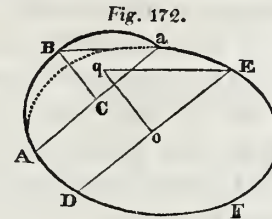
5. If through any given point in the curve of one of the radial sides a straight line be drawn parallel to the chord of the base, the straight line thus drawn will meet the curve of the other radial side.

6. Hence a cylindric surface, or cylindroidic surfaces, may be made to pass through the curves of both radial sides.

PROBLEM XI.

Given the base of a hemi-ellipsoid, the height of the axis above the base, to find a section of the solid parallel to the axis to pass along any given chord in the base.

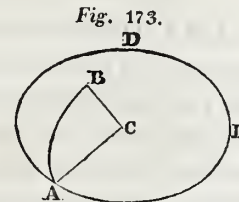
Let *AaEFD* be the base of the solid, *o* the centre of the base, and *Aa* the line of section. Through *o*, draw *oq* perpendicular to *DE*, and make *oq* equal to the height of the axis; join *Eq*. Bisect *Aa* in *C*, and draw *CB* perpendicular to *Aa*. Make the angle *CaB* equal to the angle *oEq*. Upon the axis *Aa* and semi-axis *CB* describe the semi-ellipse *ABa*, which is the section of the solid required.



PROBLEM XII.

Given the base of a hemi-ellipsoid and the height of the axis, to find the radial section upon any given line in the base of the solid.

Let *ADE* be the base of the solid, *AC* the base line of the radial section. Draw *CB* perpendicular to *CA*, and make *CB* equal to the height of the axis. Upon the semi-axes *AC* and *CB* describe the elliptic arc *AB*, which is the quadrant of the whole curve, and *ABC* is the radial section required.



DOMOID.

DEFINITIONS.

1. If a solid has one plane side, and if any portion of the solid contained between two planes which meet each other in a line perpendicular to that side be cut by any two planes parallel to that same side, and if the two sections be similar figures, the solid is called a *domoid*.

2. The perpendicular is called the *axis* of the domoid.
3. The side of the solid to which the axis is perpendicular, is called the *base* of the domoid.
4. A portion of a domoid contained between two planes passing along the axis of the solid, is called the *sectroid* of a domoid.
5. The two sides of the sectroid of a domoid which meet the axis of the solid, are called the *radial sides* of that sectroid.
6. The plane of the sectroid which is perpendicular to the axis, is called the *base* of the sectroid.
7. The two straight edges of the base which meet in the axis, are called the *radials* of the base of the sectroid.
8. The portion of the surface of a domoid which is not the base, is called the *curved surface* of that domoid.
9. If a domoid be cut by a plane parallel to the plane of the base, the portion of the solid comprehended between the two parallel planes is called a *truncated domoid*.
10. In each radial side of the sectroid of a domoid, the curve at the meeting of that radial side and the curved surface of the solid is called the *curve* of that radial side.
11. If the curve of the radial section of a domoid be convex without the figure, and the base be a circle greater than any of its parallel sections, the solid is called a *cupoloid*.

PROBLEM XIII.

Given the base of a domoid, a radial section, and the position of the line of section in the base, to find another radial section passing through any given line in that base.

Let $AaDEF$ be the base of the domoid, $ambC$ the given section, aC its line of section; and let AC be the line in which the required radial section meets the base. Join Aa . Draw pP parallel to Aa , meeting aC in p , and AC in P . Draw pm perpendicular to aC , and PM perpendicular to AC ; make PM equal to pm , and M is a point in the curve.

Hence, if the curve of the given section amb be the quadrantal arc of an ellipse, the curve AMB to be found will also be the quadrantal arc of an ellipse or of a circle; and therefore the curve may be described by a trammel, the axis CB being equal to Cb .

But if the curve has no peculiar property, we may find as many points as will be necessary to trace the curve in the same manner as the single point was found.

Fig. 174, No. 1.

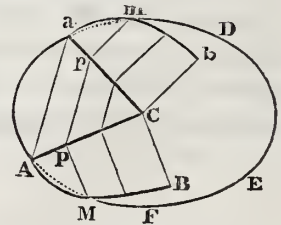
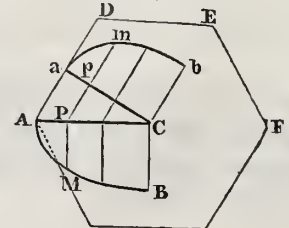


Fig. 174, No. 2.



U N G U L U S.

DEFINITIONS.

1. THE portion of a cylindroid comprehended between two planes which meet each other in a point of the axis, and which have their line of concurrence parallel to one of the axes of the ends, is called an *ungulus*.

2. If an ungulus be cut into two parts by a plane perpendicular to the line of concurrence, each of the parts is called a *segment* of an ungulus.

3. The side of the segment of an ungulus which is perpendicular to the line of concurrence, is called the *base* of the solid.

4. The two sides which are perpendicular to the base, are called the *radial sides*.

5. The line of concurrence in which the radial sides meet, is called the *axis* of the ungulus or of the section of the ungulus.

6. The straight line in which the curved surface and the base meet each other, is called the *line of subtense*.

7. When the two radial sides of an ungulus or of the segment of an ungulus are equal, the solid is called an *isosceles ungulus*, or an *isosceles segment* of an ungulus.

8. When an ungulus is cut by a plane parallel to its base, the part of the solid contained between the two parallel planes is called the *frustum* of an ungulus.

ORTHO PROJECTION.

THE position of a point with regard to a line is given, when its distance from two fixed points in that line is known.

The position of a point in space with regard to a plane is given, when its distances from any three points in that plane are known.

The position of a point in space with regard to a plane is also given, when that point is in another plane of which its inclination to and the line in which it meets the first plane are known, and when the position of the point with regard to the line of section is given.

The point in which a perpendicular from a given point in space to a given plane meets that plane, is called the *seat* of that point in space.

The perpendicular is called the *height* of that point from its seat.

DEFINITIONS.

1. Orthopjection is a method of finding the representation of a object by drawing straight lines from every point of that object perpendicular to a plane given in position to meet that plane.

2. The plane which meets the perpendicular is called the *plane* of projection.

3. The point where any perpendicular meets the plane of projection, is called the *seat* of that point of the original object from which the perpendicular is down.

4. The seat of a line is a line in the plane of projection in which a perpendicular drawn from any point in that line will terminate.

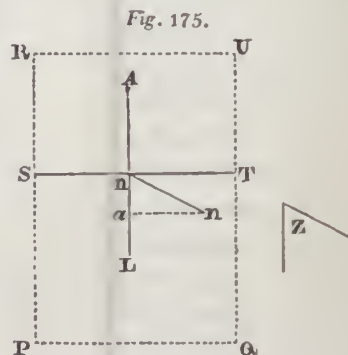
PROBLEM I.

Given the inclination of the planes, and the situation of any point in the original plane, to find the seat of that point.

Let A be a given point in the plane $RSTU$, and Z the angle of inclination of the planes, and let $SPQT$ be the plane of projection, and their line of section be ST .

Draw Aa perpendicular to ST , meeting ST in m . At the point m in the straight line am , make the angle amn equal to Z , and make mn equal to mA . Draw na perpendicular to am , and a will be the seat of the original point A .

Coroll. If the angle amn is a right angle, the seat a of the original point A will coincide with the point m .



DEMONSTRATION.

Suppose the triangle amn to be turned round upon ma , so that its plane may be perpendicular to the plane $SPQT$, and let the plane $RSTU$ be turned round ST , until it is incline in the angle Z given: then the line mA will be in a plane perpendicular to ST , the line of common section; and since the plane amn is by supposition also perpendicular to ST , mA must fall upon mn ; and because mn is equal to mA , the point A will fall upon the point n . And because an is perpendicular to am , and the plane amn is by hypothesis perpendicular to the plane $SPQT$, the straight line an will, on this supposition, be also perpendicular to the plane $SPQT$. Then A , now supposed to coincide with n , will be the original point in space in respect to the plane $SPQT$; and an being supposed perpendicular to the plane $SPQT$, the point a will be the seat of the original point A , by the definition.

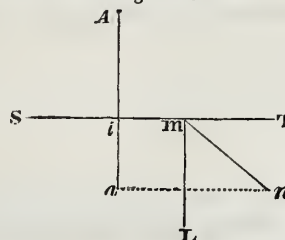
SCHOLIUM.

Hence, because the angle made by two straight lines drawn from the same point perpendicular to the line of common section of two planes, one line in each plane, is equal to the angle contained by two other straight lines drawn from any other point perpendicular to the line of common section of two planes, one line in each plane; it will not be necessary to make one leg of the angle amn in the straight line perpendicular to the original point A , for any other line perpendicular to ST will answer the same purpose.

Thus, -Draw Aa perpendicular to ST , meeting ST ; and from an convenient point m in ST , draw mL parallel to Aa . Make the angle Lmn equal to the angle of the inclination of the planes, and make mn equal to ia . Draw na parallel to ST ; the a will be the seat of A , as required.

It is evident that the same angle Lmn will serve for obtaining the seats of any number of points whatever.

Fig. 176.

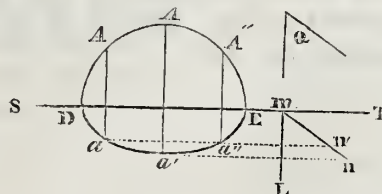


EXAMPLES.

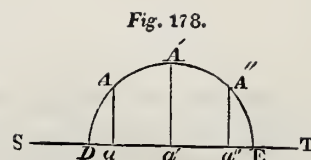
Ex. 1. Find the seats of any number of points in a given curve in a plane at a given angle Q with the plane of projection.

Here the points a, a', a'' are the seats of the points of the original points A, A', A'' . In this example the original figure is a semicircle, and the original plane intersects the plane of projection in the diameter DE .

Fig. 177.



Ex. 2. If the diameter DE be in the line of section ST , and the angle Q a right angle, we have nothing more to do than draw $Aa, A'a', A''a''$, &c. perpendicular to ST or DE , and the points a, a', a'' , &c., are the seats of the original point A, A', A'' , &c.



PROBLEM II.

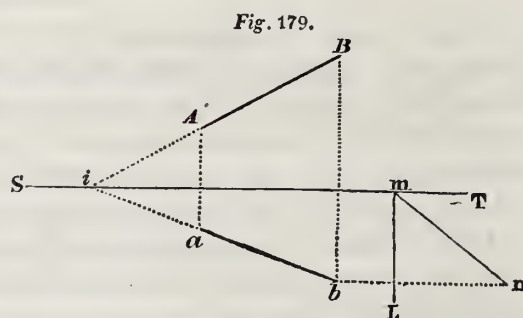
Given the magnitude of a line and its position to the plane of projection, to find the seat of that line.

CASE 1.

Find the seat b of the original point B , and produce BA to meet ST in i . Join bi , and draw Aa parallel to Bb ; then will ab be the seat of the original line AB .

Or thus:—

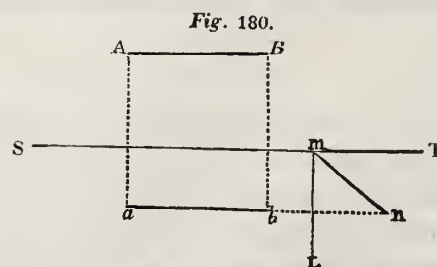
Find a and b , the seats of the original points, and join ab .



CASE 2.

When the original line is parallel to the line of section.

Find a , the seat of the original point A , and draw ab parallel to AB , and draw Bb parallel to Aa ; then ab is the seat of the original line AB .

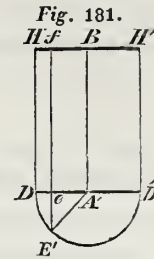


PROBLEM III.

Find the seat of a straight line on the curved surface of a cylinder upon a given axial section, the point in the circumference of the base, in which the original line terminates, being given.

Let $DD'H'H$ be the axial section, AB being the axis itself. On DD' as a diameter describe the semicircle DED' , and let E be the given point in the semi-circumference DED' .

Find e , the seat of the point E , the semicircle being supposed to be raised upon DD' perpendicular to the plane $DD'H'H$. Draw ef parallel to AB , meeting HH' in f ; then ef is the seat of the line required.



PROBLEM IV.

To find the seat of a straight line on the curved surface of a cone upon a section passing along the axis, a point in the circumference of the base or end in which the original line terminates being given.

Let $DD'C$ be the given section, AC be the axis itself.

On DD' , as a diameter, describe the semicircle DED' , and let E be the given point in the semi-circumference DED' .

Find e , the seat of the point E ; the semicircle being supposed to be raised upon DD' perpendicular to the plane $DD'C$. Join eC ; then eC is the seat of the line required.

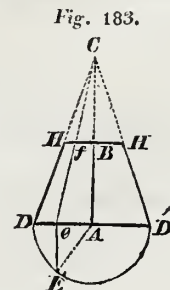


PROBLEM V.

Find the seat of a straight line on the curved surface of the frustum of a given cone upon a section passing along the axis, a point in the circumference of one of the ends in which the original line terminates being given.

Let $DD'H'H$ be the given section, and AB the axis.

Produce DH , $D'H'$ to meet each other in C . Then, by the preceding Example, find eC the seat of the straight line passing through the point E in the base, and through C the vertex of the cone, and let eC meet HH' in f , and ef will be the seat of the line required.



PROBLEM VI.

Given the seat of a line, and the distance of each extremity of its original from each seat, to find the length of the original line.

Draw two straight lines at a right angle. Make one of the legs of the right angle equal to the seat of the line given, and the other leg equal to the difference of the distances of the seat of each end of the line from its original point; then join the extremities of these two lines which do not meet, and the hypotenuse will be the length of the original line.

EXAMPLE.

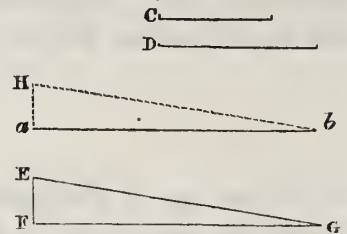
Let ab be the seat of an original line, and let C be the distance between a and its original point, and D the distance between b and its original point.

Construct the right angle EFG , and make one of the legs FG equal to ab , and the other FE equal to the difference between C and D , and join EG ; then EG is the length of the straight line, of which ab is the seat.

Or thus:—

Draw aH perpendicular to ab , and make aH equal to the difference between C and D , and join bH ; then will bH be the length of the line which is the original of ab .

Fig. 184.



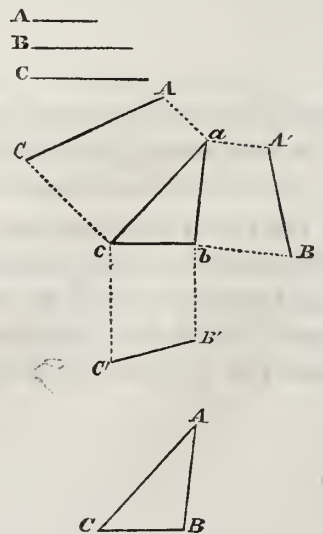
PROBLEM VII.

Given the ^{seat} ~~trace~~ abc of a triangle, and the height of each original point of concurrence of that triangle from its trace, to find the original figure.

Draw two straight lines from the extremity of each side perpendicular to that side. Upon each two lines drawn from each point of concurrence, set the height from that point; join the two extremities of each pair of parallels, and with the three lines thus joined, describe a triangle.

Here the corresponding parts of each two figures are referred to by similar letters.

Fig. 185.



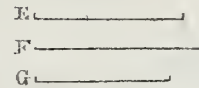
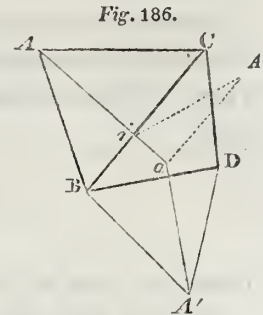
PROBLEM VIII.

Given the distances of a point in space from three fixed points in the plane of projection, to find the seat of that point and the distance between the point and its seat.

Let the three given points be B, C, D, and let the distances of these points from the original point be equal to the three straight lines E, F, G respectively.

Join BD, BC, CD. From B, with a radius equal to the line E, describe two arcs; and from C, with a radius equal to the line F, describe an arc meeting one of the arcs described from B in A; and from D, with a radius equal to the line G, describe an arc meeting the other arc described from B in the point A'. Draw the straight line Aa perpendicular to BC, and draw the straight line A'a perpendicular to BD; then the point a will be the seat of the original point required.

Let Aa meet BC in i. Draw aA'' parallel to BC. From i, with the radius iA, describe an arc meeting aA'' in A''. Then aA is the distance of the original point from its seat a. This will be evident by turning up the three triangles till the three points A, A', A'' coincide.



PLAN AND ELEVATION.

DEFINITIONS.

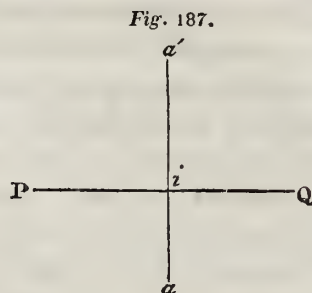
1. WHEN two planes are at a right angle with each other, and when an original object between them is first represented upon one of the planes and then on the other, the two representations are called the *seats* of that object.
2. The first seat of the object is called the *plan* of that object.
3. The second seat is called the *elevation* of that object.
4. The plane on which the plan is made, is called the *primary plane*.
5. The plane on which the elevation is made, is called the *plane of elevation*.
6. The planes on which the two seats are made, are called the *orthographic planes*, or *planes of projection*.
7. The line of separation between the orthographic planes, is called the *base line* of elevation, or the *base* of the plane of elevation.

PROBLEM IX.

Given the plan of a point, and the distance of the original point from its plan, and base line of the plane of elevation, to find the elevation of the point.

Let a be the plan of the point, and PQ the base line of the elevation.

Draw aa' perpendicular to PQ , meeting PQ in i . Make ia' equal to the distance of the original point from its plan. Then a' will be the elevation of the point of which a is the plan.

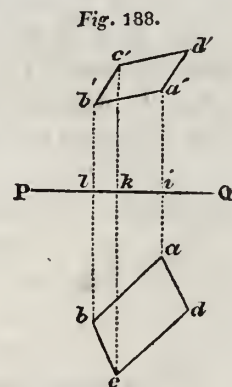


PROBLEM X.

Given the seats of three of the angular points of a parallelogram in the plan, and the distances of the points from their seats, to find the elevation of the parallelogram, and to complete the plan.

Let a, b, c be the seats of the three angular points in the plane.

Draw aa', bb', cc' perpendicular to PQ , meeting PQ in the points i, k, l . Make ia' equal to the height of the angular point above a , lb' equal to the height of the angular point above b , and kc' equal to the height of the angular point above c ; join ab, bc in the plan. Draw cd parallel to ba , and ad parallel to bc , and $abcd$ will be the plan. Again, join $a'b', bc$. Draw $a'd'$ parallel to $b'c'$, and $c'd'$ parallel to $b'a'$, and $a'b'c'd'$ will be the elevation.

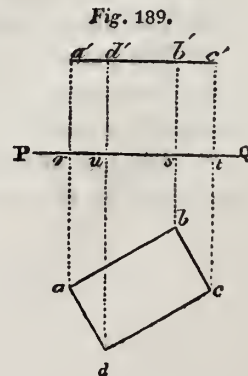


PROBLEM XI.

Given the seat of one of the edges of a given plane figure in the primary plane, to find the elevation, and to complete the plan of that figure, the plane of the figure being parallel to the primary plane, and its distance from it known.

Let ab be the seat of the given side in the primary plane.

On ab describe the figure $abcd$ equal and similar to the original figure, and $abcd$ will be the plan required. Draw aa' perpendicular to PQ , meeting it in r . Make ra' equal to the height of the figure above the primary plane, and draw $a'c'$ parallel to PQ . Draw bb', cc', dd' parallel to aa' , meeting $a'c'$ in b', c', d' , and the straight line $a'c'$ will be the elevation.

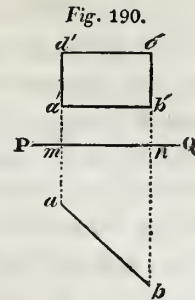


PROBLEM XII.

Given the seat in the primary plane of one of the edges of a given rectangle of which one edge is parallel and the face perpendicular to the primary plane, to find the elevation of the rectangle, the height of the edge above its seat being given.

Let ab be the given seat. Draw ad' perpendicular to PQ , meeting PQ in m . In md' make ma' equal to the height of the edge above its seat, and $a'd'$ equal to the vertical dimension of the rectangle. Draw bc parallel to ad' , also draw $a'b'$, $d'c'$ parallel to PQ , ab is the plan and $a'b'c'd'$ the elevation required.

If ab be perpendicular to PQ , then both plan and elevation will be projected into straight lines.

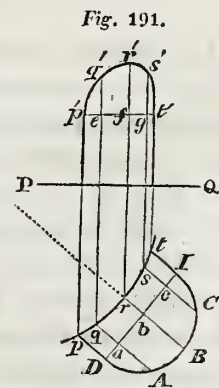


PROBLEM XIII.

To find the elevation of a circular archoid intersecting the surface of a cylinder, the base of the archoid being parallel, and the axial plane of that archoid and the axes of the cylinder perpendicular to the primary plane, the plan being given.

Let $pqrst$ be the intersection of the surface of the cylinder with the primary plane, Dp and It the seats of sides of the archoid.

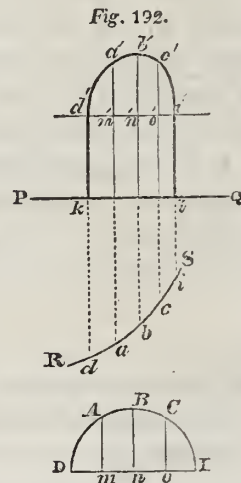
Draw Dp perpendicular to DI ; on DI describe the semicircle $DABCI$. In the semicircular arc take any number of points A , B , C ; then suppose the arc $DABCI$ to be perpendicular to the primary plane. Find the seats a , b , c by Prob. i, p. 116, of the points A , B , C . Draw aq , br , cs , &c. parallel to Dp . Find the elevation pt of the base of the archoid. Make eq' , fr' , gs' , &c. respectively equal to aA , bB , cC , &c., and describe the curve $pqrst$, which will be the elevation required.



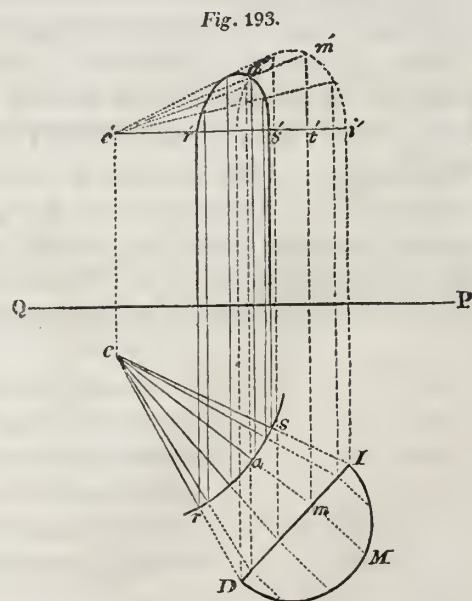
PROBLEM XIV.

Find the elevation of the arc of a semicircle or other curve bent into a surface which is perpendicular to the primary plane, the diameter or chord being in a plane parallel to the primary plane; given the intersection of the surface and the seats of the extremities of that diameter or chord, and the heights of the two seats.

Let RS be the intersection of the surface, and the two points d, i be the seats of the extremities of the diameter. Draw dd' and ii' perpendicular to PQ , meeting PQ in k and l . Make kd and li' each equal to the height of the diameter above the primary plane, and draw $d'i'$ parallel to PQ . Divide the curve di of the intersection into any number of equal parts, as here into four, and set the distances da, ab, bc, ci in the straight line DI from D to m , from m to n , and from n to o . Draw mA, nB, oC perpendicular to DI , meeting the arc in A, B, C . From the points a, b, c in RS , draw $aa', bb', cc', \&c.$ meeting $d'i'$ in m, n, o . Make $m'a', n'b', o'c'$ respectively equal to mA, nB, oC , the ordinates of the semicircle, and through the points d', a', b', c', i' draw the curve $d'a'b'c'i'$, which is the elevation required.



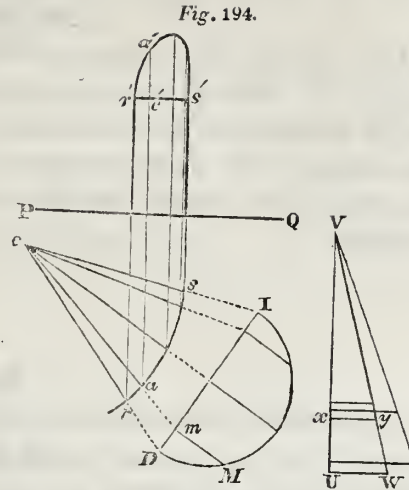
The method of finding the elevation of the section of a cone cut by a surface perpendicular to the primary plane, the axis of the cone being parallel to the primary plane, and the plan of the axial section being given, is to find the elevation of the entire cone; then find plans and elevations of the lines on the conic surface, as $mc, m'c'$, and the perpendicular to PQ , drawn from a to meet $m'c'$, will give the elevation a' of the point above a in the plan.



SCHOLIUM.

In finding the elevation of the whole cone, two sets of perpendicular lines are introduced; one set, however, would only be necessary, provided the perpendicular heights of the seats in the line rs of a sufficient number of points in the conic surface were known. This object being obtained would lessen the labour, and prevent the unnecessary confusion of lines in the construction of complex elevations.

To obviate this labour, construct a right angle VUW . Make UV equal to mc , UW equal to mM , and join VW . In VU , take Vx equal to ca , and draw xy perpendicular to UV . Make $c'a'$ equal to xy ; then the point a' will be the elevation of the point in the conic surface of which a is the plan. In this manner, as many points as will be necessary for drawing the curve may be found.

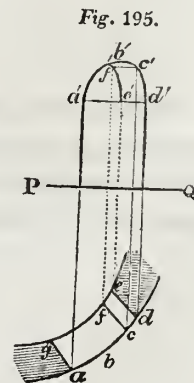


PROBLEM XV.

Given the plan and elevation of a circular arch in a circular wall, to describe the line of intersection by the interior surface of the wall and the surface of the arch.

Let $abcdefg$ be the plan of the intrados of the arch, $abcd$ being the seat of the exterior surface of the wall, and efg that of the interior surface, de and ag the jambs, and let $a'b'c'd'$ be the elevation of the intersection of the intrados of the arch, and the exterior convex surface of the wall, and let the straight line $a'd'$ be the representation of the plane on which the arch rests.

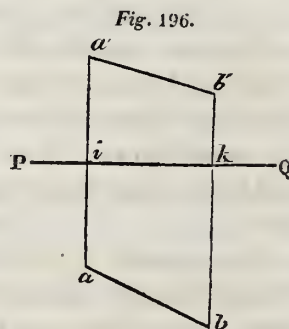
Draw ee' perpendicular to PQ , meeting $a'd'$ in e' . From any point c in the seat of the exterior surface, draw cf parallel to de in the plan. Draw cc' parallel to ee' , meeting the elevation of the intersection of the arch and the exterior surface of the wall in c' , and draw $c'f'$ parallel to PQ , and ff' parallel to ee' . Through the points e' , f' , &c. draw the curve $e'f'$, which will be the inner edge of the intrados of the arch, as required.



PROBLEM XVI.

Given the plan of a line, and the distance of each extremity of that line from its seat, to find the elevation of the line.

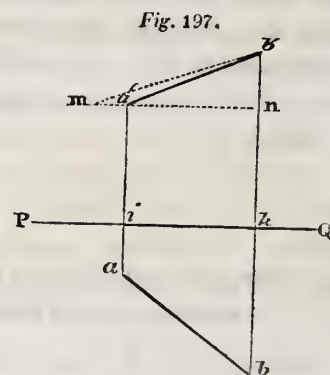
Draw aa' , bb' perpendicular to PQ , meeting PQ in the points i , k . Make ia equal to the height of the one extremity of the original line above its seat a , and make kb' equal to the height of the other extremity of the original line above its seat b , and join $a'b'$, and $a'b'$ is the elevation of the line required.



PROBLEM XVII.

Given the two seats ab , $a'b'$ of a straight line, to find the length of the original line.

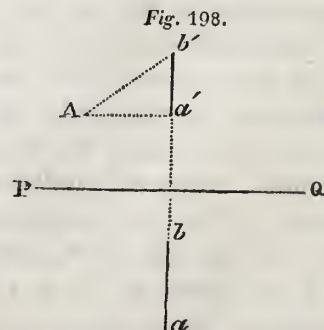
Through a' , one extremity of the elevation, draw mn parallel to PQ , meeting kb' the perpendicular from the other in the point n ; make nm equal in length to ba , the plan of the line; then the distance of the points m , b' will be the length of the original line.



PROBLEM XVIII.

When the original line is in a plane perpendicular to the line of common section.

Through a' , one extremity of the elevation of the line, draw $a'A$ parallel to PQ . Make $a'A$ equal to ab , and the distance $b'A$ is the length of the original line as required.



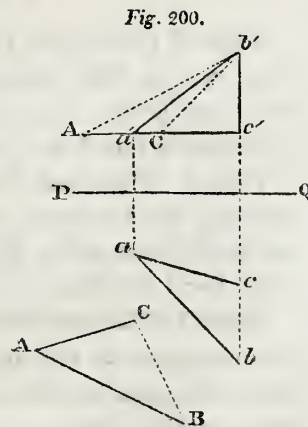
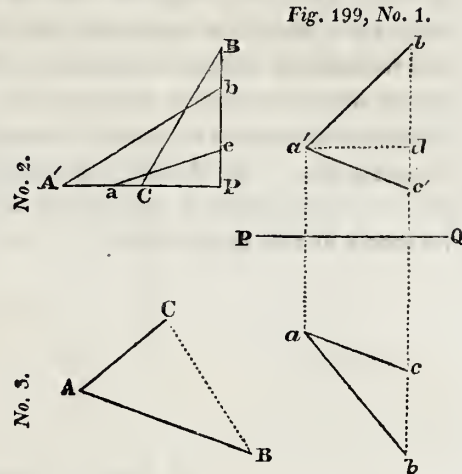
PROBLEM XIX.

Given the two seats bac , $b'a'c'$ of two straight lines meeting in a point, to find the angle contained by the two original lines.

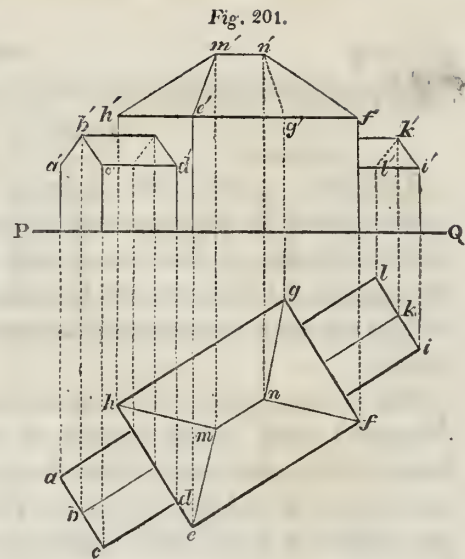
Find Ab , ac , BC , the lengths of the three original lines, by the preceding Problem, and with the lengths of these three lines describe the triangle BAC , *No. 3*, and the angle BAC will be the angle contained by the two original lines.

The separate figure *No. 2* might have been dispensed with, as the lengths of the three traces on the plan might have been applied upon the horizontal lines of the elevation; but the confusion which this application would have introduced in order to explain the method is avoided.

However, if one of the seats ac' be parallel to the primary plane, the originals Ab' , $b'C$ of the two seats ab , bc on the plan may be found in the elevation without any confusion. Then in the triangle BAC , the side AB is equal to Ab' , BC equal to $b'C$, and AC equal to ac on the plan.



Coroll. If the two seats of an object be given, the object itself may be determined; for the lengths of the lines may be found by Prob. xvii, p. 126, and every angle by the last Problem, p. 127. Thus the length of the hip $e'm'$, the angle $e'm'n'$ which the hip makes with the ridge, and in short the distance between any two fixed points whatever may be found, as the original distance represented by $e'i'$ on the elevation, and ei on the plan. In Masonry, Carpentry, Joinery, &c. every original length and angle may be found by this simple idea.



PLATES ILLUSTRATING ORTHOPROJECTION.

Plate 3 (Plan and Elevation, Orthographical Projection) shows the application of Prob. xiii, p. 123, in describing the elevation of semicircular arches in a circular wall. The seat of the thickness of the arch is given at st on the plan, and its seat on the elevation is found at st , as in Prob. xv, p. 125.

Plate 4 (Plan and Elevation, Orthographical Projection) exhibits the manner of describing the elevation of arches with conic intrados. The elevations of the two arches on the left side of the plate are described by Problem xiv, pp. 124, 125, and that on the right hand by Prob. xiv, p. 125.

Plate 5 (Plan and Elevation, Orthographical Projection) exhibits the manner of finding the elevation of the intersection of a cylindric surface with the surface of a rotative solid, the axes of both solids being parallel, and the bases of the solids being parallel to the primitive plane.

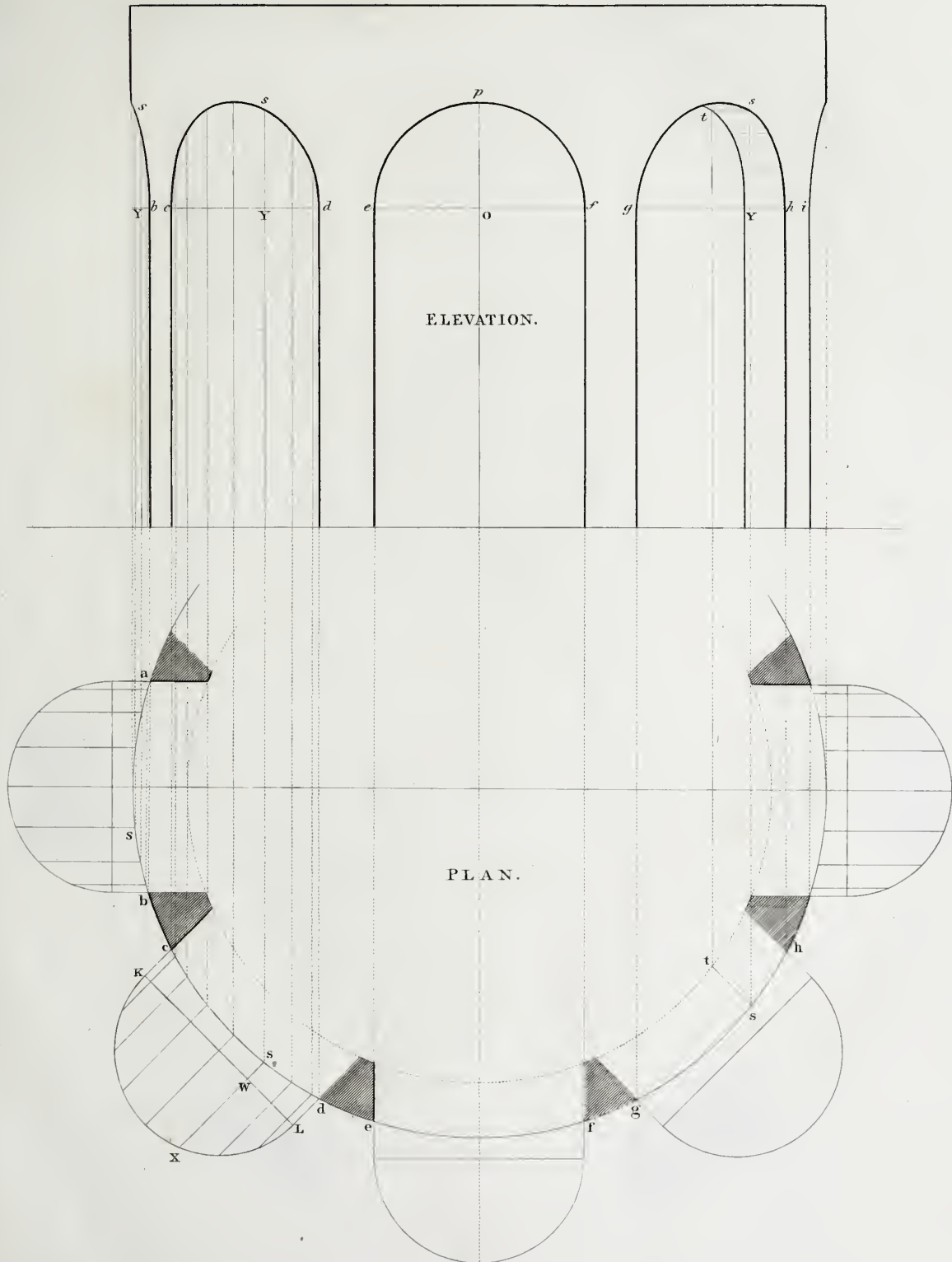
One general principle of finding a point in the elevation is this:—

Draw ie perpendicular to ad , meeting the curve of the base in e . In the curve line acd , which represents the intersection of the cylindric surface with the base of the solid, take any point b , and draw bb perpendicular to the base line ad of elevation, meeting ad in K . Join Ib , and produce Ib to meet the curve acd (which is the intersection of the curved surface of the solid and its base) in the point f : join ef , and draw bG parallel to fe , meeting Ie in G . Draw GH perpendicular to Ie , meeting the curve dHe in H ; $IeHd$

ORTHOGRAPHICAL PROJECTION.

PLAN AND ELEVATION.

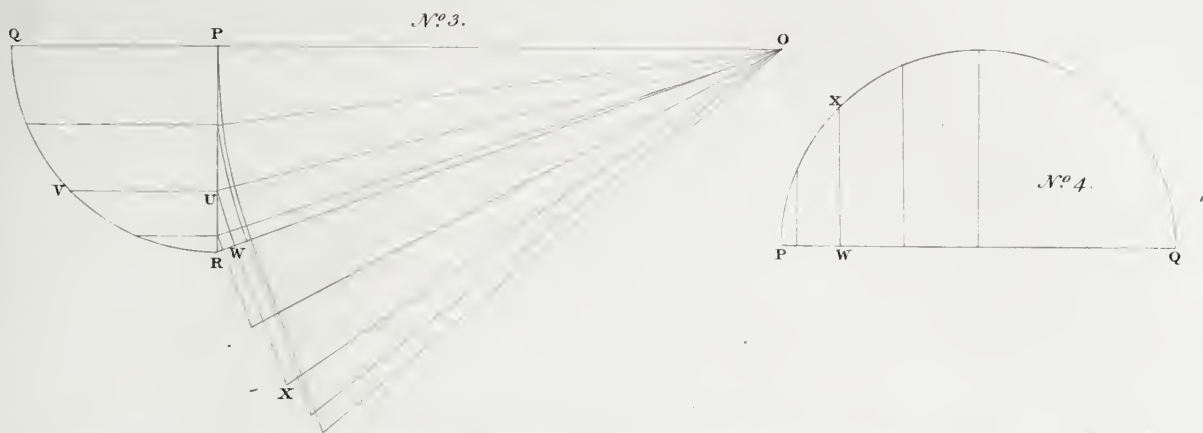
PLATE III.



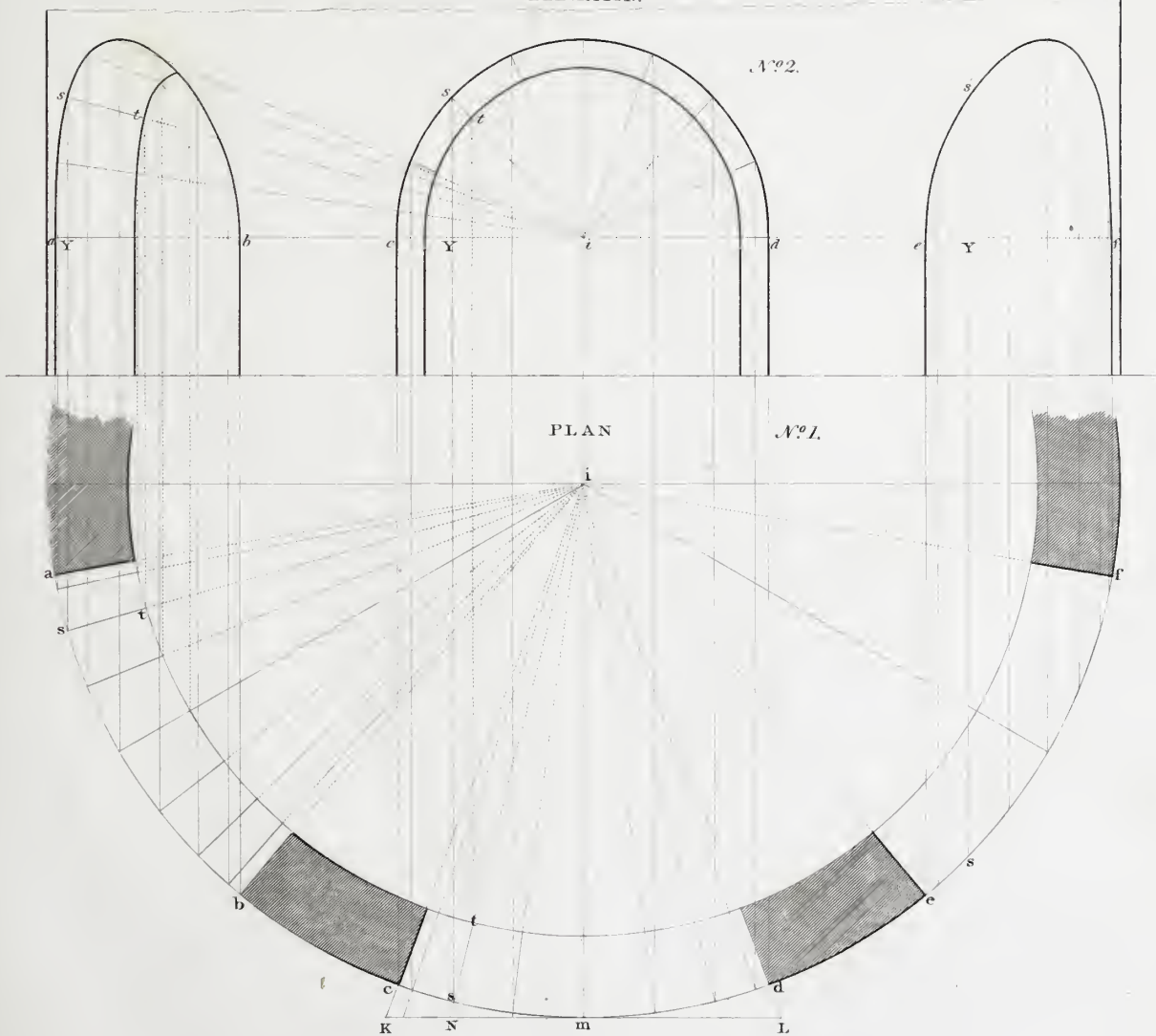
ORTHOGRAPHICAL PROJECTION.

PLAN AND ELEVATION.

PLATE IV.



ELEVATION.



ORTHOGRAPHICAL PROJECTION.

PLAN AND ELEVATION.

PLATE V.

Fig. 1.

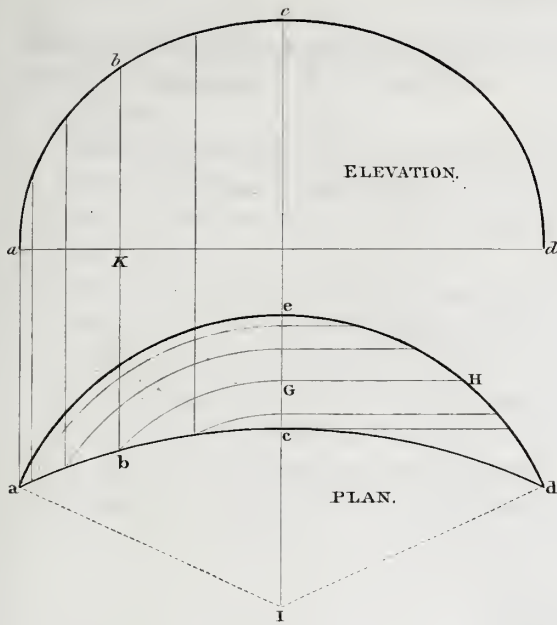


Fig. 2.

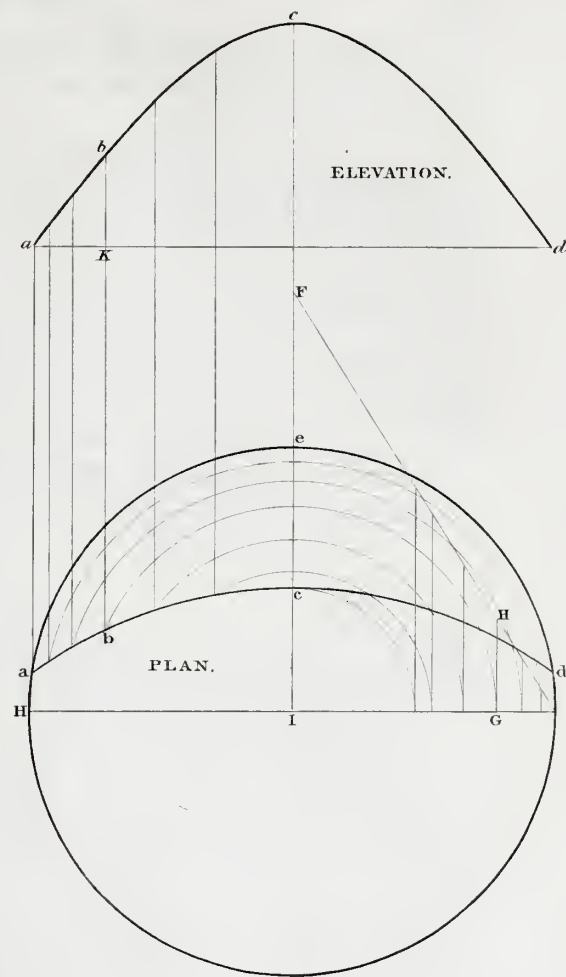


Fig. 3.

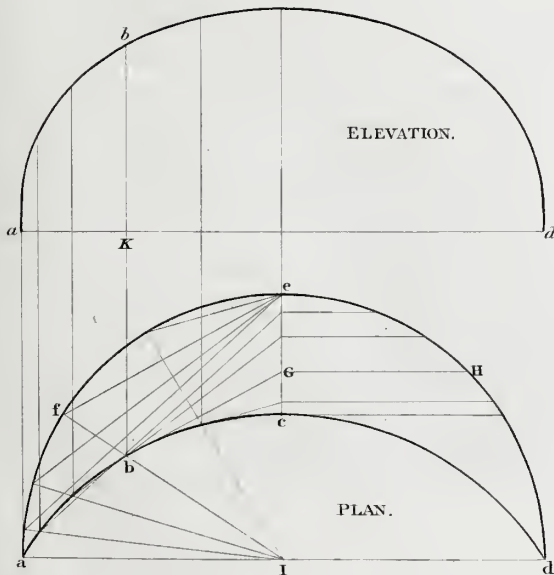
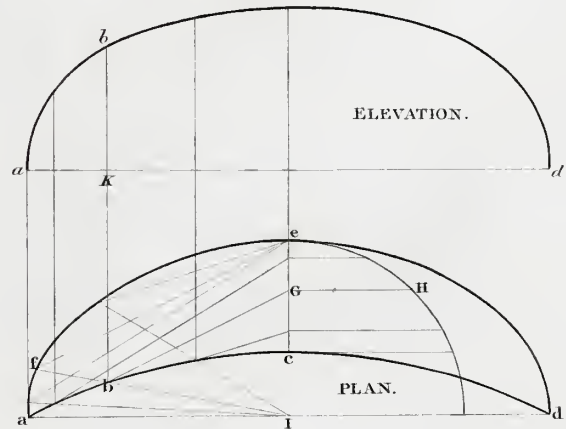


Fig. 4.



being considered as a section of the solid upon Ie perpendicular to ad , the chord of the base, or to the plane of the base itself. Make Kb equal to GH ; then b is a point in the elevation.

These figures apply to the elevation of niches in circular walls.

Figs. 1, 2, 3 have circular bases; *fig. 4* has an elliptic base, and the solid is generated round an axis parallel to the primary plane; *fig. 2* is a cone with its axis perpendicular to the base.

Though the method of finding the elevation of the two surfaces is general, yet, when the perimeter is circular, the method may be more simple; thus:—

In *fig. 1*, take any point b as before. From the centre I with the radius Ib describe the arc bG , meeting Ie in G . Draw GH perpendicular to Ie , and, bb being drawn as before, make Kb equal to GH , and b is a point in the elevation.

In *fig. 2*, let IFP be a section of the cone. From I , with the radius Ib , describe the arc bG meeting HP in G . Draw GH perpendicular to IP , meeting PF in H . Make Kb equal to GH , and b is a point in the elevation.

PROBLEM XX.

To find the equation of the curve which is the seat of the intersection of two semi-cylinders or two circular archoids, on a plane passing along each of their axes.

Let CO , $C'O$ be the axes of the two solids.

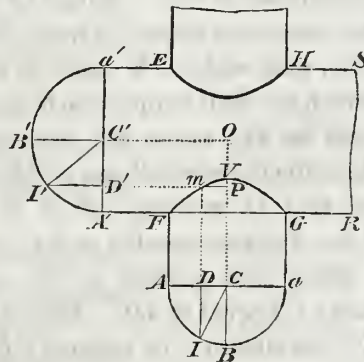
Through any point C in CO , draw Aa perpendicular to CO , and, with a radius equal to the radius of the less circular archoid, describe the semicircle $AIBa$. Draw AF and aG parallel to CO .

Again, through any point C' in $C'O$, draw $A'a'$ perpendicular to $C'O$, and, with a radius equal to the radius of the greater circular archoid, describe the semicircle $A'TB'a'$. Draw $A'R$ parallel to $C'O$, meeting AF in F , and AG in G .

In the lesser archoid, imagine the semicircle $AIBa$ to be turned perpendicular to the plane of its base, and a plane drawn parallel to the axis CO , and perpendicular to the base, meeting the end $AIBa$ in the line DI , and $AFGa$ in Dm .

In like manner, in the archoid which has the greatest radius, imagine the semicircle $A'TB'a'$ to be turned perpendicularly to the plane of its base, and a plane drawn parallel to the axis $C'O$, and perpendicular to the plane of the base, meeting the end $A'TB'a'$ in $D'I$, and the plane $A'RSa'$ in $D'm$, so that ID' may be equal to ID . Join $C'I$ and CI . Produce $D'm$ to meet CO in P .

Fig. 202.



Let $C'I = R$, $CI = r$, $D'I = DI = z$, $OP = C'D' = x$, and $Pm = CD = y$.

Then by the semicircle $\begin{cases} A'B'a' & - & - & - & - & - & - & - & - & - & R^2 - x^2 = z^2 \\ ABa & - & - & - & - & - & - & - & - & - & z^2 = r^2 - y^2 \end{cases}$

Therefore, eliminating z by adding these equations together, we have $R^2 - x^2 = r^2 - y^2$, from which it appears that the curve FmG is an equilateral hyperbola, because the co-efficients of x and y are each unity, and have like signs.

Let $y = 0$, and $R^2 - x^2 = r^2$, or $x^2 = R^2 - r^2$; whence it appears that x is the base of a right angled triangle, of which the hypotenuse is R , and the base r .

PROBLEM XXI.

To find the equation of the curve which is the seat of the intersection of a circular and an elliptic archoid, upon a plane passing along the axis of each solid, and coincident with each base.

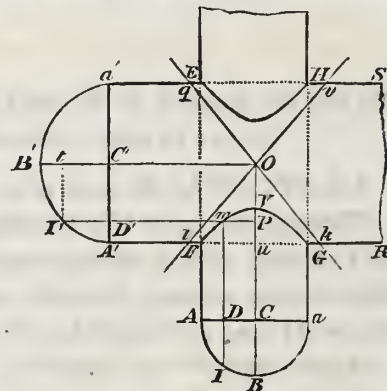
Let CO , $C'O$ be the axis of the two solids.

Through any point C in CO , draw Aa perpendicular to CO . From C , with a radius equal to the circular archoid, describe the semicircle $AIBa$. Draw AE and aH parallel to CO .

Again, through any point C' in CO , draw $A'a'$ perpendicular to $C'O$. Make CA' , $C'a'$ each equal to the semi-axis major. Draw $C'B'$ perpendicular to $A'a'$, and make $C'B'$ equal to the semi-axis minor, which we shall suppose to be greater than CB ; then with the axis major $A'a'$, and semi-axis minor $C'B'$, describe the semi-ellipse $A'I'B'a'$. Draw $a'S$ parallel to $C'O$, meeting AE in E , and aH in H , and draw $A'R$ also parallel to $C'O$, meeting AE in F , and aH in G .

Draw Dm parallel to CO , and produce mD to I , and let D meet Aa in D . In $C'B'$ make $C't$ equal to DI . Draw tI' parallel to $A'a'$, meeting the elliptic arc in I' , and draw $I'P$ parallel to $C'O$, meeting CO in P , and Dm in m ; then m is a point in the curve.

Fig. 203.



DEMONSTRATION.

For suppose the semi-ellipse $A'I'B'a'$ and the semicircle $AIBa$ to be each raised perpendicular to the plane $EFGH$; then the plane passing through $D'I'$ and $D'm$, and the plane passing through DI and Dm , would intersect each other in a line perpendicular to the plane $EFGH$ from m ; and the distance between m and the surface on the perpendicular would be equal to $D'I'$ or DI .

Let $C'A' = C'a' = a$, $C'B' = b$, $CA = r$, $OP = C'D' = x$, and $Pm = CD = y$.

Then by the semi-ellipse $A'IB'a'$, $a^2z^2 = b^2(a^2 - x^2)$,
and by the semicircle $AIBa$, $z^2 = r^2 - y^2$.

Multiply the second equation by a^2 , and $a^2z^2 = a^2r^2 - a^2y^2$; therefore

$$a^2r^2 - a^2y^2 = a^2b^2 - b^2x^2,$$

which is an equation to an hyperbola. Now, making x infinite, y will also be infinite, and the equation will become $bx = ay$; therefore $a : b :: x : y$; so that in OC , making $Ou = a$, and in AR , making uk and ul each equal to $b = C'B$, and drawing lv and kq through O , the straight lines lv and kq will be the asymptotes of the curve.

The transverse axis will be found by making $y = 0$ in the equation $a^2r^2 - a^2y^2 = a^2b^2 - b^2x^2$, and finding the value of x , which is $x = \frac{a}{b} \sqrt{b^2 - r^2}$.

PROBLEM XXII.

If the surface of an elliptic archoid meet the surface of a hemisphere, and if the bases of the two solids be in the same plane, the figure which is the seat of the intersection of the two surfaces in the plane of their bases will be a conic section.

Let $EFGH$ be the base of the hemisphere, $AaGF$ the base of the archoid, and $AIBa$ the semi-ellipse which forms the end of the archoid, and FmG be the seat of the intersection of the two curved surfaces.

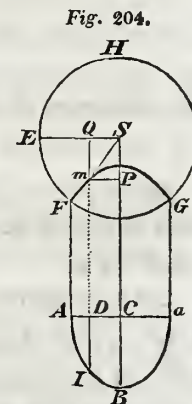
Suppose a plane drawn perpendicular to the base $AaGF$ of the archoid along the axis CS , meeting ABa in the semi-axis CB , and let a plane be drawn through m , parallel to this plane, meeting the semi-ellipse $AIBa$ in the line DI .

Also through the centre S of the hemisphere, suppose a third plane to be drawn perpendicular to the last two planes, and to the plane of the bases of the two solids, meeting the base $EFGH$ in SE .

Draw Pm perpendicular to CS , meeting CS in P , and draw mQ perpendicular to SE , meeting SE in Q , and join Sm .

Then $mQ = PS$ will be equal to the distance of the original point whose seat is m in the intersection of the curved surfaces of the two solids from the plane passing through SE . In like manner Pm or SQ will be equal to the distance of the same original point from the plane passing CS .

Let r be the radius of the hemisphere, $h = CA = Ca$ the horizontal semi-axis of the archoid, and $p = CB$ the perpendicular axis.



Also let $SP = Qm = x$, $Pm = y$, and let $z = DI$ be the distance of the seat m from its original in the intersection of the two surfaces.

By the semi-ellipse ABa , $- - - - -$ $h^2 z^2 = p^2(h^2 - y^2)$
 and since $(Sm)^2 = x^2 + y^2$ $- - - - -$ $z^2 = r^2 - x^2 - y^2$.

Multiply the second of these two equations by h^2 , and the first two sides will become identical, and consequently the second sides equal; therefore $h^2 r^2 - h^2 x^2 - h^2 y^2 = p^2 h^2 - p^2 y^2$, and, by transposition, $(p^2 - h^2)y^2 = h^2 x^2 + p^2 h^2 - h^2 r^2$, which is an equation to an ellipse or hyperbola.

Coroll. If in the equation $(p^2 - h^2)y^2 = h^2 x^2 + h^2 p^2 - h^2 r^2$ of the figure in the preceding Proposition, p be less than h , the curve FmG , which is the seat of the intersection of the two solids, will be an ellipse; and if p be equal to h , the curve will become a straight line; but if p be greater than h , the curve will become an hyperbola.

For if p be less than h , the co-efficient of y^2 will be negative and in opposition to x^2 , and therefore the curve will be an ellipse; and if p and h be equal, the first side of the equation will vanish, and consequently $h^2 x^2 + h^2 p^2 - h^2 r^2 = 0$; therefore x will be a constant quantity, and the curve FmG will become a straight line; but if p be greater than h , the co-efficient of y^2 will be affirmative as well as that of x^2 , and therefore the curve will be an hyperbola.

PROBLEM XXIII.

To determine the transverse axis and asymptotes of the hyperbola, which is the locus of the equation $(p^2 - h^2) = h^2 x^2 + h^2 p^2 - h^2 r^2$.

Make $y = 0$, and we shall have $x^2 = r^2 - p^2$. It therefore appears that the transverse axis is one of the legs of a right angled triangle, of which the hypothenuse is r , and the other leg p .

Let $x = \text{infinity}$, and consequently y will be also equal to infinity, and therefore $(p^2 - h^2)y^2 = h^2 x^2$, that is $hx = (p^2 - h^2)^{\frac{1}{2}}y$; whence $x : y :: \sqrt{(p^2 - h^2)} : h$; therefore the ratio of x to y is constant, or x is to y as the leg of a right angled triangle, of which the hypothenuse is p , and the other leg h , to the height of the semi-axis of the end of the archoid.

In *Plate X, fig. 1*, Orthoprojection (subtitled the Intersection of the Surfaces of Two Solids), is exhibited the method of finding the seat of the intersection of a hemispheric surface with that of an archoid, by supposing both solids cut by planes at the same height.

Fig. 2 shows the projection, by first finding the summits of the hyperbolas, and describing the curves from having the axis and a point in the curve given. Thus:—

Draw any line OQ and OV perpendicular to OQ . Consider OVQ the radial section of the hemisphere. Let OZ , passing through the centre of O of the hemisphere, be the axis of the archoid. In OV , make Os equal to the height of the semi-ellipse which forms the end of the archoid. Draw sh parallel to OQ , meeting the arc QV in h . Draw hi parallel to OV , meeting OQ in i . In OZ , make OD equal to Oi , and let A or B be the points where the edges of the base of the archoid meet the edges of the base of the hemisphere. Now, having the semi-axis OD and the point A in the curve, we may find the conjugate axis by Prob. ii, p. 79, and describe the curve by Prop. vi, p. 81; or having found the conjugate axis, we may then find the asymptotes, Prob. iv, p. 80, and describe the curve by Prob. i, p. 78.

PROBLEM XXIV.

To determine the place and magnitude of the axis of the ellipse which is the locus of the equation $(p^2 - h^2)y^2 = h^2x^2 + h^2p^2 - h^2r^2$.

Make $y = 0$; then will $x = \pm \sqrt{r^2 - p^2}$; from which it appears that the axis-major passes through the centre of the hemisphere, and that the semi-axis minor is the leg of a right angled triangle, of which the hypotenuse is r and the other leg p .

Make $x = 0$; then will $(p^2 - h^2)y^2 = h^2(p^2 - r^2)$, and since by hypothesis h and r are greater than p , each side of the equation will be negative, and consequently $(h^2 - p^2)^{\frac{1}{2}}y = h(r^2 - p^2)^{\frac{1}{2}}$; whence it appears that y is a fourth proportional to the leg of a right angled triangle, of which the hypotenuse is h and the other leg p , and to the leg of another right angled triangle of which the hypotenuse is r and the other leg p , and to the horizontal semi-axis h of the archoid.

Fig. 3, Plate 3³, Orthoprojection (subtitled the Intersection of the Surfaces of Two Solids), shows how the intersections of the two surfaces of the hemisphere are found by taking sections parallel to the base at the same heights.

Fig. 4 shows how the curve may be described by the semi-axis, and a point in the curve being first found. Thus:—Let the quadrant OVQ be a radial section of the hemisphere, OV being the axis. In OV make Os equal to the height of the elliptic end of the archoid, and draw sh parallel to OQ , meeting the quadrantal arc VQ in h ; draw hi parallel to OV , meeting OQ in i . Let OD be the axis of the archoid, and let A or B be the points where the sides of the base of the archoid meet the circumference of the base of the hemisphere. Through O , draw EF perpendicular to OD . Now, having the semi-axis OD and a point A in the curve, we may describe the ellipse by Prob. iv, p. 70.

In *fig. 5*, the end of the archoid is a semicircle; the curve ADB is therefore a straight line.

Plate 2⁴, Orthoprojection (subtitled the Surfaces of Two Solids), exhibits the me-

thods of finding the intersection of a circular archoid and an elliptic conoid : it will not be necessary to repeat the process, as the method is in every respect the same as that applied to *Plate 1*.³ In *figs. 1* and *2*, the vertical semi-axis is less than the semi-diameter of the base ; the curve of the seats of the intersection of the two surfaces is an hyperbola, as before.

In *fig. 3*, the radial section OVQ is an ellipse, in which the vertical axis OV is greater than the diameter of the base ; the curve of the seat of the intersection of the two surfaces is an ellipse.

In *fig. 4* the two solids are of the same height.

In *fig. 5*, the curve of the section through the axis of the elliptic conoid is similar to the end of the archoid.

Plate 3,⁴ Orthoprojection (subtitled the Surfaces of Two Solids), exhibits the projection of the intersection of the surfaces of different solids.

Fig. 1 is that of a hemisphere and cylinder, the diameter of the cylinder being less than the radius of the sphere, and the axis of the cylinder parallel to the base of the sphere, and passing through the vertical axis.

Fig. 2 is that of a cone and cylinder, the diameter of the cylinder being less than the altitude of the cone, and the axis of the cylinder parallel to the base of the cone, and passing through its axis.

Fig. 3 is that of a sphere and cuneoid, the end of the cuneoid being a circle.

Fig. 4 is that of a cone and cuneoid.

Plates 4 and *5*, Orthographical Projection, exhibit various examples of constructing the seats, which will be understood by tracing the lines by the eye in each diagram.

In every figure we need only draw sections parallel to the base at the same height, and find the seats of these sections, and draw a curve through the intersections of every two seats ; the curve thus drawn will be the seat of the intersections of the two curved surfaces.

PROBLEM XXV.

Given in one plane the bases of any two solids intersecting each other, to find the seat of the line of common section of their curved surfaces on the plane of their bases.

Find a number of seats of the sections of both solids by planes parallel to their bases, every section of the one solid having a corresponding section of the other at the same height ; then a curve being drawn through all the corresponding points of the intersections of the seats of both sides, will be the seat of the intersection of the surfaces of the two solids.

If either of the two solids be an archoid, the edges of every section which meet the curved surface will be projected into parallel lines.

OF THE INTERSECTION OF THE SURFACES OF TWO SOLIDS.

Fig. 1.

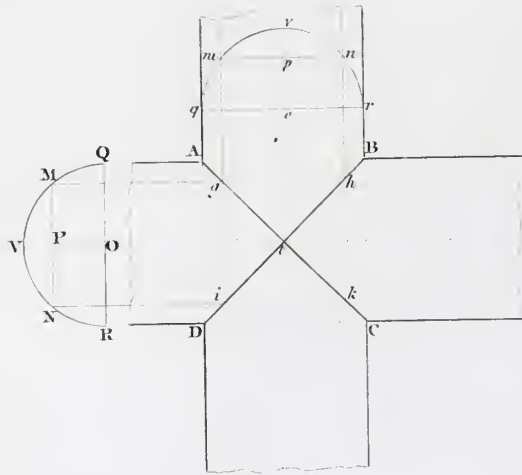


Fig. 2.

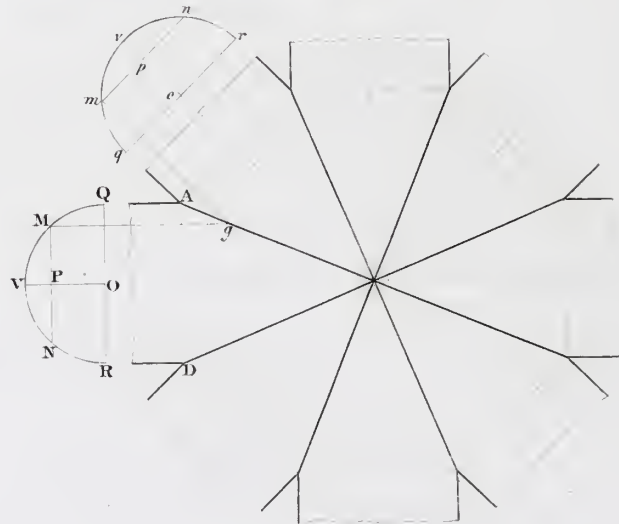


Fig. 3.

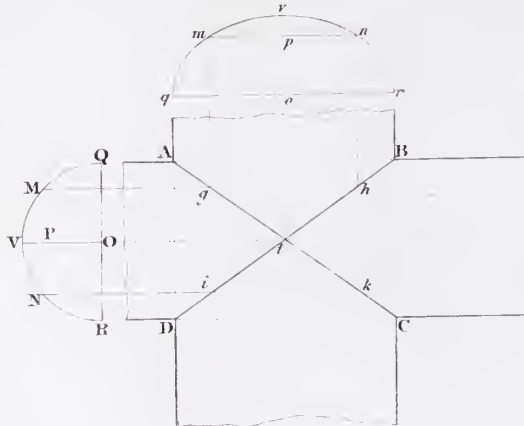


Fig. 4.

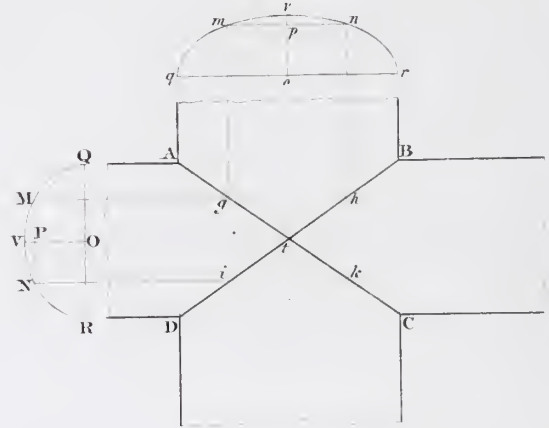


Fig. 5.

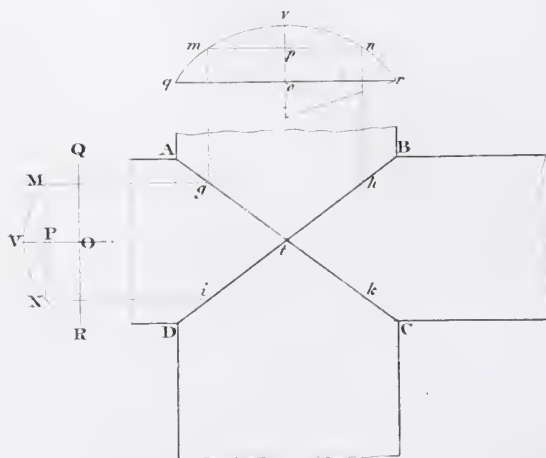
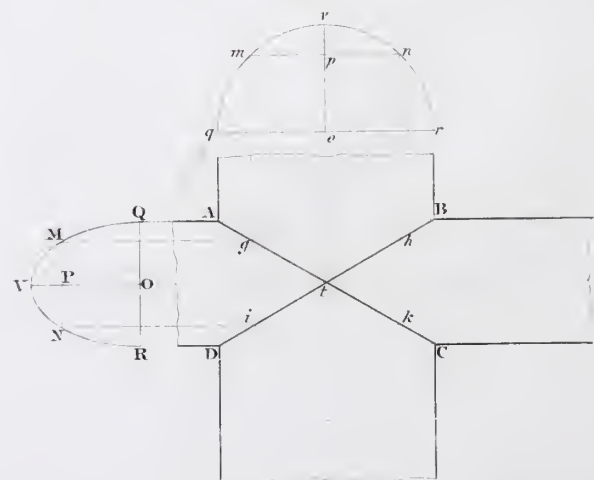


Fig. 6.





OF THE SURFACES OF TWO SOLIDS.

Fig. 1.

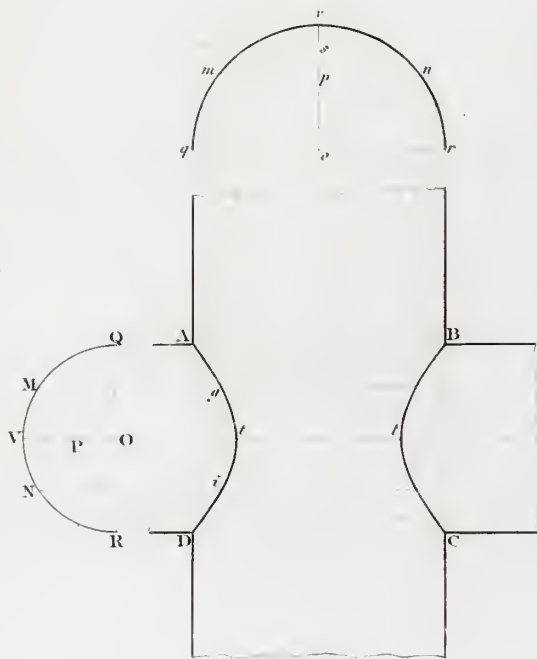


Fig. 2.

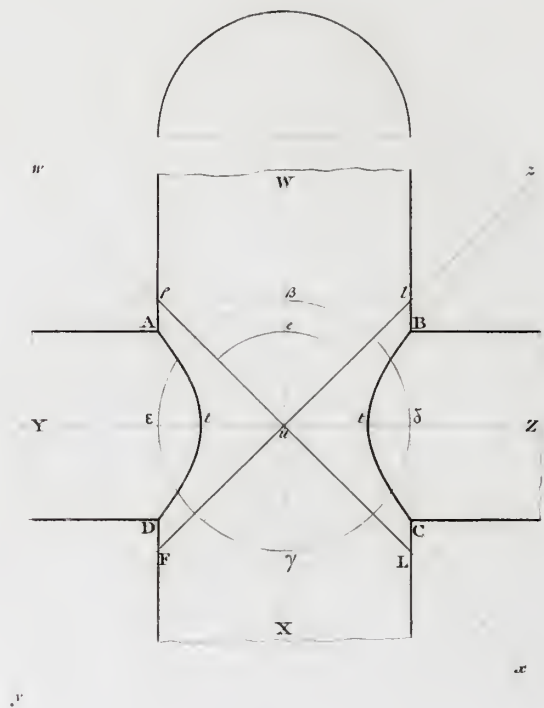


Fig. 3.

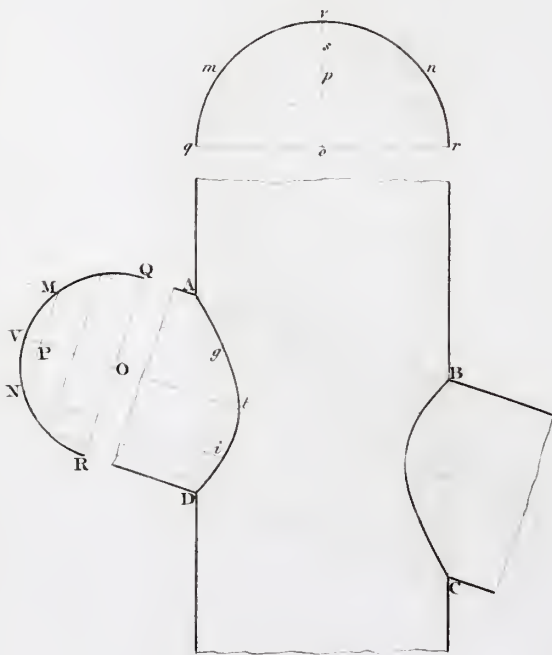
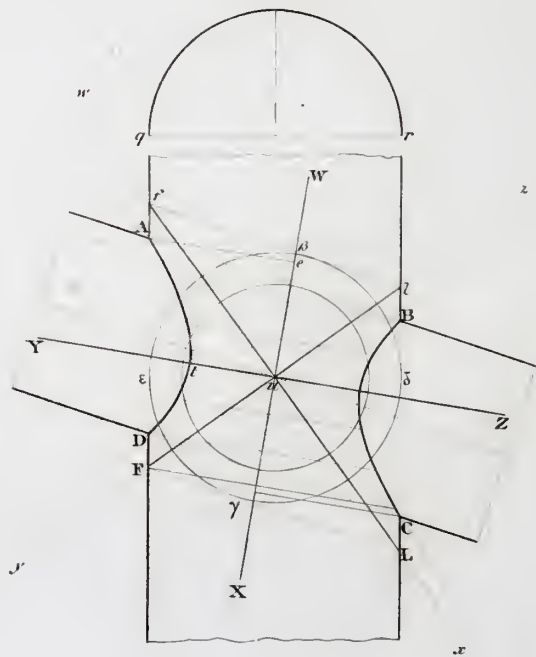


Fig. 4.



Engraved by C. Armstrong.

London, Published by John Day & Co. Octo. 20. 1822.

The edge of the section of every conoid and of every domoid with a circular base will be projected into a circle.

The edge of the section of every domoid with an elliptic base will be projected into a similar ellipse, which will be similarly situated with that base.

The edge of the section of every cone projected on a plane parallel to the axis will be an hyperbola, and the section of the cone through the axis will be the asymptotes of that hyperbola.

EXAMPLES.

In *Plate 1* (Orthoprojection of the Intersection of the Surfaces of Two Solids), let *QVR*, *fig. 1, 2, 3, 4, 5, 6*, be the end of the one solid, *qvr* the end of the other, their base lines being *QR*, *qr*.

Let *O* bisect *QR*, and *o* bisect *qr*, and let *OV* be perpendicular to *QR*, and *ov* perpendicular to *qr*. In *OV* take any point *P*, and make *op* equal to *OP*. Through *P*, draw *MN* parallel to *QR*, and draw *Mg* parallel to *QB* or *RC*; also through *p* draw *mn* parallel to *qr*, and draw *mg* parallel to *qD*; then *g* is a point in the intersection *Agt*, the points *A, B, C, D* being those in which the sides of the base meet and form the parallelogram *ABCD*.

In the same manner we may find as many points *g* as we please.

If *mg* is prolonged to *i*, and if *Ni* be drawn parallel to *RC*, *i* will be another point in the intersection *Dit*.

In the same manner we may find as many points *i* as we please.

If *Mg* and *Ni* are prolonged to *h* and *k*; then if *nh* be drawn parallel to *rC*, *h* will be a point in the diagonal *Bht*; and if *nh* be prolonged to *k*, *k* will be a point in the diagonal *Ckt*.

Besides the general method here given for describing the seats of the intersections of the two solids, the following method will apply to figures where each solid is a semi-cylinder, having its axis in the base.

From *u* (*plate 2, fig. 4*) the centre of the parallelogram *ABCD*, with a radius equal to that of the greatest cylinder, describe the circle $\beta\delta\gamma\epsilon$, which will touch the lines *AD* and *BC* in ϵ and δ . Draw *fl* parallel to *AB* or *DC*, touching the circle in β , and meeting *qD* in *f*, and *rC* in *l*; also draw *FL* parallel to *AB* or *DC*, touching the circle in γ and meeting *qD* prolonged in *F*, and *rC* prolonged in *L*.

Through the points *f, L* draw *wx*, and through the points *F, l* draw *yz*, which will intersect *xy* in *u*. Draw *WX* bisecting the angle *wuz*, and *YZ* bisecting the angle *wuy*. Draw *Ae* perpendicular to *WX*, meeting *WX* in *e*; from *e*, with the distance *eA*, describe an arc meeting *YZ* in *t*. Then with the asymptotes *wx, yz*, and the semi-transverse axis *tu*, describe the hyperbola *AtD*.

In the whole of the diagrams contained in the seven plates entitled Orthopjection, and subtitled the Surfaces of two Solids, the seats of the intersections of the surfaces of each solid are found by one uniform method; we shall, therefore, only refer to such of the diagrams as appear best adapted to illustrate the principle, and for explanation to such of the others as have any peculiarity in their construction.

Let VOQ , *fig. 1, Plate 4*, be a radial section of the solid. In OV , make OP equal to op , and draw PM parallel to OQ , meeting the curve VQ in M , and draw MI parallel to VO , meeting OQ in I ; from the centre O describe the arc Ikg , meeting mg in g ; then g is a point in the curve, and thus we may find as many points as will be necessary to trace the curve which is an hyperbola, and may therefore be described as follows:—

In OV , *fig. 2*, make Qs equal to ov , and draw sh parallel to OQ , meeting the curve VQ in h , and draw hi parallel to VO , meeting OQ in i . Draw ZY through O parallel to AB , and WX through O perpendicular to ZY . From O , with the radius Oi , describe an arc meeting ZY in t . Through t , draw pu perpendicular to YZ . Let AB meet WX in e ; from e , with the radius eA , describe an arc meeting pu in k . Draw ek , meeting Ot in f . Make tp and tu each equal to Of . Through O and p draw wx , and through O and u draw yz ; then with the asymptotes wx , yz , describe the hyperbola AtD .

In *fig. 3* one of the solids is a semi-cylinder, and the other an elliptic conoid, of which the axis perpendicular to the base is longer than the revolving axis. As the seat of the intersection of the two solids is a portion of an ellipse, the figure may be described as follows:—

In OV make Os equal to ov , and draw sh parallel to OQ , meeting the curve QV in h . Draw hi parallel to VO , meeting OQ in i . Through O , draw tt parallel to AD or BC . From O , with the radius Oi , describe an arc meeting tt in the points t, t ; then the curve passes through the points A and B , and through t : therefore the semi-axis Ot , and a point A or B in the curve, is given to describe the ellipse; but, to carry the reader one step farther,—Through O draw kl perpendicular to tt , and from A , with a radius equal to tO , describe an arc meeting kl in e . Join Ae , and prolong Ae to meet tt in f . Make Ok, Ol , each equal to Af ; then with the axis major kl , and semi-axis minor Ot , describe the semi-ellipse $kABL$.

In *fig. 4*, one of the solids is a cylinder, and the other an elliptic conoid, of which the axis round which the solid is generated is equal to the diameter of the cylinder. Here the seats of the intersections of the surfaces are straight lines.

In *fig. 5*, one body is a cylindroid, and the other an ellipsoid, and $ov : OV :: oq : OQ$; therefore the intersections of the surfaces are straight lines.

In *Plate 3*, one of the solids in each figure is a hemisphere.

In *fig. 1* the other solid is a cylindroid, of which the longest axis is perpendicular to the plane of the base. Here the seats of the intersections of the two bodies are hyperbolas, and may also be constructed as in *fig. 2*.

ORTHO PROJECTION.

PLATE 3.

OF THE SURFACES OF TWO SOLIDS.

Fig. 1.

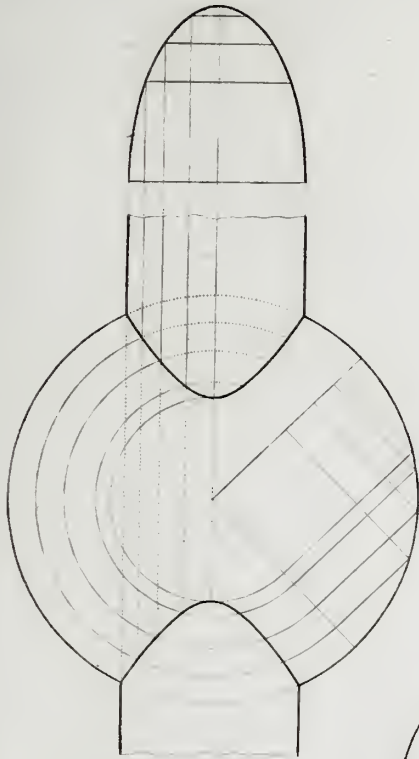


Fig. 2.

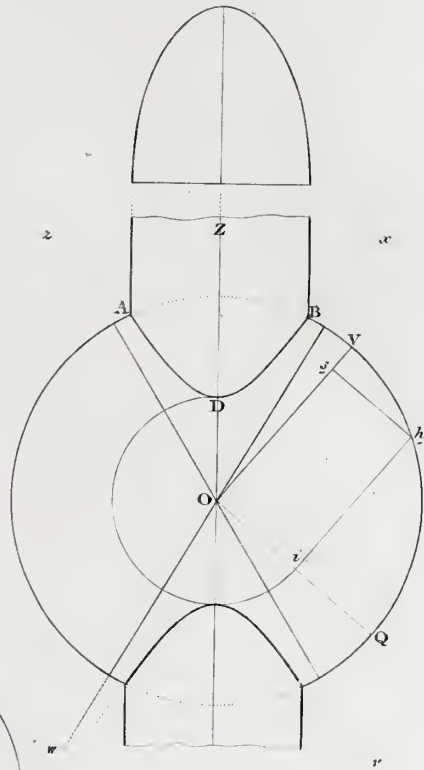


Fig. 5.

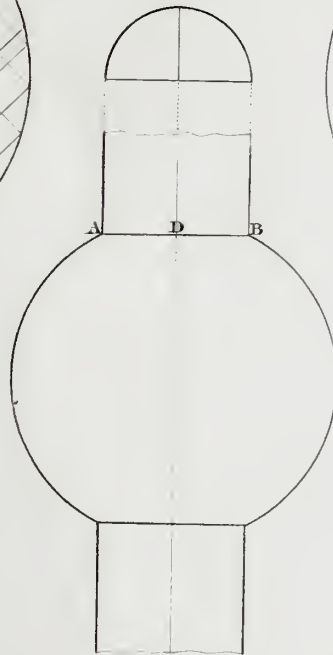


Fig. 3.

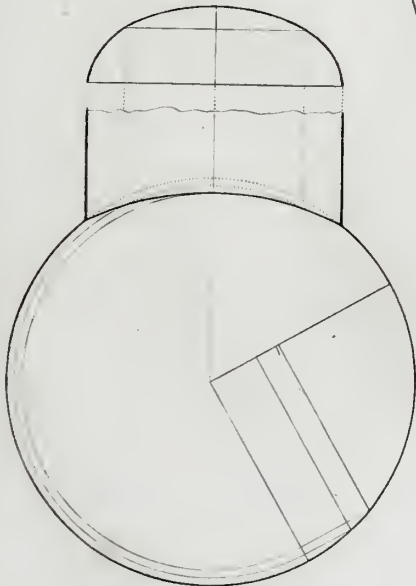
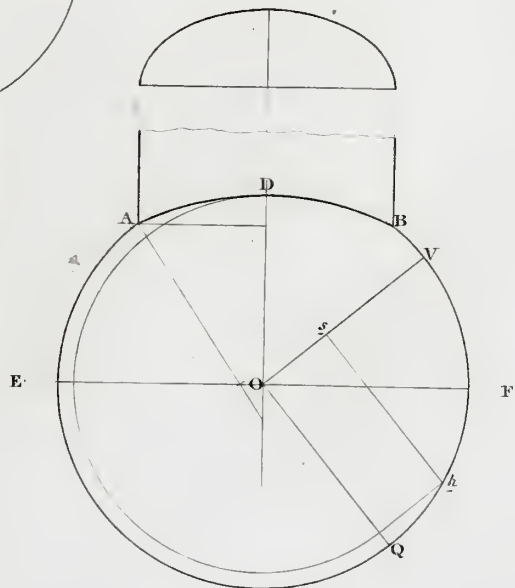


Fig. 4.



Engraved by C. Armstrong.

London Published by John Day Oct 20 1872

OF THE SURFACES OF TWO SOLIDS.

Fig. 1.

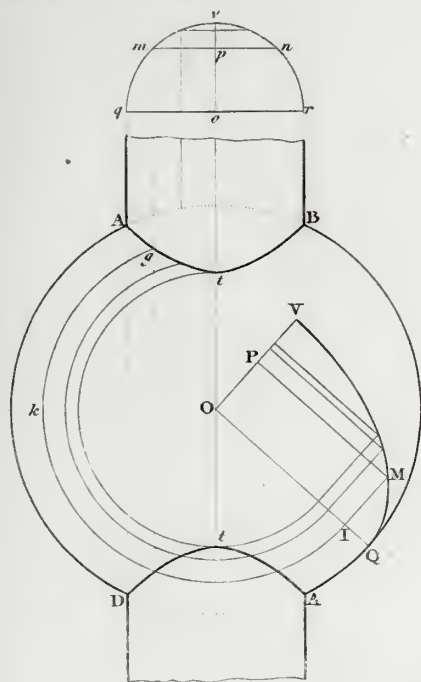


Fig. 2.

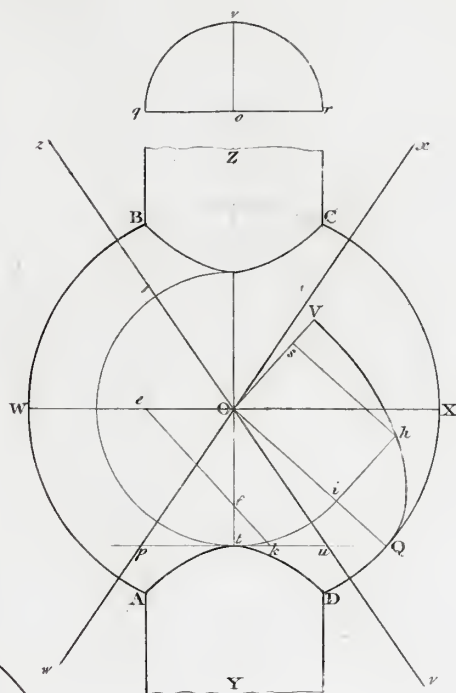


Fig. 5.

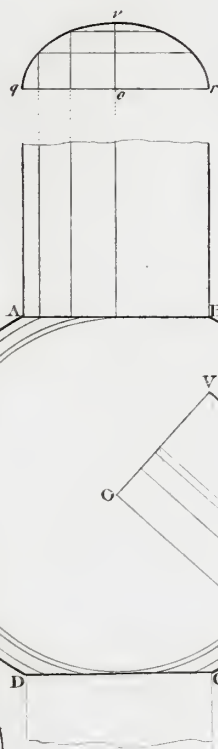


Fig. 3.

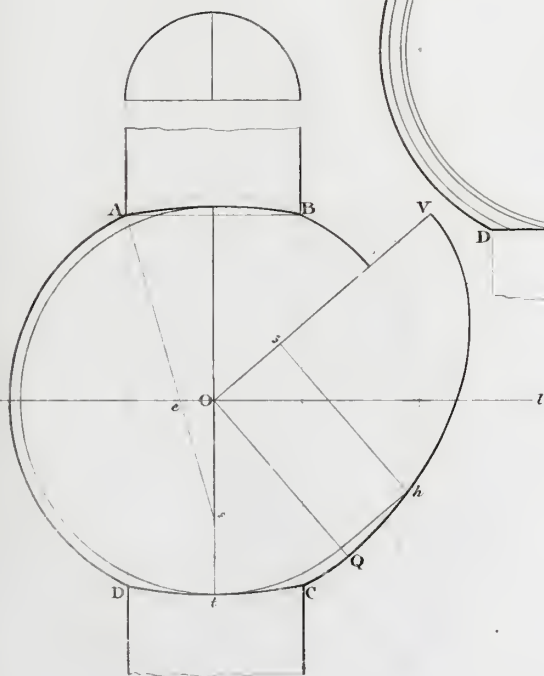
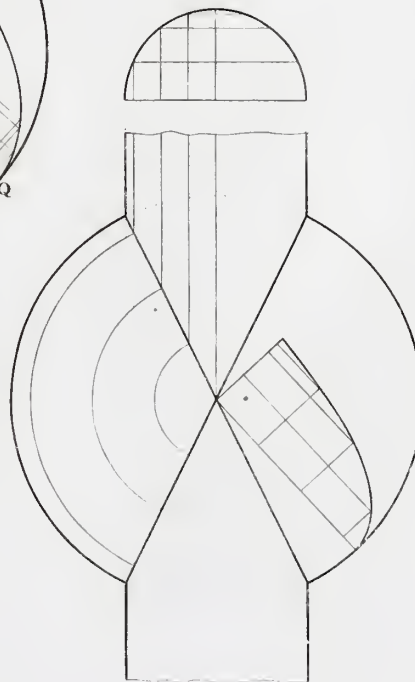


Fig. 4.





OF THE SURFACES OF TWO SOLIDS.

Fig. 1.

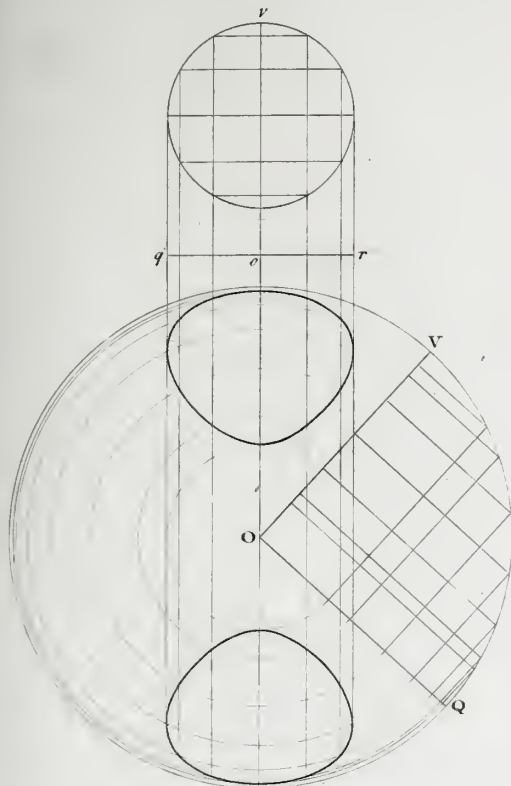


Fig. 2.

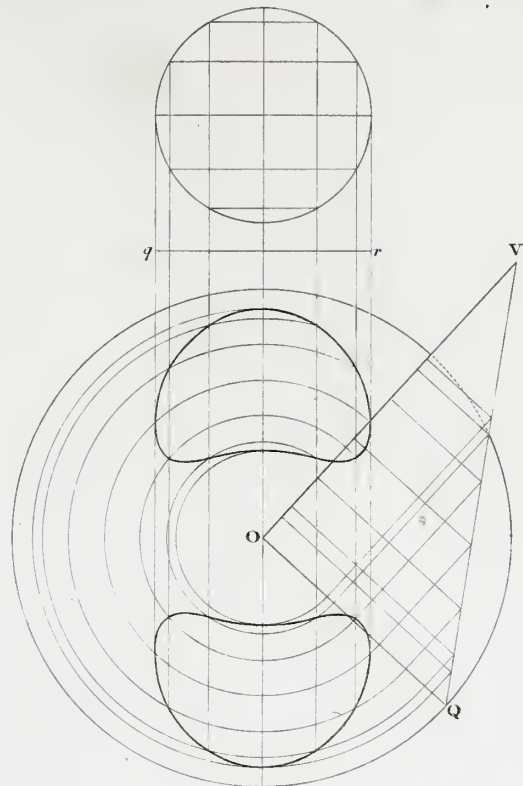


Fig. 3.

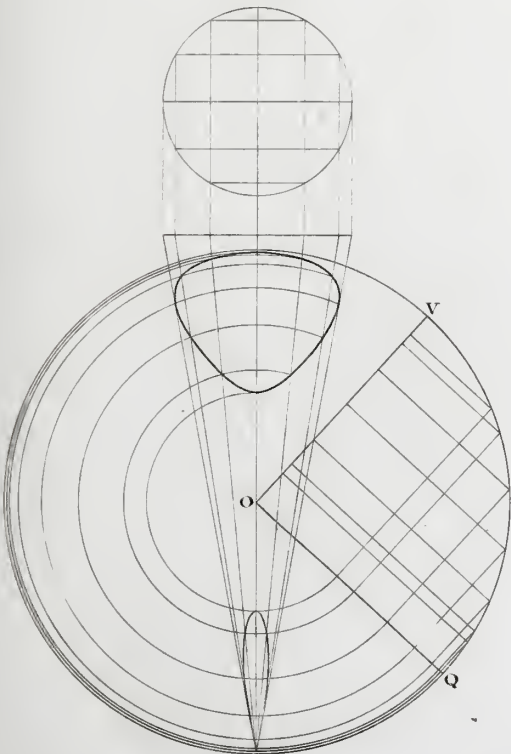
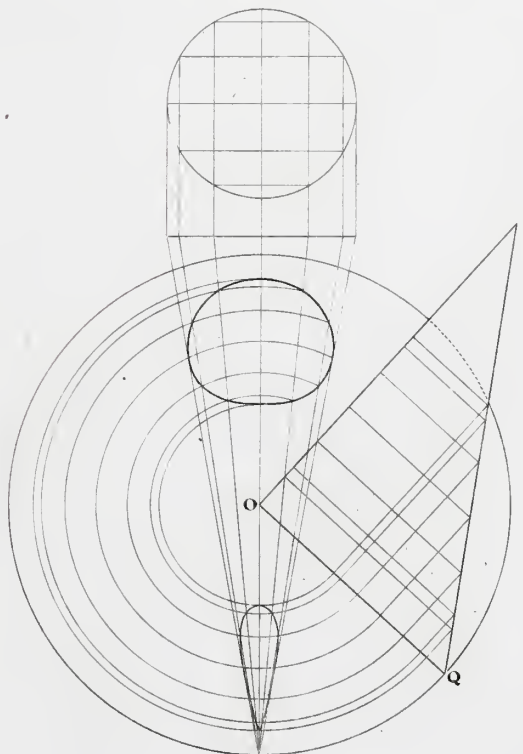


Fig. 4.





ORTHO PROJECTION.

PLATE 6.

OF THE SURFACES OF TWO SOLIDS.

Fig. 1.

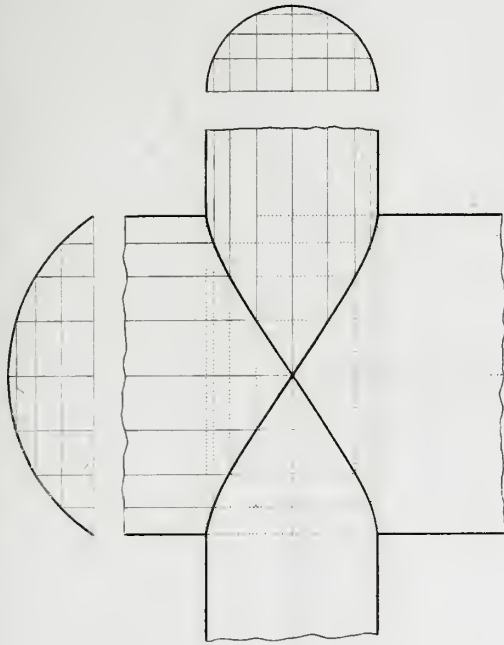


Fig. 2.

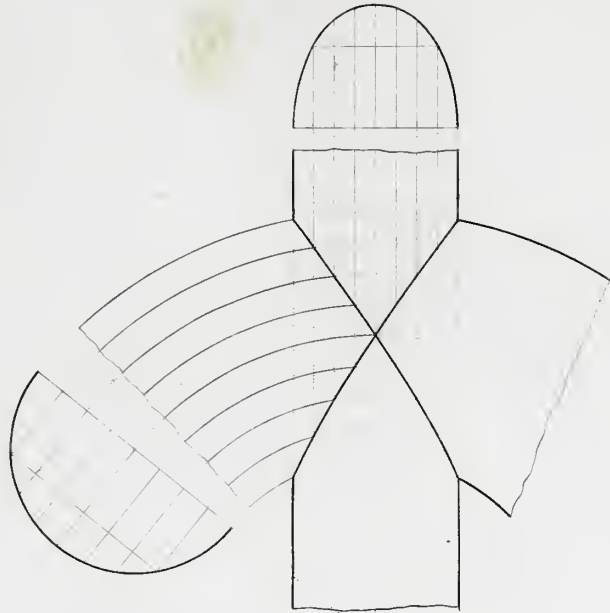


Fig. 3.

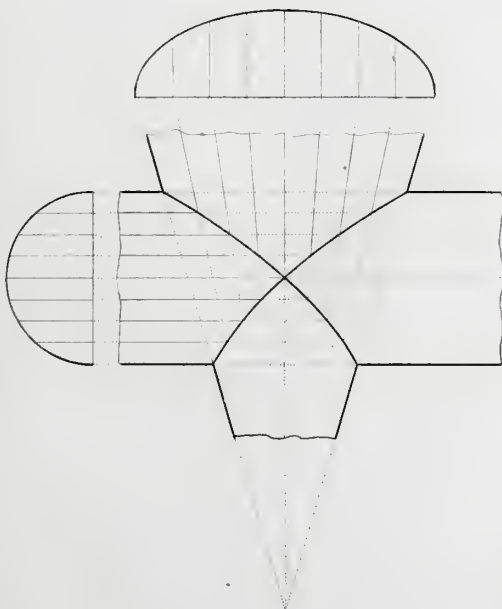
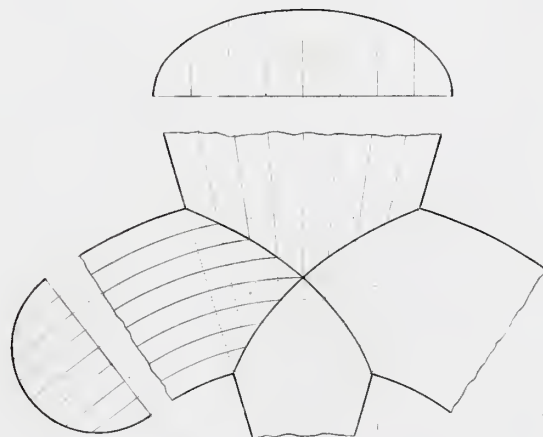


Fig. 4.



OF THE SURFACES OF TWO SOLIDS.

Fig. 1.

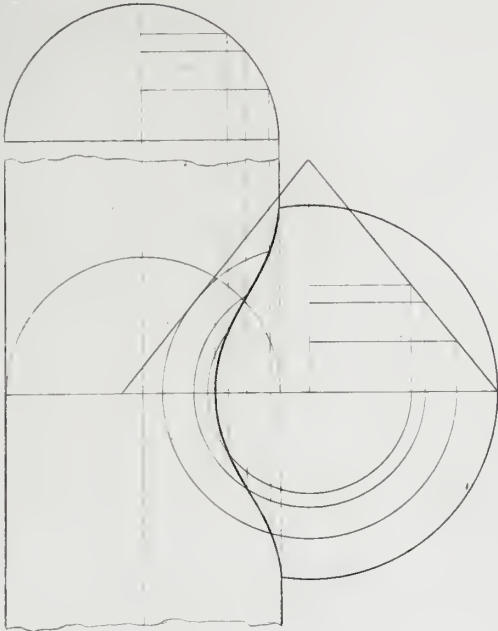


Fig. 2.

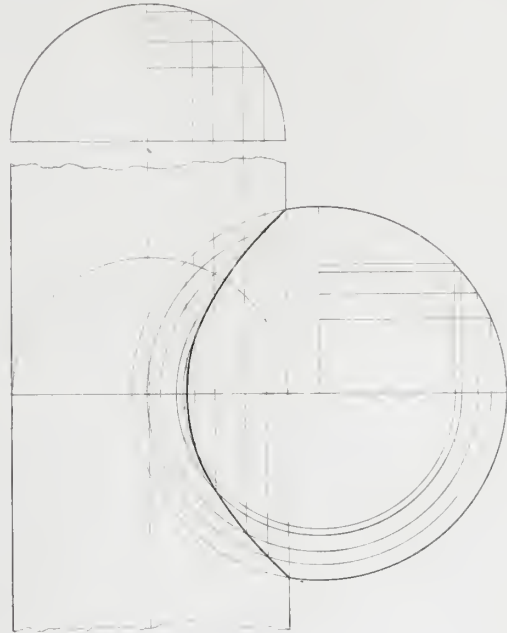


Fig. 3.

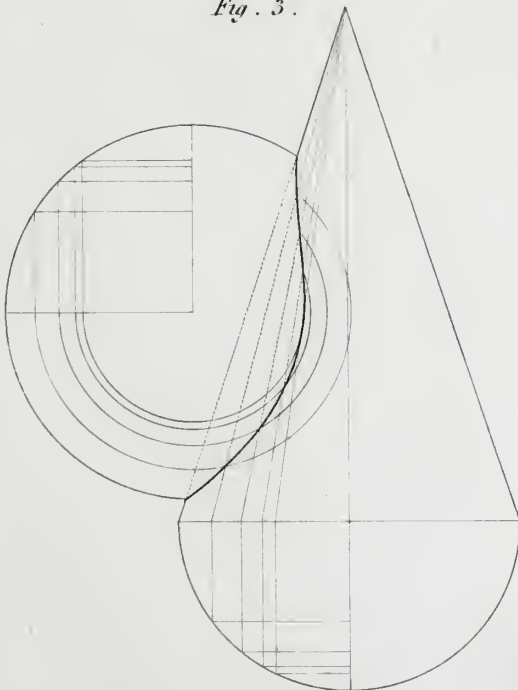
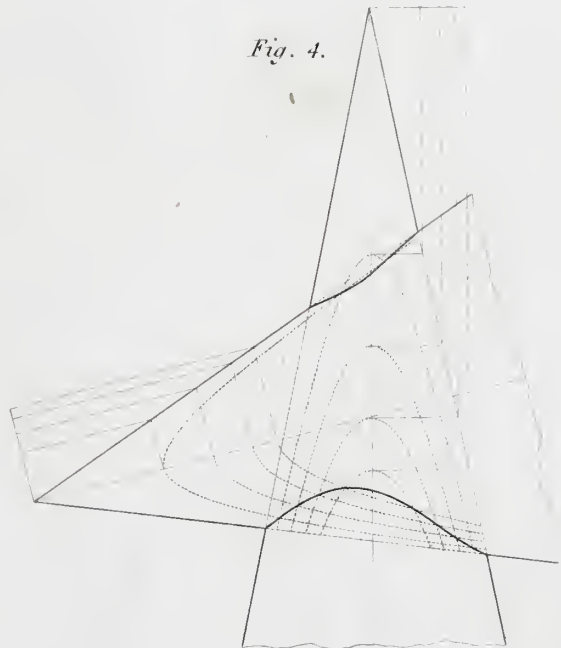


Fig. 4.



In *figs. 3 and 4* the other solid is a cylindroid also ; but the minor axis is perpendicular to the plane of the base. The seat of the intersections of the surfaces of the two bodies is an ellipse.

In *fig. 5*, the other solid is a cylinder, and the intersection of the two surfaces is a straight line.

In *Plate 5, figs. 1 and 3*, one of the solids is a hemisphere ; in *fig. 1*, the other solid is an entire cylinder, of which the surface is considerably above the plane of the base : the projections of the surfaces of the two bodies form ovals, each having the figure of an eggoid.

In *fig. 3*, the other solid is an entire cuniconoid, of which the principal rectangular plane is perpendicular to the plane of the base. Here the intersections of the surfaces of the two solids are also ovals of the eggoid form.

In *figs. 2 and 4*, one of the rotative solids is a cone ; in *fig. 2* the other solid is a cylinder ; and in *fig. 4* the other solid is a cuniconoid.

Plate 6 contains four diagrams of groined vaults or severies. In *fig. 1* the plan is rectangular : one of the generating arches is a semicircle, and the other the segment of a circle. In *fig. 2* one is a semicircular annulus, and the other an elliptic archoid. In *fig. 3* one of the solids is a circular archoid, and the other a cuniconoid ; and in *fig. 5* one of the solids is a semicircular annulus, and the other a cuniconoid.

Plate 7, fig. 1, one of the solids is a semi-cylinder, and the other a cone with its axis perpendicular to the plane of projection. Here the seat of the intersection of the two surfaces is a figure of contrary flexure.

In *fig. 2* one of the solids is a semi-cylinder, and the other a hemisphere, and the seat of the intersection of both curves is a parabola.

In *fig. 3* one of the bodies is a cuniconoid, and the other a hemisphere.

In *fig. 4*, both solids are cones, of which their axis is in the plane of projection ; therefore all the sections at an equal height are hyperbolas.

DEVELOPEMENT OF THE SURFACES OF SOLIDS.

DEFINITIONS.

1. A **PLANE** figure, which when bent on the curved surface of a solid so that every point of the figure may come in contact with the curved surface, and that the edges of the figure may coincide with the edges of the curved surface, or meet each other on the curved surface, is called the *developement* of the curved surface of that solid.

2. The developement of the curved surface of an ungulus is called a *gore*.

The surfaces of solids which may be covered by plane figures are those of cylinders, cylindroids, prisms, cylinoids, pyramids of all descriptions, cones, and solids compounded of them. The surface of a sphere, though equal to four times the area of its great circle, cannot be developed, since a plane figure cannot be bent upon the surface of a sphere. For the same reason a domoid, or the sectroid of a domoid, cannot have an exact developement.

PROBLEM I.

To find the developement of the surface of a cylinder, or of a cylindroid, or that of any sectroid; given the length of the axis, and the end of the cylinder, cylindroid, or sectroid.

Draw a straight line equal in length to the developement of the arc of the end, and from one extremity of the straight line thus drawn, draw a perpendicular, and complete the rectangle of which these two lines about the right angle are sides.

Let abc be the end. Draw the straight line AC equal in length to the developement of the arc abc , and from C , one extremity of the straight line AC , draw CE perpendicular to AC , and make CE equal to the length of the axis, and complete the rectangle $ADEC$, which will be the developement of the cylindric surface as required.

Fig. 205, No. 1.

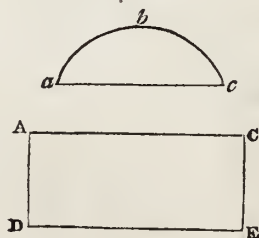


Fig. 205, No. 2.

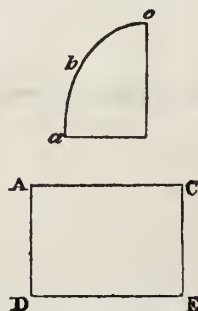
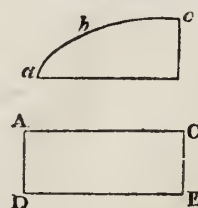


Fig. 205, No. 3.

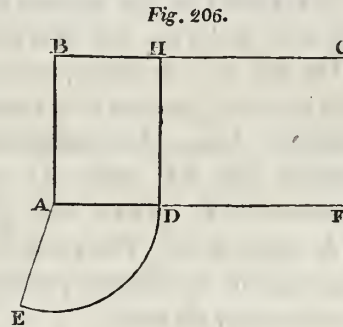


PROBLEM II.

To find the developement of the curved surface of the sectroid of a cylinder, and consequently that of the whole cylinder, the radial section and the base of the solid being given.

Let $ABHD$ be the radial section, and AED the base of the solid.

Produce AD to F , and make DF equal to the developement of the arc DE , and complete the rectangle $DFGH$, which is the developement required.

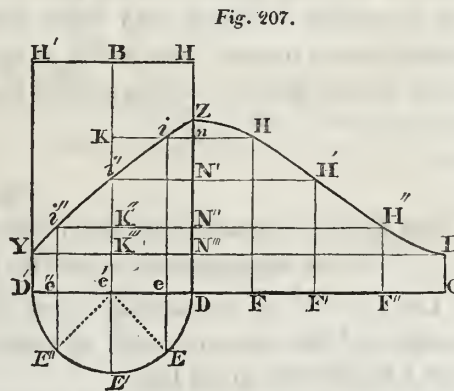


PROBLEM III.

Given the position of the line in the base of a semicircular archoid, or in the axial plane of a semi-cylinder, to find the developement of that portion of the cylindric surface contained between one of its ends and the intersection of a surface passing through the given line perpendicularly to the plane of the axial section.

Let $D'DHH'$ be the axial section of the cylinder, eB being the axis, and $D'D$ the diameter of the base, and let YZ be the line through which the surface passes perpendicularly to the plane $D'DHH'$.

Divide the semi-circumference $D'ED$ of the base into any number of equal parts at the points $E, E', E'', \&c.$ (as here into four), and find $ei, e'i', e''i'', \&c.$ the seats of the lines passing through the points $E, E', E'', \&c.$ of the base in the cylindric surface parallel to the axis. Through the points $i, i', i'', \&c.$ Y in which these seats meet the line YZ , draw the straight lines $Kn, i'N', i''N'', \&c.$ YN''' parallel to $D'D$, meeting $e'B$ the axis in $K, i', K'', K''',$ and the side HD in the points n, N', N'', N''' . Then by Prob. ii, p. 139, with the radial sections $e'DnK, e'DN'i', e'DN''K'', e'DN'''K'''$ describe the developements $DFHn, DF'H'N', DF''H''N'', DGIN'''$ of the sectroids, of which the bases are $De'E, De'E', De'E'',$ and $De'D'E$, and draw the curve $ZHH'H''I$; then will $DGIZ$ be the developement required.



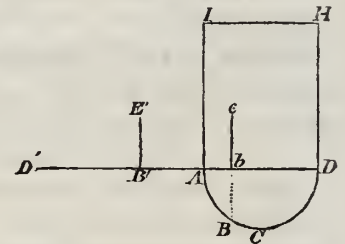
PROBLEM IV.

Given the section of a cylinder along the axis, the seat of any point on the curved surface in that section, to find the place of the point in the development.

Let $ADHI$ be the section of a cylinder passing along the axis, and e the seat of a point on its surface.

On AD , as a diameter, describe the semicircle $ABCD$, and draw eb parallel to AI or HD , meeting AD in the point b . Draw AD' perpendicular to AI , and in the straight line AD' make AB' equal to the development of the arc AB . Draw $B'E'$ parallel to AI , and make $B'E'$ equal to be . The point E' is the place in the development of the original point in the curved surface represented by its seat e .

Fig. 208.

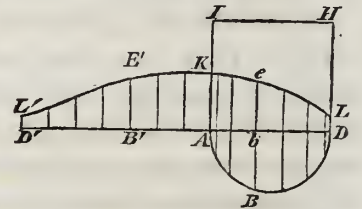


PROBLEM V.

Given the section $ADHI$ of a cylinder along the axis, the seat KeL of any line on the curved surface in that section; to find the development of the line.

Find a number of points in the development of the line, in the same manner as the point E' was found by the preceding Problem, and draw the curve $K'E'L'$ through these points; then $K'E'L'$ is the development of the line on the surface represented by the seat KeL .

Fig. 209.



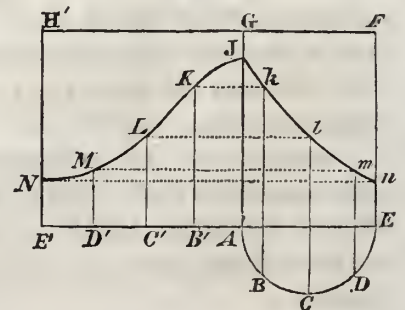
PROBLEM VI.

Given the development and end of a semicircular archoid, and the form of any line drawn in the development, to find the seat of that line in the base of the archoid

Let $AGHE'$ be the development of the curved surface of the semi-cylinder or circular archoid, and $JKLMN$ the given line.

Produce $E'A$ to E . Make AE equal to the diameter of the cylinder, and on AE describe the semicircle ACE . Divide the arc ACE into any number of equal parts (for Example 4); also divide AE' into equal parts AB' , $B'C'$, $C'D$, &c. by repeating the chord of one of the arcs in the line AE' as often as the arc ACE contains equal parts.

Fig. 210.



Draw $B'K$, $C'L$, $D'M$, &c. and Bk , Cl , Dm , &c. parallel to AG , and Nn , Mm , Ll , Kk parallel to LA , and through the points J , k , l , m , n draw a curve, which will be the seat required.

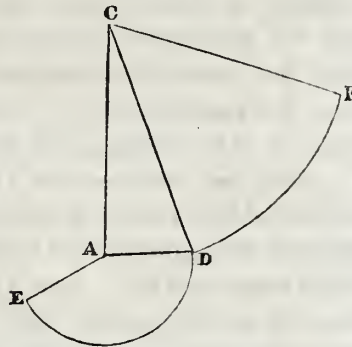
PROBLEM VII.

To find the developement of the curved surface of the sectroid of a cone, and consequently that of the cone itself; the radial section of the solid and its base being given.

Let CAD be the radial section, and AED the base of the sectroid, AD or AE be the radius of the base, and AC the axis.

From C , with the radius CD , describe the arc DF . Make the arc DF equal to the arc DE , and join CF ; then will CDF be the developement of the curved surface of the sectroid, as required.

Fig. 211.

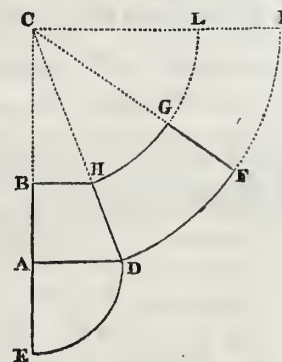


PROBLEM VIII.

To find the developement of the curved surface of the sectroid of the frustum of a cone, and consequently that of the whole frustum; given the radial section of the frustum, and the base of the solid.

Let $BADH$ be the radial section of the frustum, and AED the base of the solid. Produce AB and DH to meet each other in C ; then by the preceding Problem, having the radial section CAD , and the base ADE , find the developement CDF . Then from C , with the radius CH , describe an arc HG meeting CF in G ; then will $HDFG$ be the developement of the curved surface of the sectroid of the conic frustum as required.

Fig. 212.



If AE be perpendicular to AB , and if the arcs HG and DF be continued to K , and FK be made equal to FD , and KC be joined meeting the arc GL in L , then $HDKL$ will be the developement of the curved surface of the frustum of the half cone.

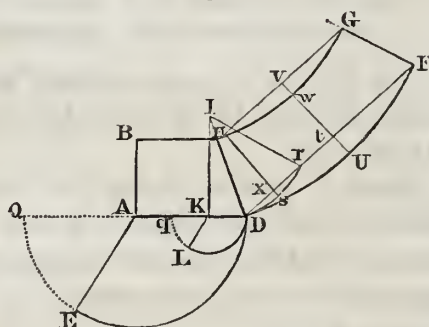
PROBLEM IX.

To find the developement of the curved surface of the sectroid of the frustum of a cone, the radial section of the frustum and its base being given, without completing the radial section of the entire cone.

Let ABHD be the radial section, and AED the base of the solid, as before.

Divide the radius AD into any convenient number of equal parts, as three, and let DK be the third part of DA. From K, with the radius KD, describe the arc DL, and draw KL parallel to AE. Draw KI perpendicular to AD, meeting DH or DH produced in I. Find the developement IDr of the curved surface of the sectroid of the frustum of a cone, of which the radial section is DKI, and the base of the sectroid KDL. Join Dr, and produce Dr to F. Make DF equal to as many times Dr as DA contains DK; therefore, since DA in this instance is three times DK, make DF equal to three times Dr. Bisect Dr in x, and DF in t, and draw xs and tU each perpendicular to DF. Let xs meet the arc Dr in s; make tU as many times xs as DA contains DK; but since DA is three times DK, make tU equal to three times xs. Describe the arc DUF, of which the chord is DF, and the versed sine tU. Draw HG parallel to DF, and produce Ut to meet HG in v. Make vG equal to vH, and make Uw equal to DH. Through the three points H, w, G describe an arc HwG, of which the chord is HG and the versed sine vw, and join FG; then DFGH is the developement of the sectroid of the frustum of a cone, as required.

Fig. 213.



SCHOLIUM.

The angle of developement DIR might have been obtained with the arcs DL and Dr by the following analogy. Find the side of the cone by an arithmetical operation, thus:—

As the difference between the bases or ends of the frustum
is to the length of the slant side of that frustum,
so is the diameter of the greater end
to the length of the side of the cone.

The diameter of the base of the cone is the same with the diameter of the greater end of the frustum; therefore we have now given the length of the side of the cone, and the

radius of its base, to find the angle of developement, which may be done arithmetically by the following analogy:—

As the length of the side of the cone
is to the radius of its base,
so is the number of degrees in the angle of the sectroid
to the angle of developement.

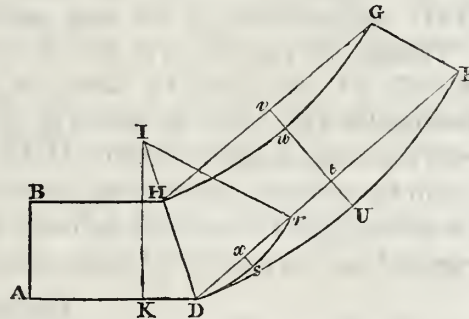
PROBLEM X.

To find the developement of the curved surface of the sectroid of the frustum of a cone ; given the radial section of the solid, and the chord of the arc of developement of the arc of the base of the solid, without having recourse to a centre in describing the arcs of developement.

Let ADHB be the radial section, and AB the axis of the conic frustum.

Divide AD into any convenient number of equal parts, as, for example, three; and in DA make DK equal to one of these equal parts. Draw KI perpendicular to AD, meeting DH in I, or DH produced in I. From I with the radius ID describe an arc Dsr. From D, with a distance which is the same part of the intended chord of the arc of developement that DK is of DA, describe another arc meeting the former at r ; that is, in this example, equal to one third. Join Dr, and produce Dr to F. Make DF the same multiple of Dr that DA is of DK; that is, since DA is three times DK, DF must be made equal to three times Dr. Bisect Dr in x , and DF in t , and draw xs and tU each perpendicular to DF. Make tU the same multiple of xs that DA is of DK; therefore, since DA is three times DK, make tU equal to three times xs . Describe an arc DUF upon the chord DF, and with the versed sine tU draw FG parallel to rI , and HG parallel to DF. Produce Ut to meet HG in v , and in Uv make Uw equal to DH, and upon the chord KG and with the versed sine vw describe the arc HwG; then will DUFwG be the developement required.

Fig. 214.



PROBLEM XI.

Given the axial section of a cone, and any line in that section, to find the developement of that portion of the conic surface contained between the vertex and the intersection of a surface passing through the given line perpendicularly to the plane of the axial section.

Let $CD'D$ be the axial section of the cone, CA being the axis, and $D'D$ the diameter of the base; and let YZ be the line of section, or that through which the surface passes perpendicularly to the axial plane $CD'D$.

Divide the circumference DED' of the base into any number of equal parts at the points $E, E', E'', \&c.$ and find $eC, e'C, e''C, \&c.$ the seats of the straight lines on the conic surface drawn from C to each of the points $E, E', E'', \&c.$ supposing the semicircle DED' perpendicular to the axial section CDD' .

Through the points $i, i', i'', \&c.$ in which the lines $eC, e'C, e''C, \&c.$ meet YZ , draw the straight lines $BK, i'L, i''N, \&c.$ parallel to $D'D$, meeting the axis AC in the points $B, i', M, \&c.$ and CD in the points $K, L, N, \&c.$ Then with the radial sections $ADHB, ADLi, ADN M, \&c.$ and the bases $ADE, ADE', ADE'', \&c.$ of the sectroids, describe the developements $DFGK, DRHL, DSIN, \&c.$ and through the points $Z, G, H, \&c.$ draw the curve ZHJ . Join CJ ; then $CZHJ$ is the developement required, and the curve ZHJ is the developement of the original of the line of section YZ .

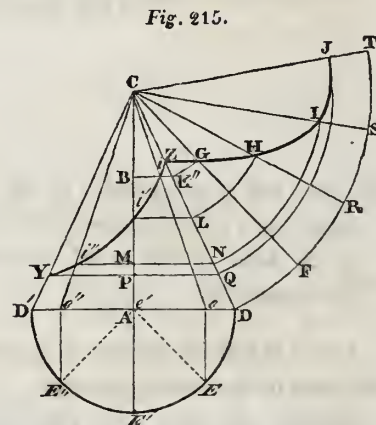


Fig. 215.

PROBLEM XII.

Given a section AVD of a cone along its axis, and the seat e of any point E' on its surface in that section, to find the place of the original point E' , on the developement of the conic surface.

Let $AB'C'D'V$ be the developement of the conic surface for the semi-cone.

Through e draw the straight line Vb , meeting AD in b ; on AD , as a diameter, describe the semicircle ABD , and draw bB perpendicular to AD . Make AB' equal to the developement of AB . Join VB' ; draw ef parallel to AV , the side of the cone, meeting AD in f , and in BV make $B'E'$ equal to fe , and E' will be the point required.

Or if AC' be the developement of the quadrantal arc AC , we might have found $C'B'$, the developement of CB , and the point E' would have been ascertained as before.

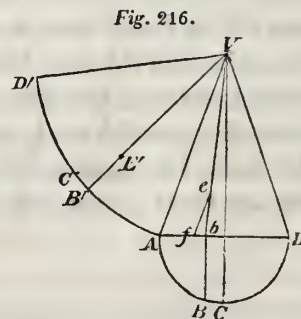


Fig. 216.

DEMONSTRATION.

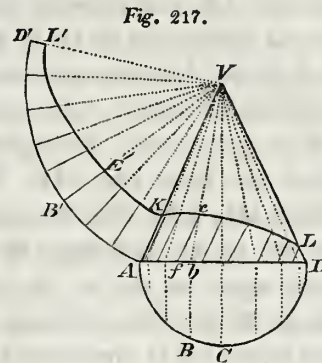
This method of finding the point E' on the developement is evident from this principle, that all the lines drawn from the vertex to the circumference of the base are equally inclined to the axis.

PROBLEM XIII.

Given the section AVD of a cone along the axis, the seat KeL of any line on the curved surface in that section, to find the developement of the line.

Find a number of points in the same manner as the point E' was found by the preceding Problem; and through all the points thus found, draw a curve $KE'L'$, which is the developement of the line of which the seat is KeL .

In this manner, the developement of any portion of a conic surface, of which seats of its boundaries are given, may be found.

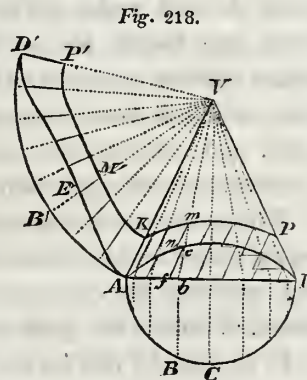


EXAMPLE.

Let it be required to find the developement of a conic surface, of which the seat is bounded by the two concentric arcs AeD , KmD , and the straight lines AK , DP .

Here we have nothing more to do, than to find the developement of the boundaries whose seats are given by the preceding Problem, and the figure $AE'D'P'MK$ will be the developement required.

Or we may find every point M in the developement thus:—draw en parallel to DA , and mn to KA ; make $E'M'$ equal to mn ; in the same manner find the other points in KMP' and $AE'D'$, and through the points thus found draw the curves, and the thing is done.



PROBLEM XIV.

Given the base and axis of a semi-ungulus, to develop the curved surface.

Let KPC be the base of the semi-ungulus.

Draw CH perpendicular to PK, meeting PK in A, or, if necessary, PK produced. Draw CB perpendicular to CA, and make CB equal to the height of the axis. On the semi-axis CA, CB, describe the quadrant ABC of an ellipse. Divide the arc AB into parts by equal chords at the points 1, 2, 3, &c. and repeat the chord as often upon the line AH as the arc AB contains one of the parts, and let the points of division be D, E, F, &c. From the points 1, 2, 3, &c. draw straight lines perpendicular to AC, meeting AC in *d*, *e*, *f*, &c. and CK in *l*, *m*, *n*, &c. and CP in *q*, *r*, *s*, &c. Through the points D, E, F, &c. draw the lines LQ, MR, NS, &c. parallel to KP. Make DL, EM, FN, &c. each respectively equal to *dl*, *em*, *fn*, &c. and make DQ, ER, FS, &c. respectively equal to *dq*, *er*, *fs*, &c. Through the points K, L, M, &c. draw a curve, and through the points P, Q, R, &c. draw another curve, and the figure PKH contained by PK and the two curves is the developement required.

As it may not always be convenient to draw the section and plan to the full size, they may be drawn in the following manner to a reduced scale, without reducing the developement.

Draw the straight line Gf. In GF take any point A, and make AG equal to one half, one third, one fourth, &c. of the base of the right radial section, and make GF equal to one half, one third, one fourth, &c. of the height of the axis, and describe the curve AbcdeF so as to be similar to the right radial section.

Divide the curve AbcdeF into any number of equal parts, and set them along the straight line Af, marking the extremity of every second, or third, or fourth, &c. part, until the straight line Af contain as many equal parts as the arc AF, so that Af will be the length of the curve at the full size.

Fig. 219, No. 1.

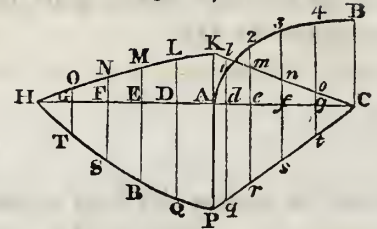


Fig. 219, No. 2.

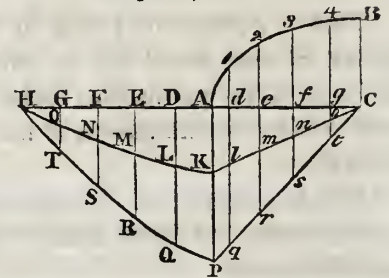
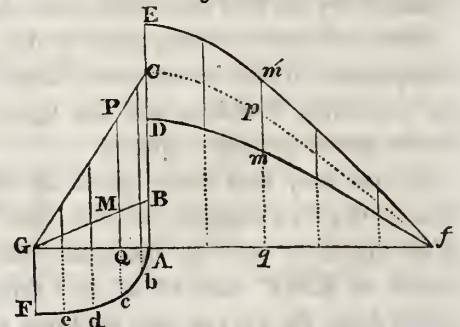


Fig. 220.



In order to find any ordinate, and consequently every ordinate:—Suppose, for instance, that corresponding to the point c in the quadrantal arc $AbcdeF$. Draw AE perpendicular to Gf . In AE make AC equal to the distance which the middle of the chord of the base is from the plane which passes through the edge of the sectroid perpendicular to AE , and in AE make AB equal to half the breadth of the chord of the base. Join GB and GC . Draw cP and qm' parallel to AE , and let cP meet GB in M , and GC in P . In qm' make pm, pm' each equal QM , and m, m' are points in each curve. In the same manner all other points will be found so as to complete the gore $fmDEmf$.

SCHOLIUM.

The developement of the surface of every solid generated by the motion of a straight line moving parallel to a fixed straight line, and passing round the edge of a given figure which is perpendicular to the fixed straight line, may be found in the same manner as the preceding developements of the cylinder, whatever may be the form of that figure, provided that a rectangle contained on one side by the intersection of the figure and the two adjoining sides, one by the line of departure and the other by the line of arrival, be given.

Or the developement of any part of that surface may be found, if the trace of that part is given in the rectangle, or any line of which the trace is given may be drawn on the developement.

On the contrary, if a line be given in the developement, its trace may be found upon the rectangle, which is the plane of position.

In illustration to what has now been observed, the diagrams exhibited in *Plates 1 and 2* of the developements of surfaces generated by a straight line parallel to itself will be evident, without any farther description, by a bare inspection of the figures, the references being made by similar letters.

These *Plates* contain the various forms of developements that occur in practice, as applicable to the intrados or soffits of arches in doors, windows, recesses, and passages.

In all the diagrams, *Plates 1 and 2*, the point E is in the middle of the arc which forms the end of the arch, Ae on the straight line is the developement of AE , ed is equal to eA , ep is equal to EP , ef is equal to ef .

Where there is not sufficient room for the trace of the solid, and the developement of its surface in one diagram, *figs. 5, 6, 7, Plate 1*, show how they may be separated.

The developement *fig. 1, Plate 3*, is the developement of the surface of a groined vault.

The developements shown in *figs. 2, 3, 4, 5, 6*, are applied in practice to polygonal domes, and are exactly the very reverse of the developement of the groined vault.

Figs. 7 and 8 are applied to any prismatic perforations made perpendicularly.

The developements of the surfaces of domoids are described in the same manner as the developements of a sectroid.

In *Plate 1* (entitled Surfaces of Solids, formed by cylinders and cylindroids under the developement of surfaces), *fig. 1* is the plan and developement of the gore of one of the sectroids of a square dome composed of four equal sectroids, the axial section perpendicular to one of the sides being the quadrant of a circle.

Fig. 2 is the plan and developement of the gore of one of the sectroids of another square dome, the axial section perpendicular to one of the sides being the semi-segment of a circle, the versed sine being the axis of the dome.

Fig. 3, the plan and developement of one of the sectroids of an octagonal dome, the axial section perpendicular to one of the sides being the quadrant of a circle.

Fig. 4, the plan and developement of the gores of the curved surfaces of two adjacent sectroids of an oblong dome, the axial section perpendicular to the least side being the quadrant of an ellipse, and the axial section perpendicular to the longest side being the quadrant of a circle.

Fig. 5, the plan and developement of the gores of the curved surfaces of three sectroids of an elliptic octagonal dome; the axial sections perpendicular to these three sides being as in *No. 2*.

In *Plate 1*, Rotative Solids generated by curves under developement of surfaces:—

Domoids with curved surfaces must be covered upon the principle of dividing the whole into sectroids, and supposing each sectroid to be an ungulus of a cylinder or cylindroid, or the lesser segment of an ungulus, according to the nature of the solid.

It is evident that, by increasing the number of sectroids, the angles of the surfaces will become more obtuse, and by continually increasing this number it will ultimately acquire a uniform curvature, and therefore will not perceptibly differ from the surface of a domoid which has a circular or elliptic base.

When the sections perpendicular to one of the axes are circular, and the axes pass through the centres of the circles, and when the sections along the axis are equal and similar ellipses, having the axis of the solid for the axis major or minor of each ellipse, the domoid ought to be divided into equal sectroids.

Fig. 1, No. 1, the elevation of a circular domoid, having all its radial sections quadrants of circles.

In *No. 2* are given the radial *ACQ* of the domoid, and the base *ECD* of a sectroid, to find the developement of the gore, the base of the sectroid being an isosceles triangle, and the point *A* its vertex.

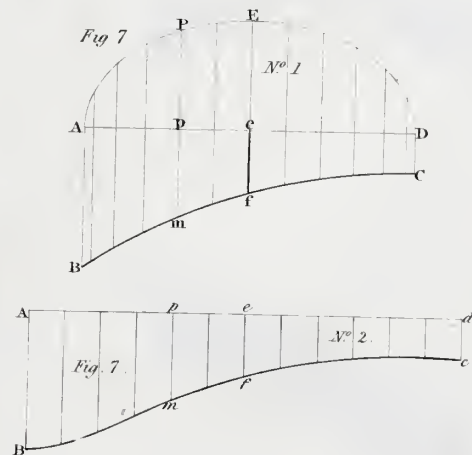
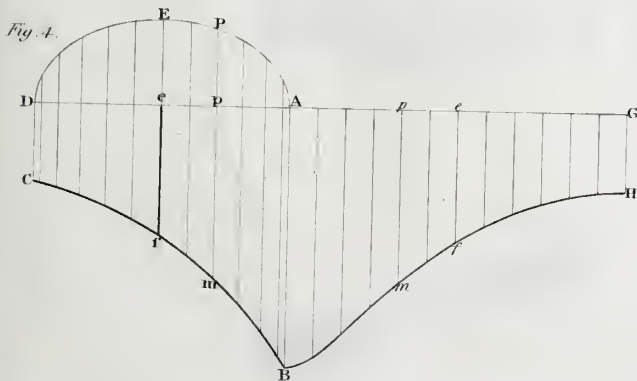
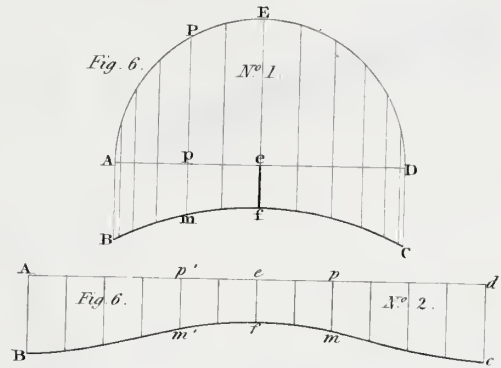
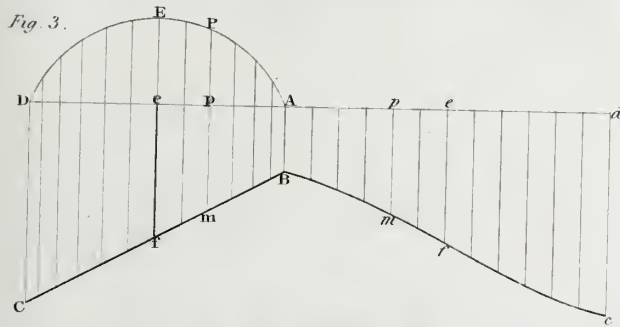
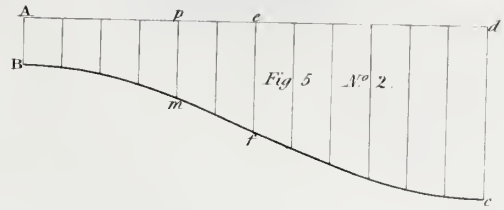
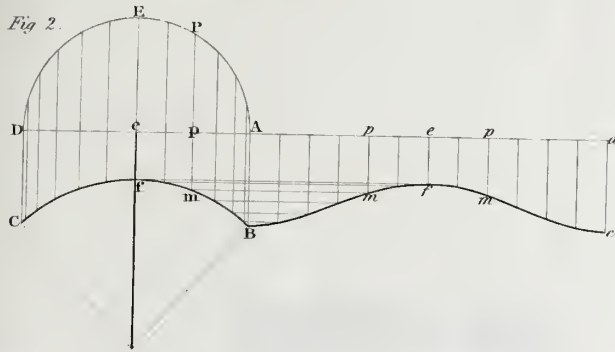
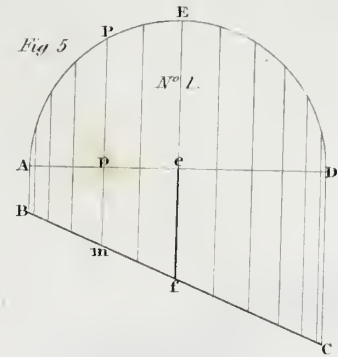
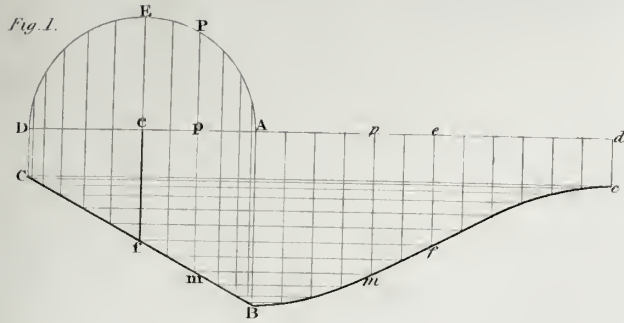
The developement *EDF* of the gore will be found by *Prob. xiv, p. 146*.

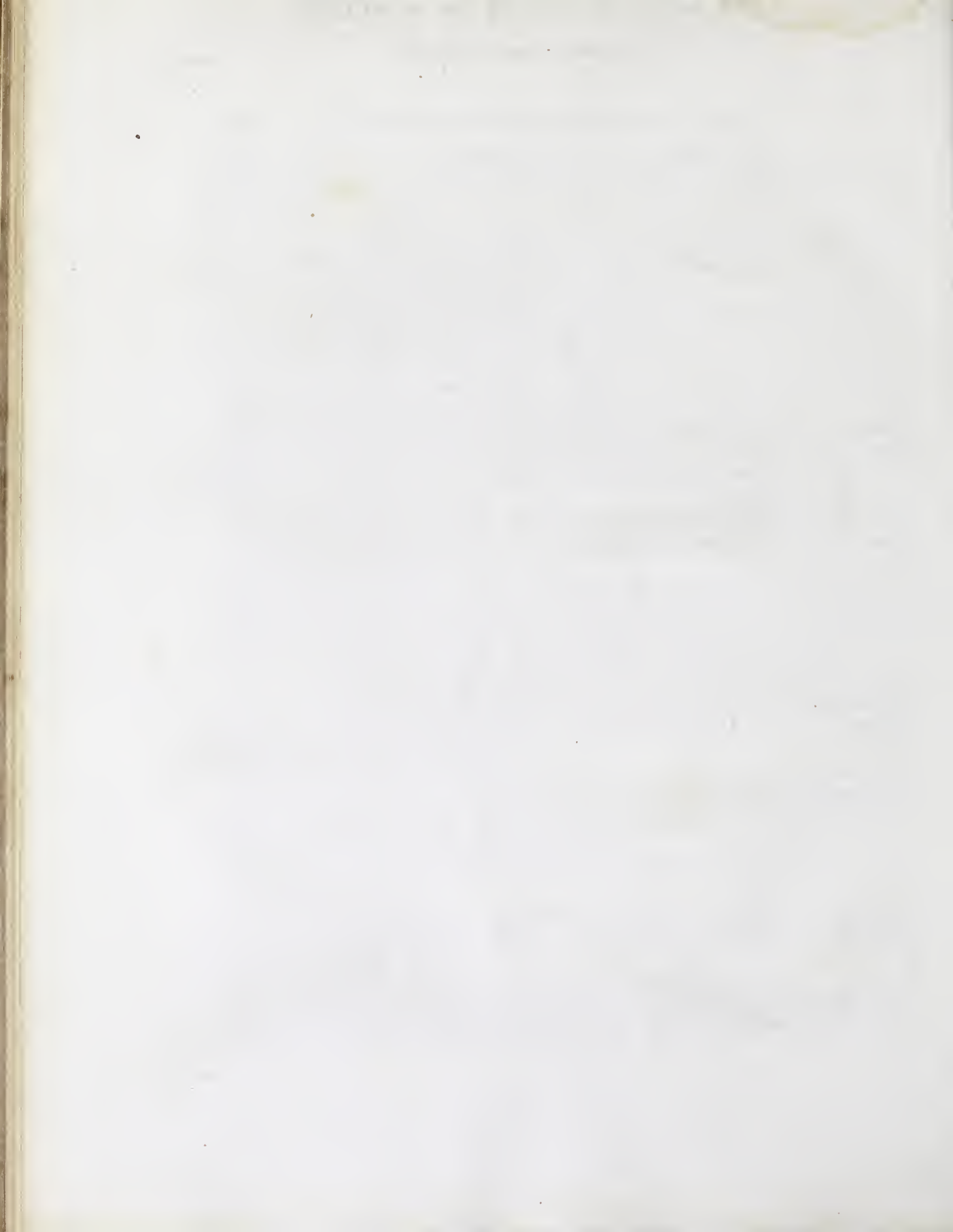
Here we have supposed the dome to consist of isosceles sectroids, of which each is the semi-ungulus of a cylinder, and that the curved surface of each sectroid is in contact only

THE DEVELOPEMENT OF SURFACES

GENERATED BY PARALLEL MOTION.

PLATE I.

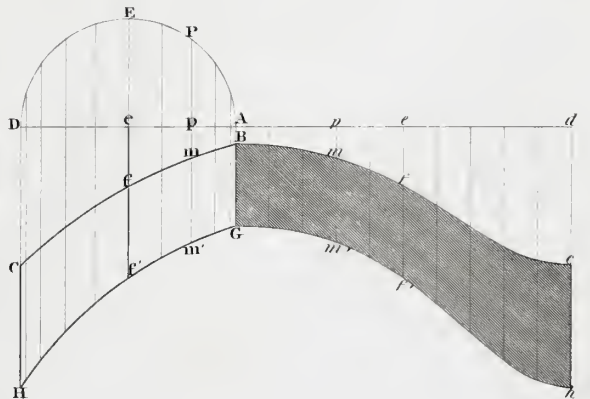
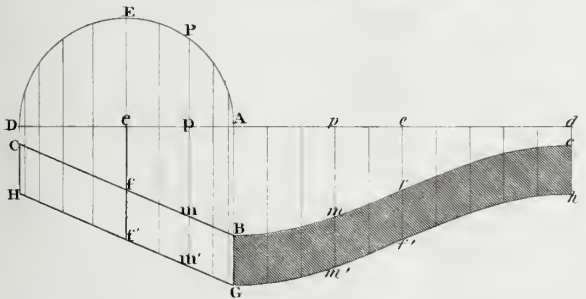
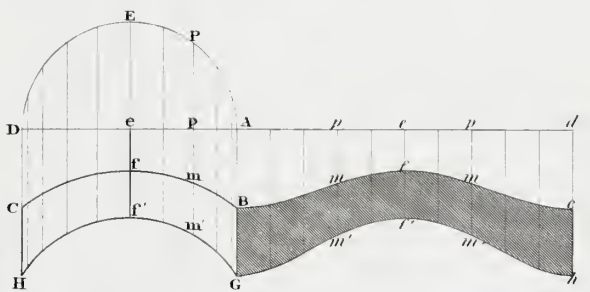
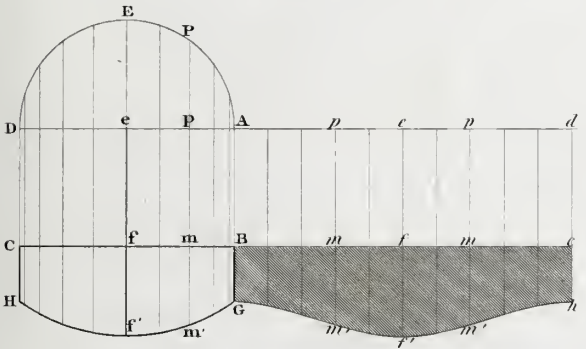
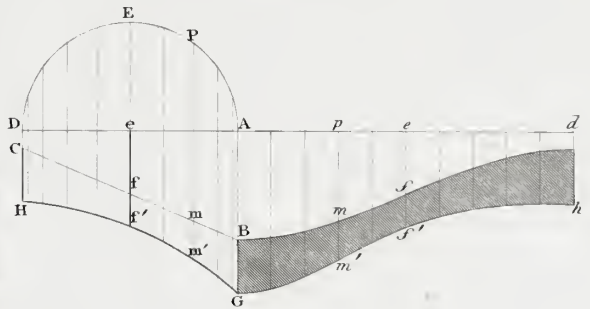
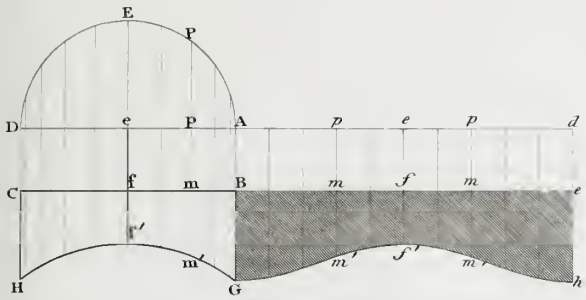
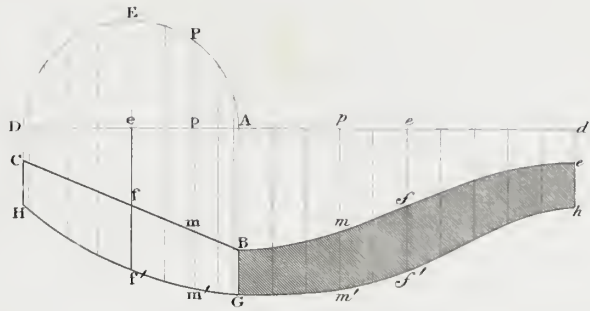
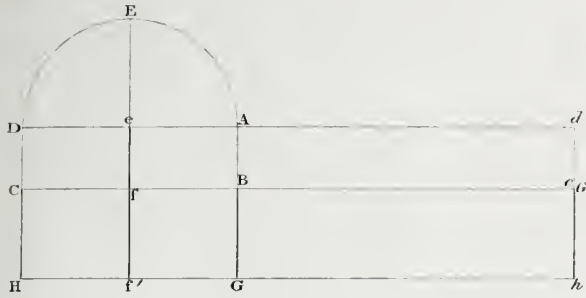


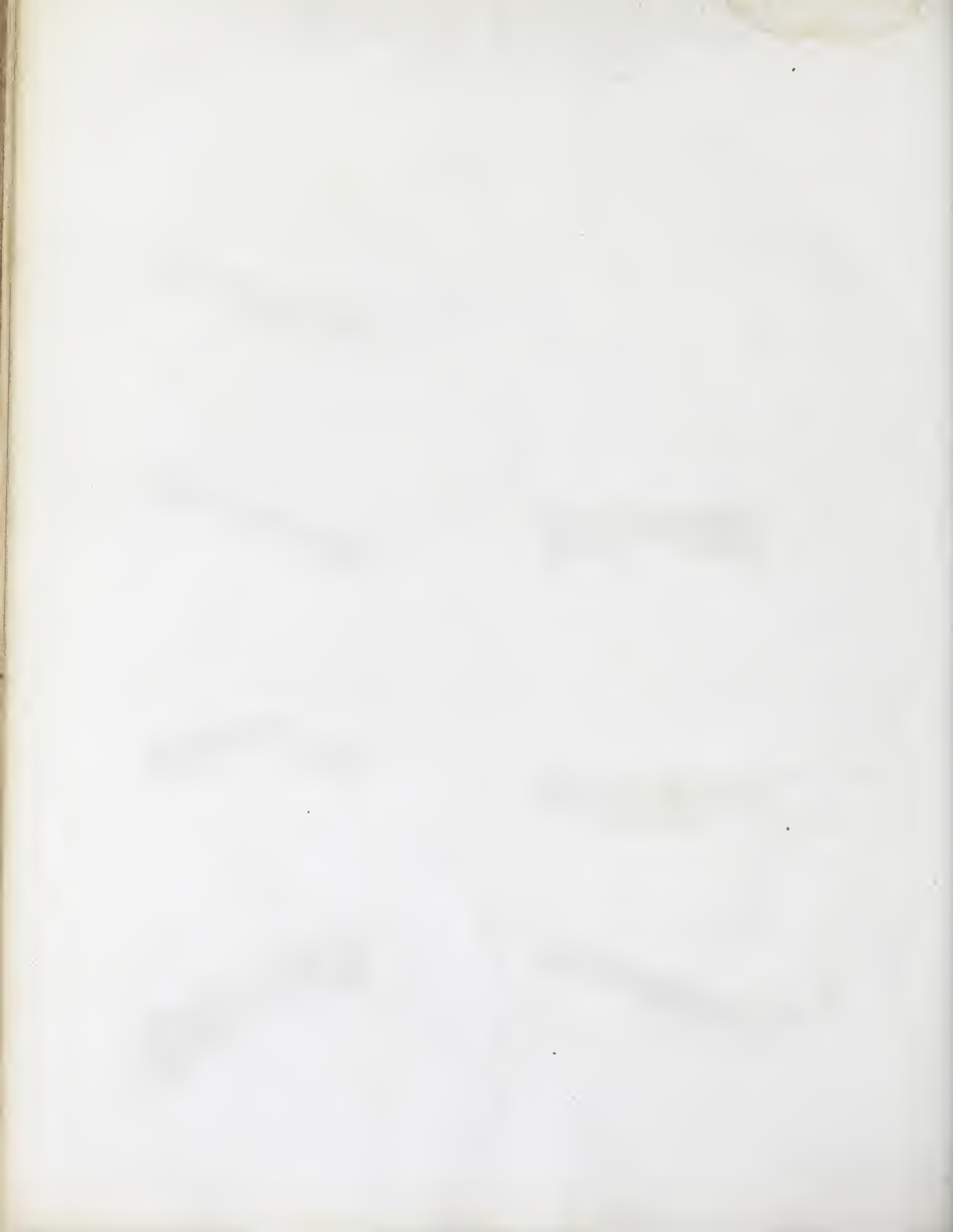


THE DEVELOPEMENT OF SURFACES.

GENERATED BY PARALLEL MOTION,

PLATE II.





THE DEVELOPEMENT OF SURFACES.

GENERATED BY PARALLEL MOTION.

PLATE III.

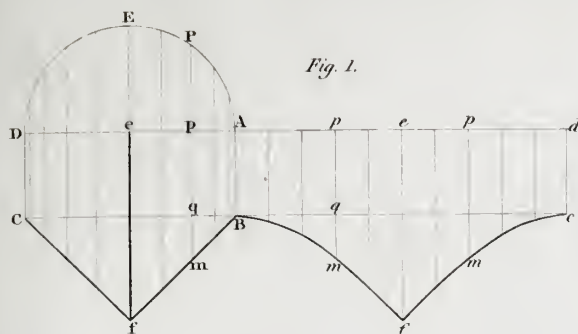


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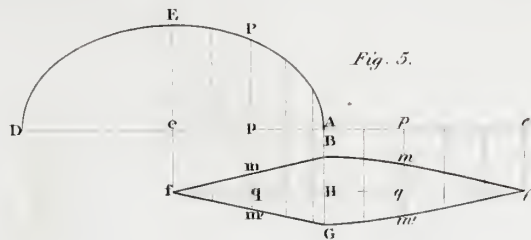


Fig. 5.

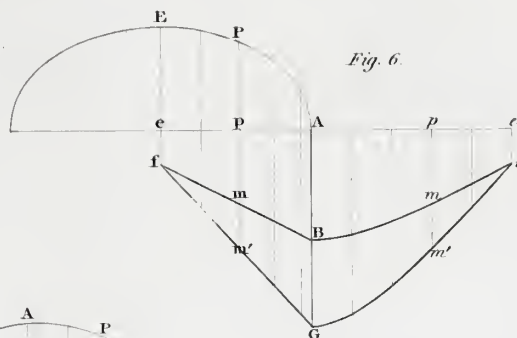


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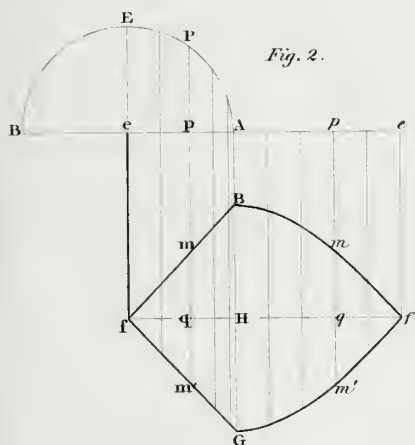


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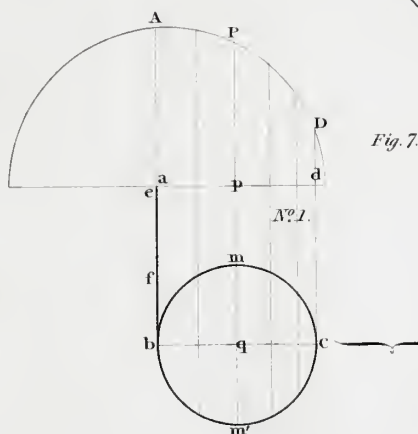


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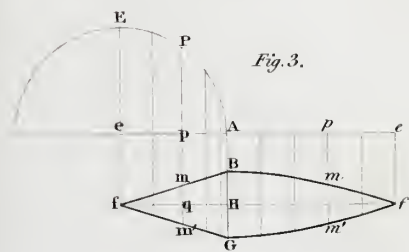


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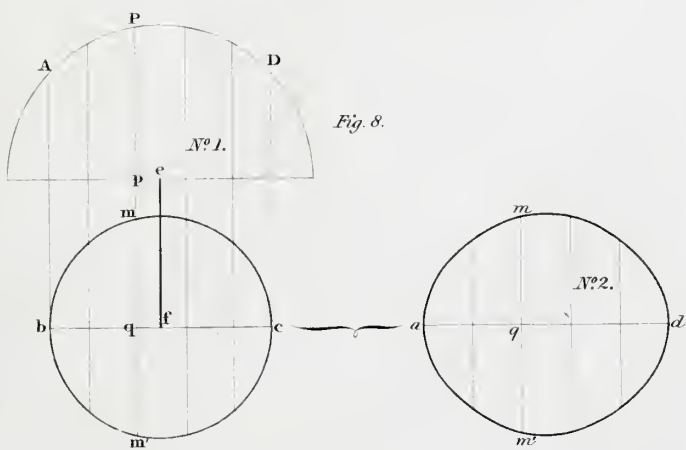


Fig. 8.

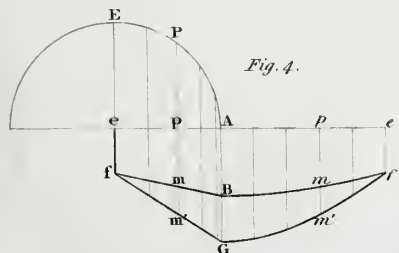


Fig. 4.

THE DEVELOPEMENT OF SURFACES.

SURFACES OF SOLIDS FORMED BY CYLINDERS & CYLINDROIDS.

PLATE I.

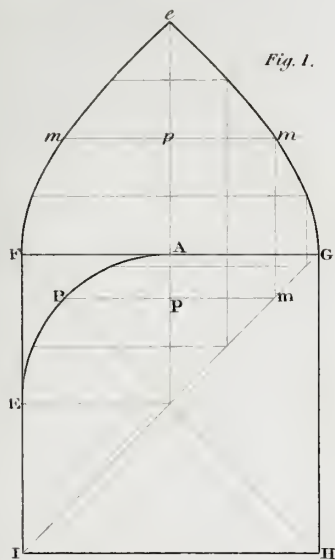


Fig. 1.

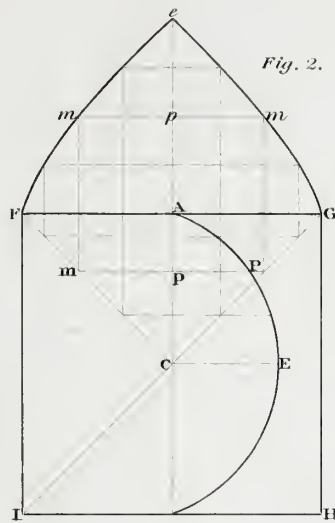


Fig. 2.

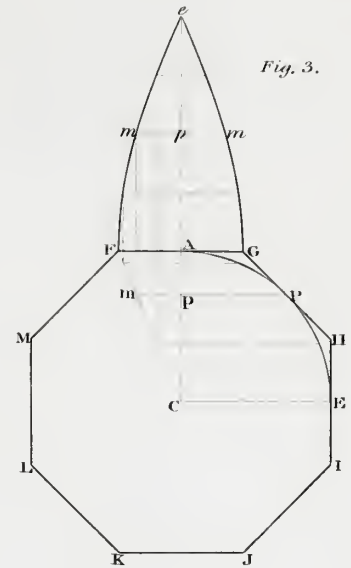


Fig. 3.

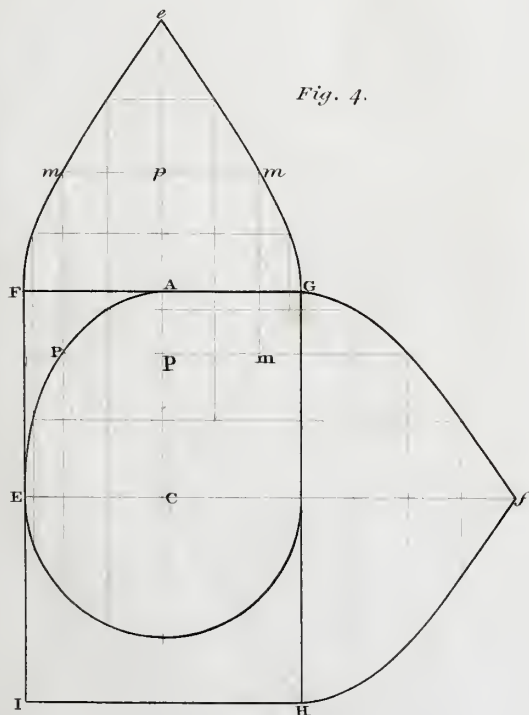
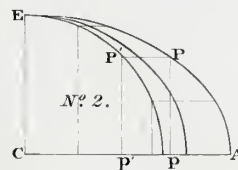


Fig. 4.



Nº 2.

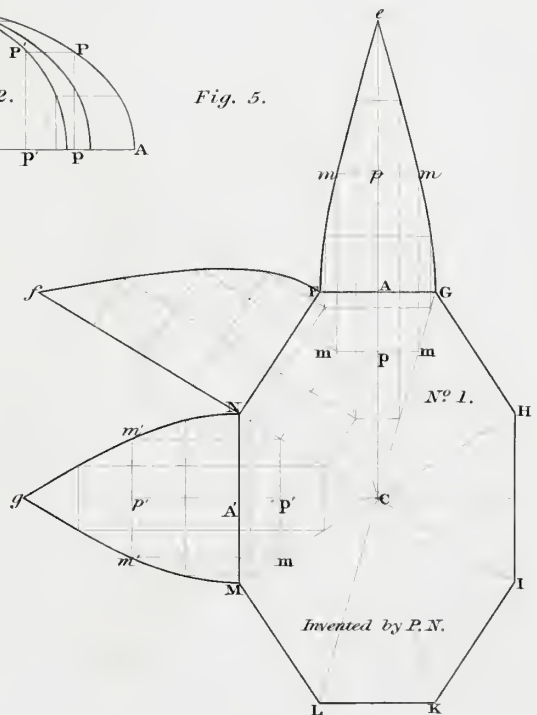


Fig. 5.

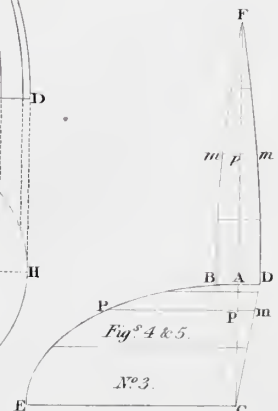
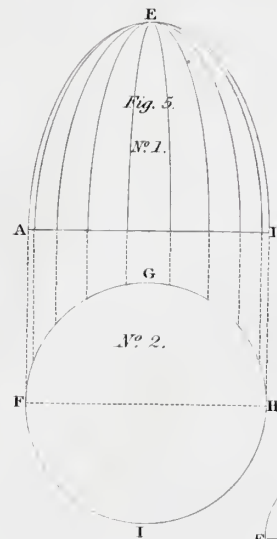
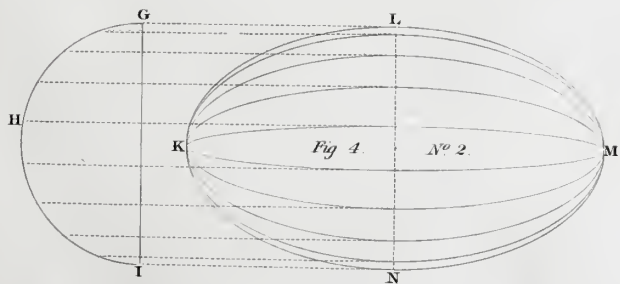
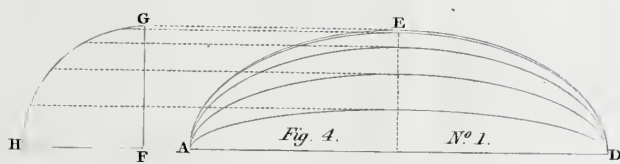
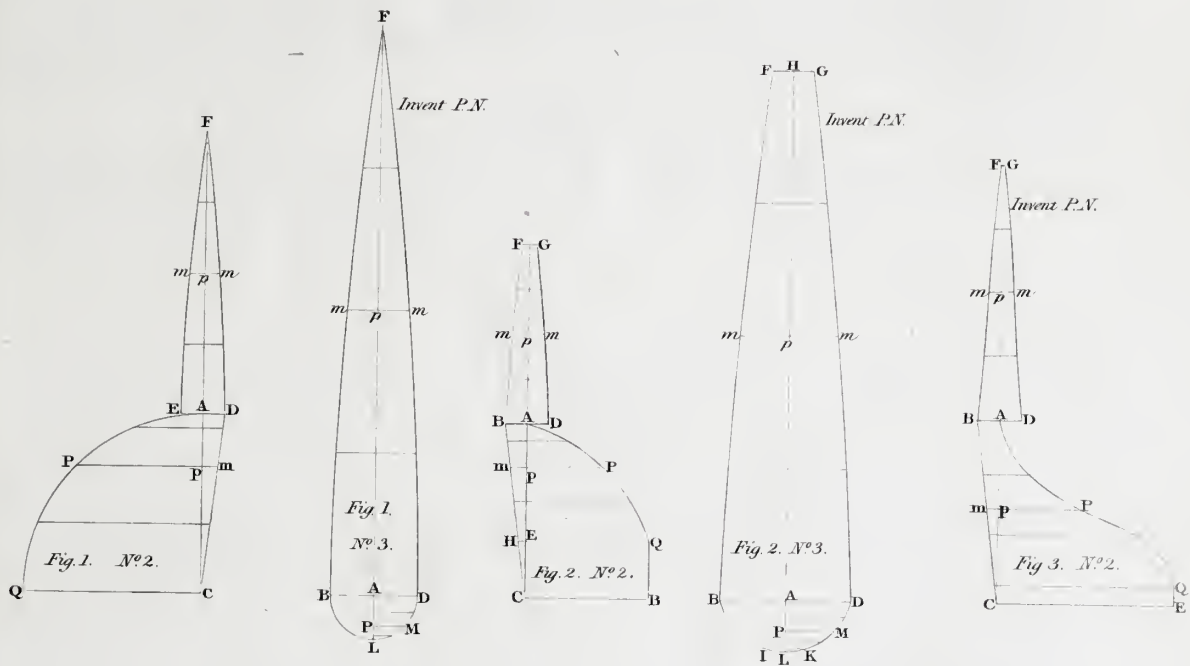
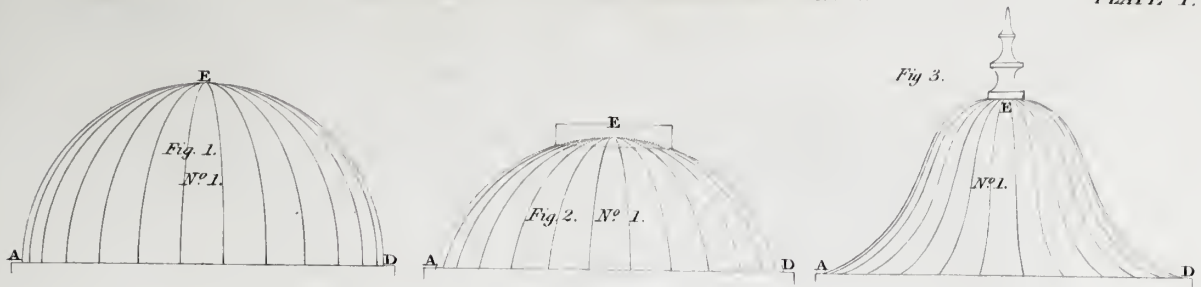
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Invented by P.N.

THE DEVELOPEMENT OF SURFACES.

ROTATIVE SOLIDS GENERATED BY CURVES.

PLATE I.



with the edge of the radial section, which bisects the angle at the axis of that sectroid; and therefore the base is a polygon circumscribing the circular base of the dome.

The developement of one of the equal and similar sectroids of a domoid which has a circular base, and has its section through the axis, a segment less than a semicircle may be found without the necessity of laying down the base of the sectroid.

It will be necessary, however, to show how the developement of the arc may be found arithmetically.

For this purpose, let z = the length of the arc, s = the sine or half chord, and v = the versed sine; then the numeral values of the sine and versed sine being substituted in the following formula,

$$z = \frac{2s(6s^2 + 5v^2)}{6s^2 + v^2}$$

will give the developement of the half arc nearly; and this is the developement of the arc of the radial section of the sectroid.

EXAMPLES.

1. Let it be required to find the length or the developement of the arc of a segment, of which the chord is 30 feet and the versed sine 15 feet.

The sine is $15 = \frac{30}{2}$, so that, in this case, the sine and the versed sine are equal; therefore the sine may be substituted in the formula for the versed sine. Whence $z = \frac{2s(6s^2 + 5v^2)}{6s^2 + v^2}$ will become $\frac{22s^3}{7s^2} = \frac{22s}{7}$. Consequently, by substituting 15 for s , we shall have $z = \frac{22 \times 15}{7} = 47\frac{1}{7}$, which is the length of the arc; and here the formula coincides with the proportion of Archimedes.

2. Let it be required to find the developement of the arc of a circle, of which the chord is 40 feet and the versed sine 15.

By substituting the numbers for their representatives,

$$z = \frac{2 \cdot 20(6 \cdot 20^2 + 5 \cdot 15^2)}{6 \cdot 20^2 + 15^2} = \frac{40(2400 + 1125)}{2400 + 225} = \frac{40 \times 3525}{2625} = 53.7 \text{ feet,}$$

which is the length of the arc, nearly.

Let it now be required to find the curve of the axial section of a circular dome, of which the diameter of the base is 40 feet and the height of the axis 15 feet.

Here the radial section is half the segment of a circle, of which the sine or half chord is 20 feet, and the versed sine 15 feet. The length of the arc of the radial section to this would be the half of 53.7, which is 26.85 feet; and this is the length of the developement of one of the gores.

PROBLEM XV.

To form the gores of a domoid without the radial section or base of the sectroid.

Find the developement of the length of the arc, and the height of the segment of a circle of which the breadth of the gore shall be its chord. Draw a straight line equal in length to the developement of the arc. Through one extremity draw a perpendicular, on which set half the breadth of a gore, and produce the straight line first drawn on the other side of the perpendicular, making the part produced equal in length to the versed sine of an arc, of which the sine will be to the versed sine as the radius of the base of the dome is to the height of that dome, and of which the sine is equal to the breadth of the gore.

Having described the arc belonging to the sine and versed sine, divide this arc into any number of equal parts, and divide the developement of the length of the gore into as many equal parts: through the points of division in the arc, draw sines to the several arcs from the summit of the versed sine, and through the points of division in the line of developement draw straight lines perpendicular to and on each side of it.

Begin at the greatest of the sines, and take their lengths respectively, and set them from each point of intersection on each opposite part of the perpendicular one after the other, beginning on the first perpendicular next to the base of the gore.

Draw a curve through the points thus found on the one side of the line of developement and then on the other, and the figure contained by the two curves and the chord of the entire segment is the developement of the gore required.

EXAMPLE.

Let it be required to describe the developement of a gore for a circular dome, of which the section along the axis is the segment of a circle having a chord of 40 feet, and a versed sine of 15 feet in the axis of the dome, so that the breadth of the gore shall be 12 inches at the bottom.

As the height of the dome is to half the diameter of its base, so is half the breadth of the gore at the bottom to the versed sine of a segment similar to the axial section of the dome. Thus,

$$\begin{array}{r} \text{ft.} \quad \text{ft.} \quad \text{in.} \quad \text{in.} \\ 20 : 15 :: 6 : 4\frac{1}{2} \\ \hline 20 \overline{)90} \\ \underline{40} \\ 45 \end{array}$$

Draw the straight line CD , and suppose CD equal to the length of the arc, viz. 26 feet 10 inches. Through C draw AB perpendicular to CD , and suppose CA and CB each to be equal to 6 inches. Produce DC to E , and make CE equal to $4\frac{1}{2}$ inches. Describe the arc $AEmB$, of which the chord is AB and the versed sine CE .

Divide the arc BE into any number of equal parts at m ; also divide the straight line CD into the same number of equal parts at P . Make CP every where as many of the equal parts of CD that BM is of BE . Draw the lines pm parallel to AB , meeting CE in p , and the lines MM' through P parallel to AB . Make PM , PM' equal to pm , and the lines



passing through the other points of division in the same manner. Through the points *M* draw a curve, which will be one edge of the gore; in the same manner draw the other edge, which will complete the developement.

In this manner the developement of a board or gore of a dome can be described, whatever may be the form of the radial section. It is only describing a figure on the base, exactly similar to the radial section of the dome.—See *figs. 1, 2, No. 3*.

In *fig. 3*, the curvature *ACEQP* of the axial section retrogrades, and the developement *BDFG* will partake of the same form; and when the dome is truncated, it is obvious that the developement of the gore must also be truncated, as in *No. 2, fig. 2, No. 3, fig. 2, and No. 2, fig. 3*.

Fig. 4 is the representation of an ellipsoidal dome, *No. 1* being the elevation, and *No. 2* the plan, exhibiting the orthoprojection of the gores or curved surfaces of the sectroids.

Fig. 5 represents the plan and elevation of a dome in the form of an elliptic conoid.

Figs. 4 and 5, No. 3, show the manner of forming the developement of both domes in *figs. 4 and 5*, which in fact are portions of the same solid; with this difference, that the quiescent axis of the solid in *fig. 4* is in the plane of the base, and in *fig. 5* it is perpendicular to the base.

GENERAL PRINCIPLES.

OF THE FORMATION OF ARCHES OF DOUBLE CURVATURE.

DEFINITIONS.

1. AN arc of double curvature is a curve line in which any three points which are not in a straight line being taken, the plane passing through these points will not pass through all the other points in that curve.

2. Arches of double curvature are those which are formed by the curved surfaces of two different solids, whose intersection forms an arc of double curvature.

3. A sphero-cylindric arch is an arch of double curvature, formed by the intersection of a sphere and cylinder, when the sphere surmounts the cylinder.

4. A cylindro-spheric arch is an arch of double curvature, formed by the intersection of a sphere and cylinder, when the cylinder surmounts the sphere.

The same is to be understood by the composition of any other two bodies, as *sphero-conic*, *cono-spheric*, &c.; the body which surmounts the other being designated by the termination *o*, and the arc which is surmounted by the termination *ic*.

Arches of double curvature may be constructed in an infinite variety of forms; but we shall limit the number, by only employing the surfaces of the six solids,—the cylinder, the cylindroid; the cone, the cuni-conoid; the sphere, and the spheroid; which being selected in twos out of these six, will form fifteen combinations, and consequently fifteen different arches.

In the construction of arches of double curvature, let it be understood, unless expressed to the contrary, that only a portion not exceeding the half, cut by a plane from each entire solid, is employed, and that in the cylinder, cylindroid, or cone, the cutting plane either passes along the axis, or is parallel to the axis; and in solids of revolution, the cutting plane is perpendicular to the axis; and that, in both solids which form an arch of double curvature, the planes formed by the cutting plane are in the same plane; and that in a cylindroid the portion of the solid is the archoid of a cylindroid.

5. When, in an arch of double curvature, the axes of both solids are entirely in the same plane, the arch is called a *right arch* of double curvature.

6. When, in an arch of double curvature, the axes of both solids are not in the same plane, the arch is said to be *oblique*.

PROBLEM I.

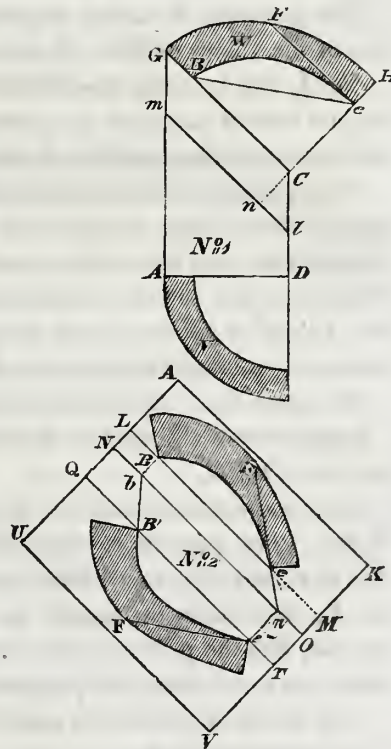
Given any two parallel lines in the determining plane of a cylinoid, and the end of the cylinoid, to construct that portion of this solid which is contained between two planes, each passing along each of the parallel lines perpendicularly to the determining plane.

Let the determining plane be $ADCG$, and let V be the given end of the cylinoid, and GC and ml be the two parallel lines.

Find W , the oblique end, in the same manner as in Prob. i, p. 99. Join Be , the two inner extremities, and draw eF parallel to BC , meeting the outer curve GFH in the intermediate point F , and draw Cn perpendicular to BC , meeting ml at n , and join Be .

In No. 2, draw the two parallel lines NO and QT at the distance Cn , No. 1, and draw ne perpendicular to NO , meeting NO at n , and QT at e . In the plane $QTVU$ draw eF , making the angle QeF equal to BeF , No. 1; and in the plane $NOKA$, and at the point n in the straight line NO , draw nF , making the angle NnF equal to the angle BeF , No. 1. In nF make ne equal to nl , No. 1, and through e draw LM parallel to NO . Let $QTON$ be considered as the edge of a plank, en its thickness, $NOKA$ the face of the plank, and $QTVU$ the under surface parallel to the face, the three sides being developed in one plane. Apply the section W of the oblique end as a mould to the plank No. 2, first to the face $NOKA$, so that the points B and e in the mould may be in the straight line LM , the point e in the mould being upon e ; then, having with a pencil drawn the shape of the mould on the top, remove and apply it to the under side $QTVU$, so that the points B and e in the mould may be in the line QT , the point e in the mould being upon e in the line QT , and draw the shape of the mould; then the figure on each side of the plank, which is identical with W , will be the two sides of the solid to be formed; and by cutting away the material on the outside of these lines, so that a straight edge parallel to the straight line bB on

Fig. 222.



the edge of the plank may coincide with the surface, connecting the outlines formed by the application of the mould, we shall have the solid contained between the two parallel planes as required.

SCHOLIUM.

By this method the edges of angle brackets, the curve edges of cove brackets, the angle ribs of groins, hand-rails of stairs, &c. are formed in a more simple manner, and out of less material than has yet been done.

This principle is further explained in *Plate 2, General Principles*, designated Formation of the Edges of Ribs, Brackets, or Hand-rails, to range in prismatic surfaces.

Fig. 1, No. 1, is the mould drawn out as if it were the section of a cylinoid; the figure may be that of angle rib of a groin, or that of the angle rib of a cove bracket. *No. 2* is the application of the mould to the plank.

Fig. 2, No. 1, may represent the face mould of a hand-rail. The face mould may be applied to the plank in any other direction besides that of bringing the points to the edge of the plank, and this will be useful either to avoid a bad part of the wood, or to match the grain in any manner which may be desired. In *fig. 2, No. 2*, suppose, for instance, that *TQUV* is the top of the plank; *TONQ* the narrow surface in which the thickness is measured, commonly called the edge, perpendicular to the plane *TQUV*; and that *ONAK* is the under face, and consequently parallel to the plane *TQUV*.

Suppose now the mould *eBqrFste* to be placed in *No. 2* in the situation required, upon the face *TQUV*.

Draw *en* perpendicular to *TQ* or *ON*, meeting *TQ* in *v*, and *ON* in *w*. Make *wn* equal to *ve*. In *eF* produced, take any point *I*; draw *IJ* perpendicular to *TQ* or *ON*, meeting *TQ* in *x*, and *ON* in *y*. Make *yJ* equal to *xI*, and join *nJ*. In *nJ* make *ne* equal to *Cn*, *No. 1*; then apply the mould, so that the line *eB*, which is drawn upon it, may be on the line *nJ*, and the point *e* in the mould upon *e* in the plank, and the plank will be properly lined out for forming the perpendicular sides of the rail.

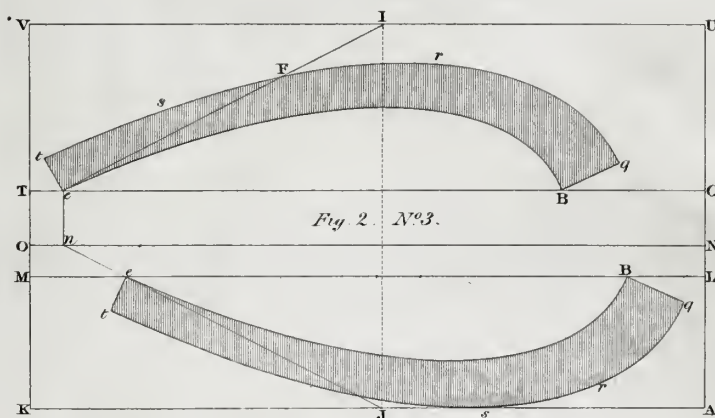
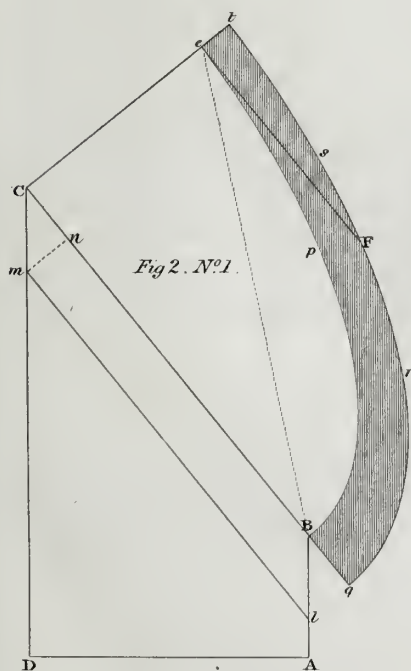
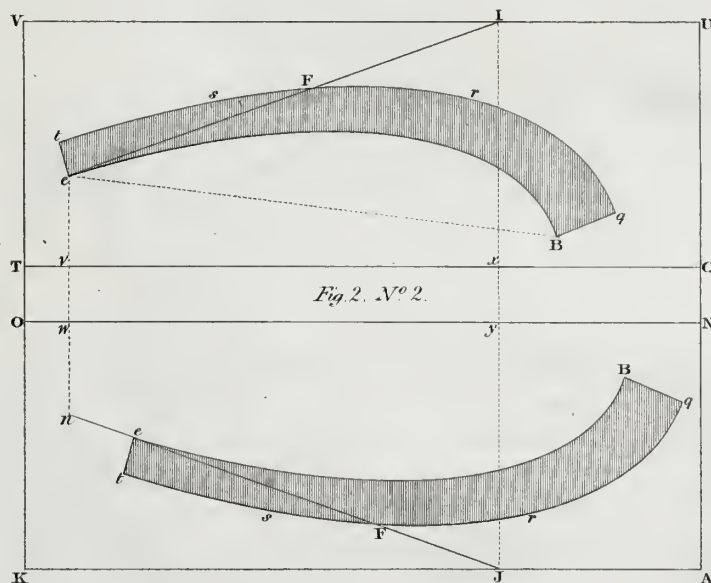
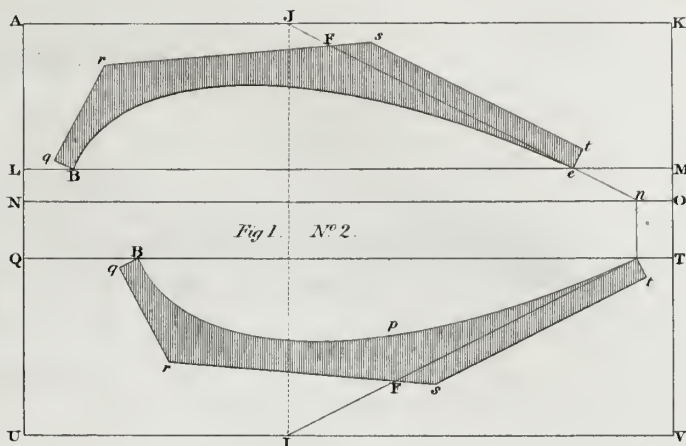
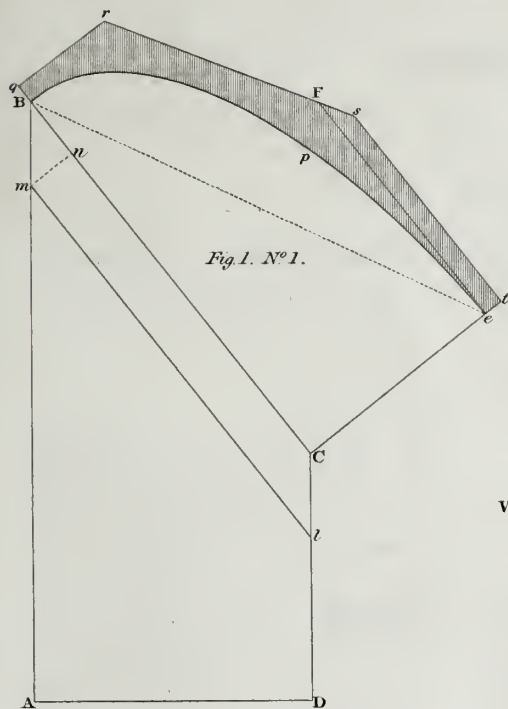
Fig. 2, No. 3, shows the same lines by bringing the points of the mould to the edge of one face of the plank.

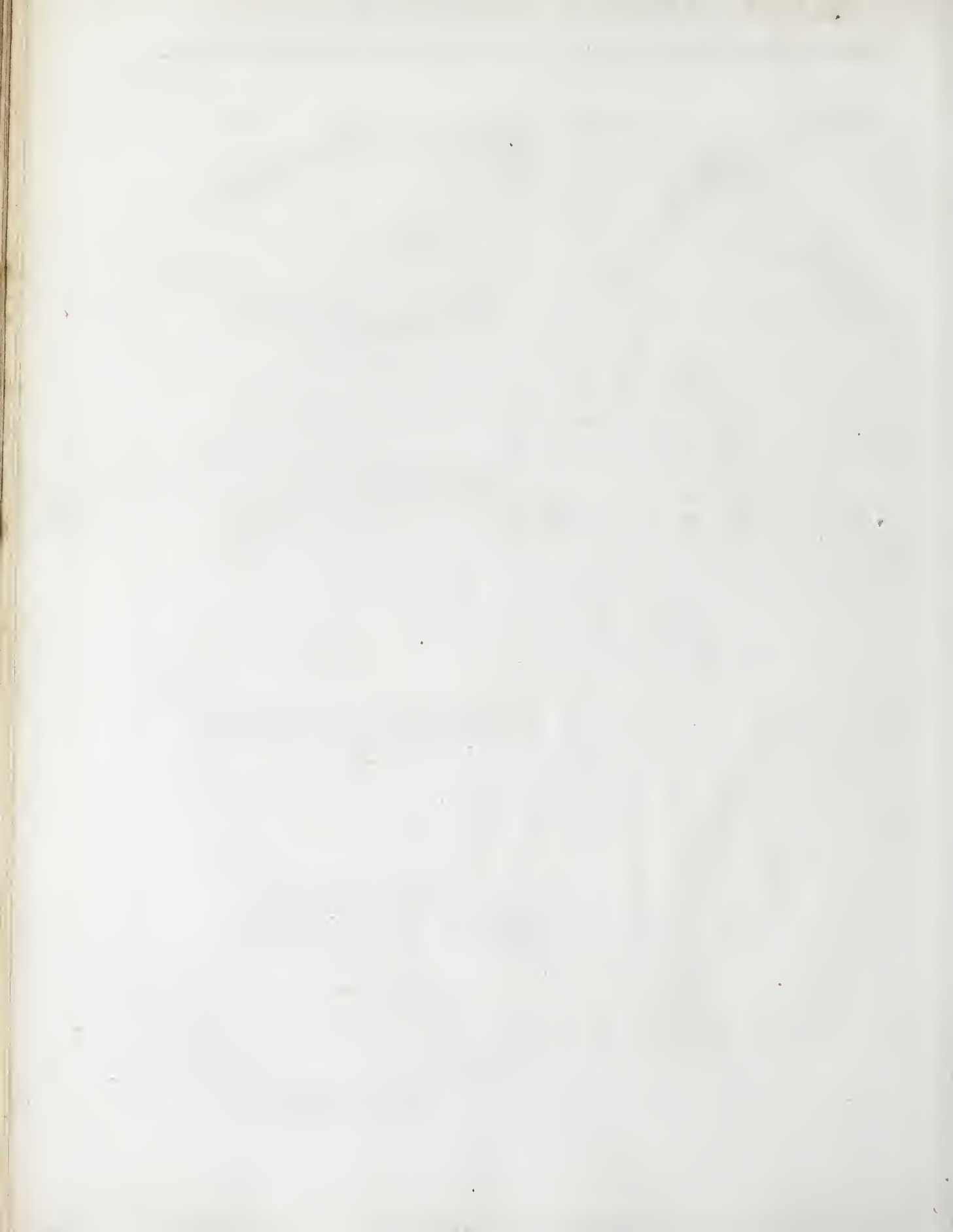
Plate 1, General Principles (subtitled Formation of the Edges of Ribs, Brackets, or Hand-rails, to range in prismatic cylindric surfaces), is to the same purpose; but, as it depends on the principles of solid angles, we shall at present pass over the description.

GENERAL PRINCIPLES,

FORMATION OF THE EDGES OF RIBS BRACKETS OR HANDRAILS TO RANGE IN PRISMATIC SURFACES.

PLATE II.





PROBLEM II.

To construct a cylindro-cylindric right arch with concentric intrados and extrados, having their axis parallel to the base, and that of the other which is of greater diameter perpendicular to the base.

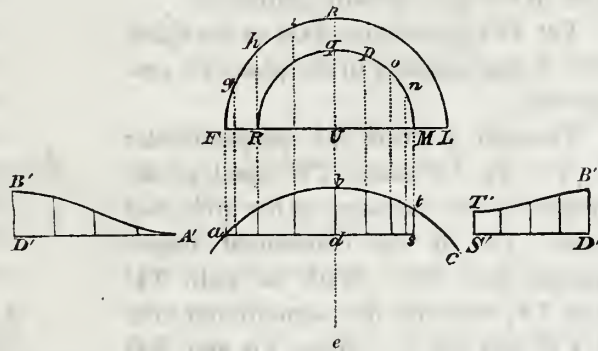
METHOD 1.

Let abc be the arc, and consequently the base of the cylinder, which has its axis vertical, and let e be its centre.

Through e draw any line eU , meeting the arc abc in b . Through any point U in eU , draw FL perpendicular to eU .

From U , with the radii of the cylindric surfaces which form the extrados and intrados of the arch, describe the arcs FkL , RqM , meeting FL in R , M , as well as in F and L . Draw Fa and Ms parallel to eU , and let MS meet abc in t . Draw as perpendicular to eU , meeting eU in d . Find the developement $A'D'B'$ of that portion of a cylindric surface represented by its seat adb , the radius of the cylinder being UF ; also find the developement $S'T'B'D'$ of that portion of a cylindric surface represented by its seat $stbd$; then if a hollow semi-cylinder with its end perpendicular to the axis be made, and the developement $A'D'B'$ be applied on the outside with the straight edge $A'D'$ close to the end, and the point A in the base; and drawing a line by the curved edge, first upon one side and then upon the other, and the developement $S'T'B'D'$ be applied to the intrados in the same manner to each half, and lines drawn to each application; then, cutting away the superfluous wood between the lines on the upper and under surface, the arch required will be formed.

Fig. 223.



SCHOLIUM.

The method of constructing a cylindro-cylindric right arch is well adapted to a small arch made in one piece, or even to a large arch, where the versed sine BD of the segment of the base formed by the intersection of the two cylinders on the plane of their bases is

very small. Suppose, for instance, that a circular bow in the wall of a house has a chord of 18 feet, and versed sine 6 feet, and that a window 4 feet wide with a semicircular head is to be inserted in the bow, the versed sine answering to a chord of 4 feet will not exceed $2\frac{1}{2}$ inches. In this case, the waste of material will not be of much consequence; but where the arch is large and the versed sine considerable, the following method will be more eligible:—

METHOD 2.

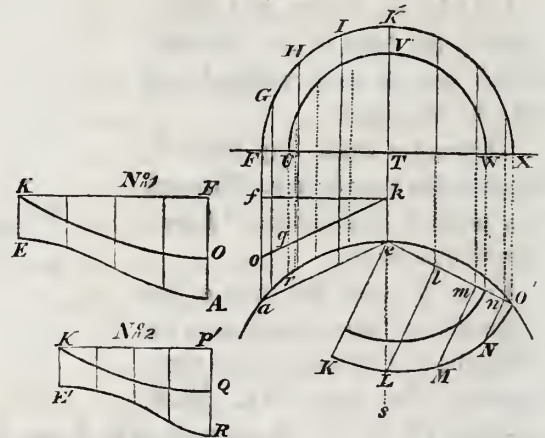
Let aeO be the seat of the intersection of the two cylindric surfaces.

Let Ts represent the axis of the cylinder, being parallel to the plane of projection.

Through T draw FX perpendicular to Ts . In FX make TU equal to the radius of the intrados of the arch, and make TF of any convenient length greater than TU . With the radii TU and TF , describe the semicircular arcs UVW and FKX . Draw Fa and XO parallel to Ts . Join the half chords ae and eO . Draw ok at any convenient distance which the thickness of stuff might require parallel to ae , meeting Fa in o , and Ts in k .

We have now given two parallel lines ae and ok in the determinating plane $FTea$ of a cylindroid, and the end of the cylindroid FKT or TKX to construct that portion of the solid between two parallel planes passing along each of the parallel lines perpendicular to the determinating plane $FTea$. This being done by Prob. i, p. 153, the next thing is to find the curved surfaces of the horizontal cylinder: this will be done by the two developements $OAEK$, $QREK$, No. 1 and 2. The curve OK' in No. 1, and QK' , No. 2, agreeing with ok and qk , will coincide with the face of the plank, and the curves AE' , No. 1, and RE' , No. 1, will be the face of the wall when each of these moulds are bent, one upon the exterior and the other into the interior cylindric surface: then the superfluous wood being cut away will form the solid for one half of the arch; the other half will be found in the same manner by reversing the application of the moulds.

Fig. 224.



Here the vertical side of the front rib is a plane surface. The straight side *FH* of the bevel *HFG* is applied to the plane of the inside of the front rib, and the stuff is cut away from the inner arc to the cylindric surface on the outside, until the curved edge *FG* coincide in all parts, and thus form the intrados of the front rib in the portion of a spheric surface.

Plate 1, General Principles (subtitled Cono-cylindric Arch), and *Plate 2*, General Principles (subtitled Sphero-cylindric Arch), are given for the exercise of the learner.

The first of these plates shows how an arch in a circular wall with a conic intrados may be executed in three parts; the bevel shown at the end belongs to the middle piece. In the execution of this, the learner must first understand how the section of a semi-cone perpendicular to a plane passing through the axis is to be found.

The second of these two plates only shows a different method of executing a spherical niche in a circular wall by forming the intrados in a conic surface instead of a cylindric surface.

DEFINITIONS OF ARCHITECTURAL SOLIDS.

1. THE art of combining the various geometrical figures and solids in the formation of the exterior and interior parts of an edifice, is called the *geometry of architecture*.

2. Plane figures which may be each so divided into two parts, that when the one part is applied to the other their boundaries may coincide, are called *symmetrical figures*.

3. The straight line which divides a symmetrical figure into two equal and similar figures, is called the *line of symmetry*.

4. When one side of a room or the front of an edifice is so constructed that it can be cut into two such parts by a vertical plane, and that a straight line passing from any point in that front perpendicularly to that plane will meet another point of that front on the other side of the plane at the same distance from it as the first point, that side or front is said to be *symmetrical*.

5. The plane which thus divides the front of a building into two equal and similar parts, is called the *plane of symmetry*.

In the decoration of walls, and in the construction of the front of an edifice, the plane of symmetry is always vertical.

6. The surface of a room which is opposite to the floor is called the *ceiling*.

7. When the ceiling of a room is composed of one or more concave surfaces of one or more geometrical solids, the ceiling is called a *vault* or *vaulted ceiling*.

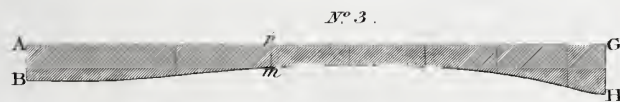
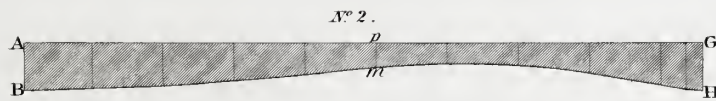
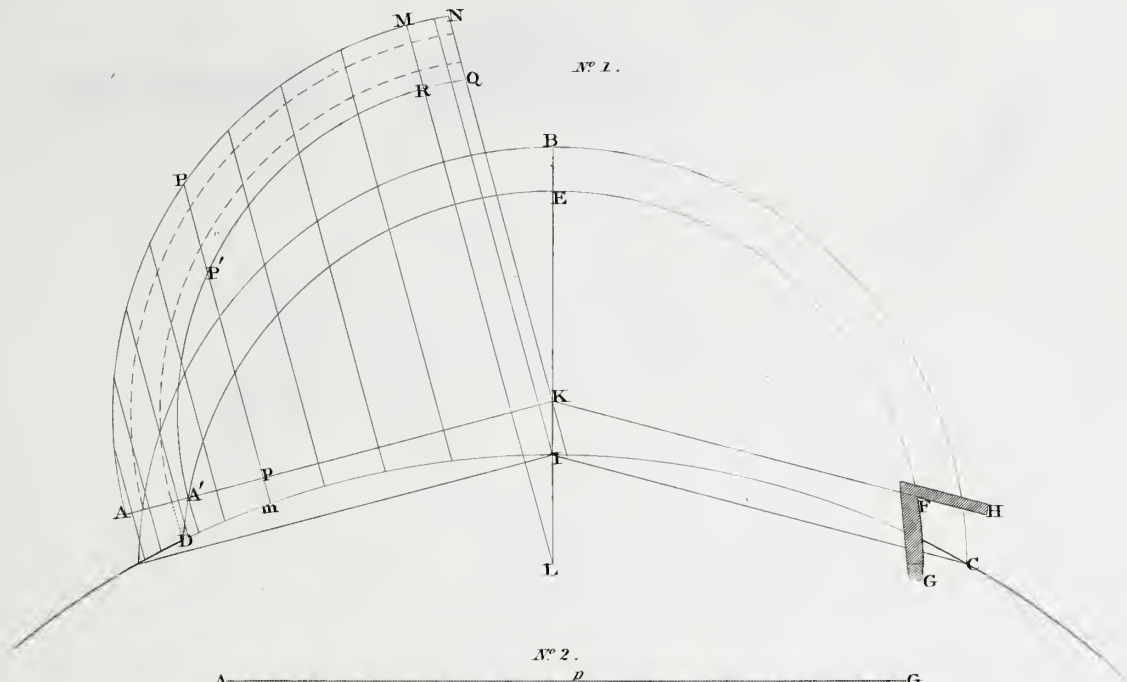
8. When the vaulted ceiling is that of the concave surface of an archoid having its axial plane vertical, the ceiling is called an *arched ceiling*.

9. When the ceiling is the concave surface of a domoid having a circular or elliptic base, and its axis in a vertical position, the ceiling is called a *dome* or *cupola*.

GENERAL PRINCIPLES.

CONSTRUCTION OF RIBS OF DOUBLE CURVATURE.

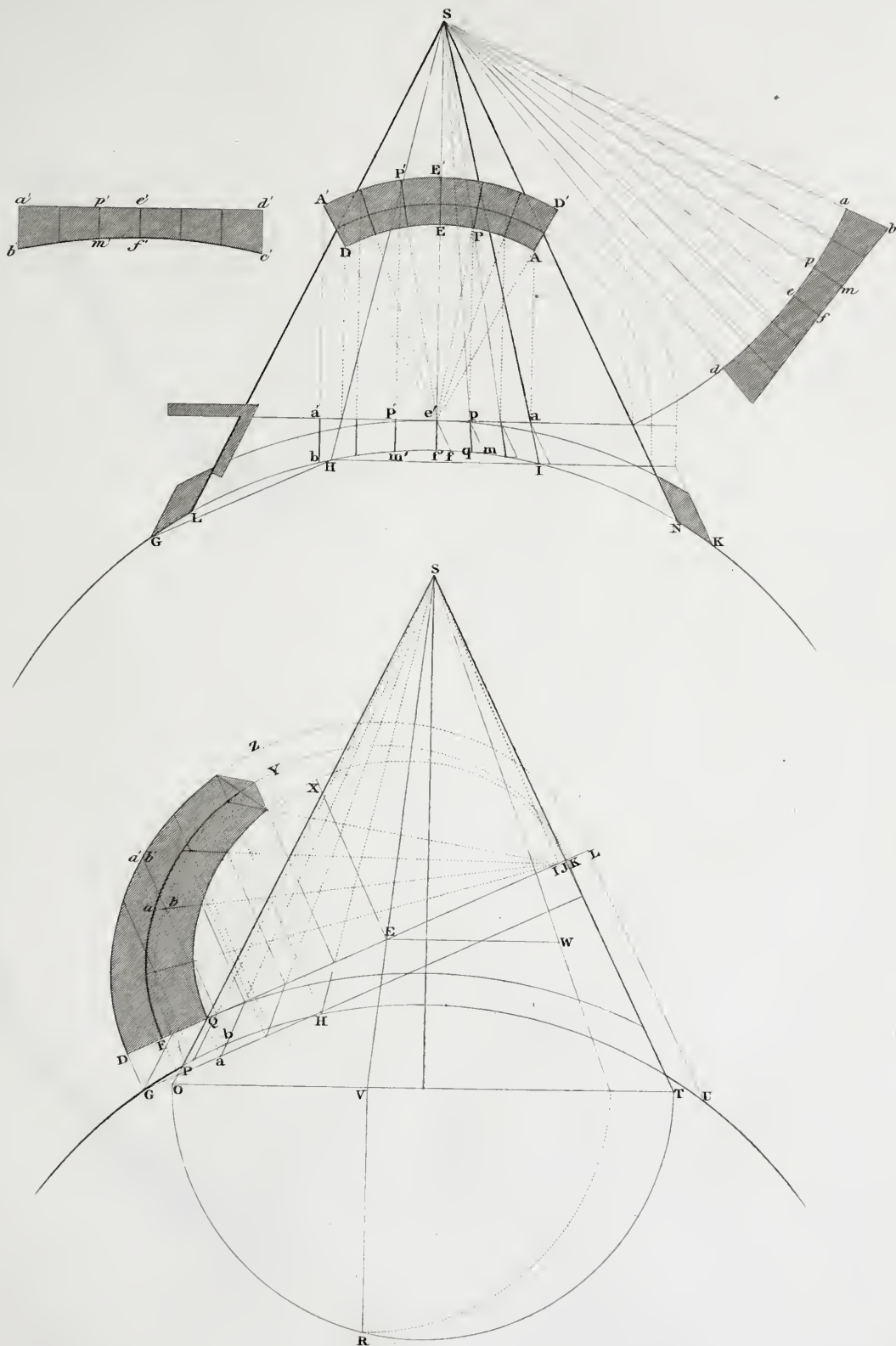
PLATE II.



GENERAL PRINCIPLES.

CONO-CYLINDRIC ARCH.

PLATE I.

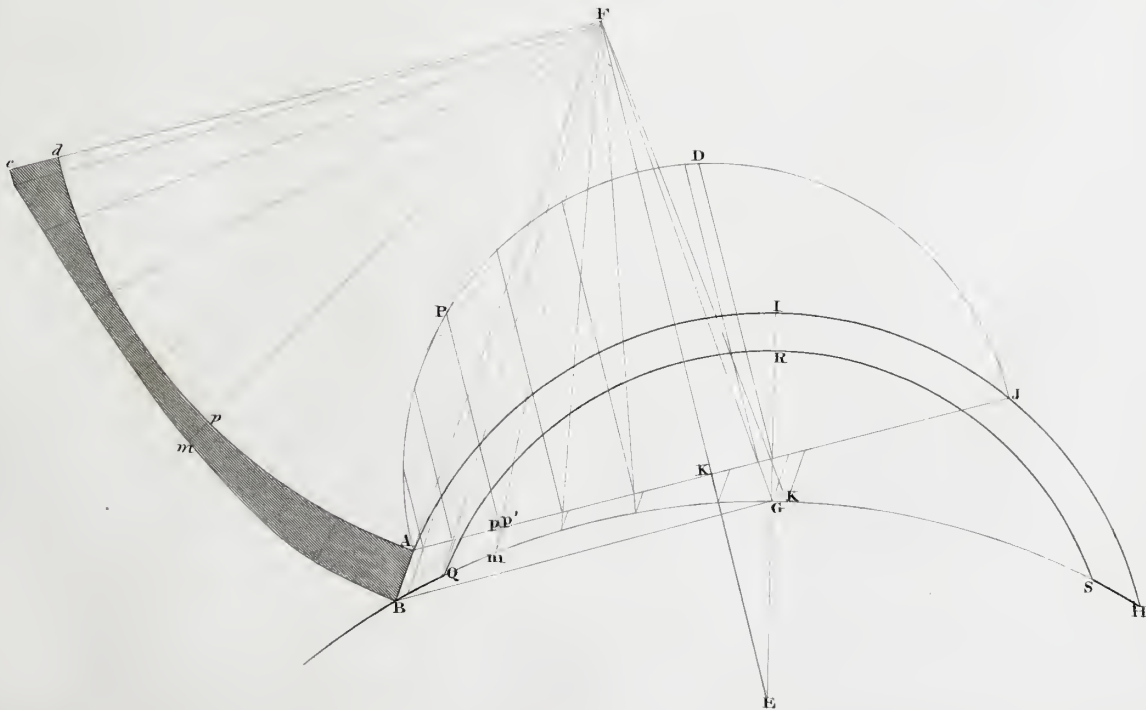
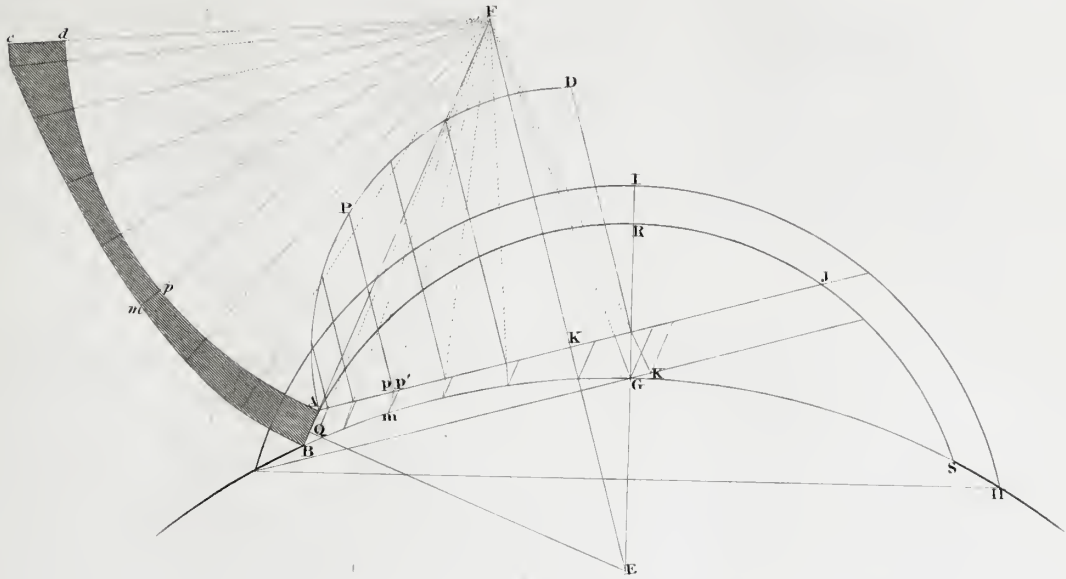




GENERAL PRINCIPLES.

SPERO-CYLINDRIC ARCH.

PLATE II.





10. When the ceiling is of the form of a pendentrix, it is called a *pendented* ceiling.

11. The parts which are terminated by the sides and the base of the solid, are called *pendentives*.

12. When the ceiling is in the form of a groinoid, it is called a *groined* ceiling, or *cross vault*.

13. When a ceiling which meets each of the walls in the concave surface of the segment of an archoid, and the inner edges of the surfaces of the archoids meet the edges of a plane surface forming the middle of the ceiling, so that the bases of the archoids may be horizontal, and each axial plane parallel to each wall, the ceiling is called a *coved* ceiling, and the archoidal surfaces *coves*.

PLATE I.

Fig. 1, called an archoid, considered as a geometrical solid, but when applied to vaulting it is called an *arch vault*.

Any perforation in a building covered with an arch vault, is called an *arch-way*; but when the ends are shut and form a room, that room is said to be *arch vaulted*.

Fig. 2, called domoids when considered as geometrical solids; but when applied to building they represent the external appearance of domes. Domes upon elliptic plans are only used internally with regard to their concave surface. Polygonal domes upon octagonal planes are sometimes used both externally and internally, but those upon any other plan very rarely.

Figs. 2, 3, 4, called domoids when considered as geometrical solids; but in their application to architecture are called domes, whether used externally with the convex surface, or internally with the concave surface.

All circular domes are comprehended under the general name of conoidal domes; of these the hemispheric dome exhibited in *fig. 2* is frequently used, as also the lesser segments of spheres and frustums of spheres.

Figs. 3 and 4 represent domes upon polygonal planes; of these octagonal domes are most frequently executed.

Fig. 3 represents a dome upon a square plan.

Fig. 4, a dome upon an octagonal plan. Domes upon elliptic plans, or ellipsoidal domes, are but rarely employed in the exterior of an edifice; but in the interior, where the figure can be entirely seen, they are both elegant and convenient for the vaultings of oblong rooms.

Figs. 5, 6, 7, 8, geometrically called pendentoids, from the hanging portions of the curved surface that are terminated by the sides, or by the sides and base of the solid. These solids have no technical name in architecture, at least as used by our architects. Alberti, one of the famous restorers of Roman architecture, calls such a solid as *fig. 5*,

6, or 8, when employed in the internal construction of an edifice, a *vellar cupola*, from its resembling the sail of a ship when fully extended by the wind. One of the pendentives of each of the solids represented by *figs. 5, 6, 8*, is marked *abcde*, and the pendentive in *fig. 7* is marked *abcdef*.

The vaultings of the nave and aisles of St. Paul's Cathedral Church, London, are formed by a combination of concave pendentoids.

Fig. 7 occurs mostly in archoid passages piercing a dome.

Figs. 9 and 10, geometrically called groinoids, are termed in building *groined vaults*. These figures are either applicable to rooms of four or eight sides, or to cross passages. *Fig. 9* is also frequently employed in the vaultings of cellars as well as in magnificent apartments. Here the lines in which the curved surfaces of the arched vaults meet each other, are called *groins*.

Those that reside in London may avail themselves of the opportunity of seeing groined vaults, such as *fig. 9*, executed in stone in the passage under the building called the Horse Guards, passing from Charing Cross to St. James's Park.

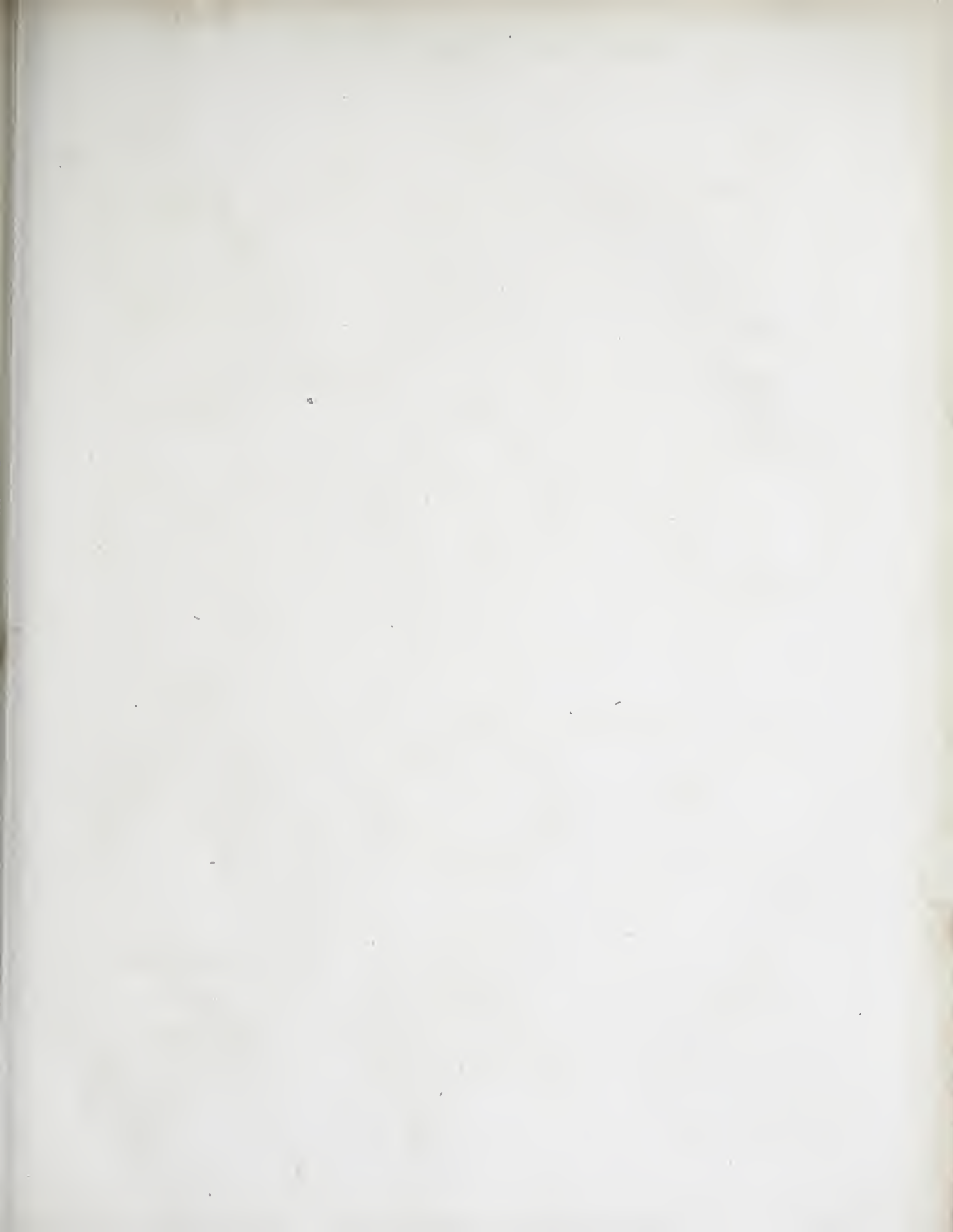
Fig. 11, geometrically called a lunoid, from the lesser arched windows, called lunettes, penetrating a large arched vault. This species of groined vaults is vulgarly called by our workmen Welsh groins. Lunette is the French name of an arched window penetrating an arch ceiling.

Such groins may be seen executed in stone under the gate, Somerset House, leading from the Strand to the court.

Fig. 12, geometrically called a lunetted domoid, from the lunetted windows piercing a dome.

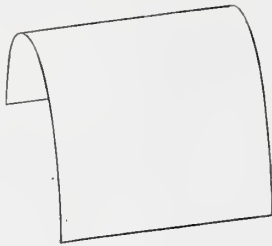
Polygonal domes and groined vaults are of the same species, but male and female; for if the surfaces of all the archoids, that have their axial planes of the same breadth that form an equal pitched groined vault, were to remain entire, the space that is completely enclosed by the curved surfaces of the archoids is a domoid or dome in architecture*; and if the external curved surfaces remain, and the curved surfaces forming the dome be cut away, the interior cavity will form a groined ceiling or vault: so that the solid domoid might be exactly comprehended between the groins or angles of a groined vault; and thus, if the cubature or solidity of the dome be known, that of the vacuity of the groin ceiling would be ascertained.

* In the mathematical names of the architectural solids, the termination *oid* from the Greek, signifying likeness, has been added to the architectural names which have been given to these solids, or to certain parts of them when the entire solid was without a name. The termination *atrix* might be applied to the internal cavities formed by these solids, as domatrix, groinatrix, &c.



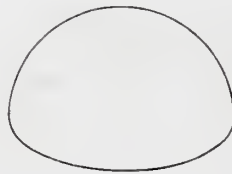
ARCHOID.

Fig. 1.



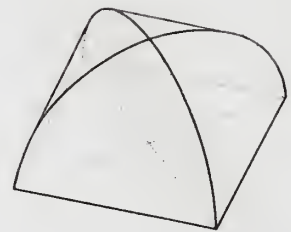
DOMOID.

Fig. 2.



DOMOID.

Fig. 3.



DOMOID.

Fig. 4.

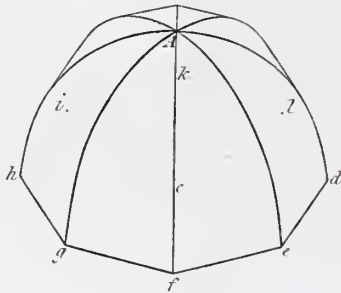
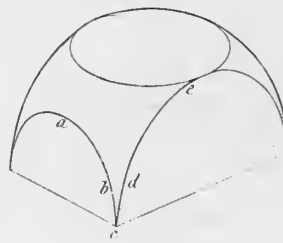


Fig. 5.



PENDENTOIDS.

Fig. 6.

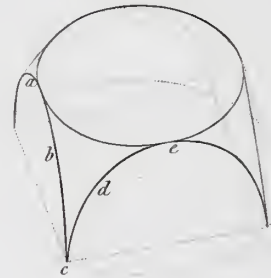


Fig. 7.

PENDENTOIDS.

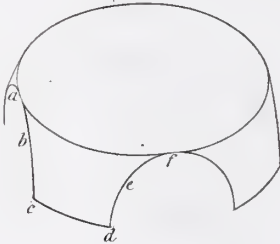
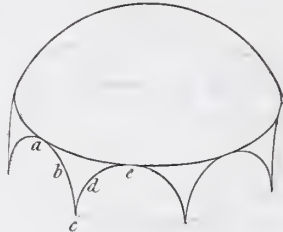
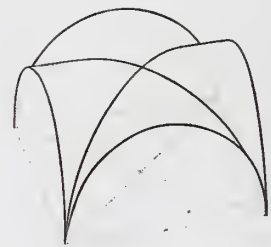


Fig. 8.



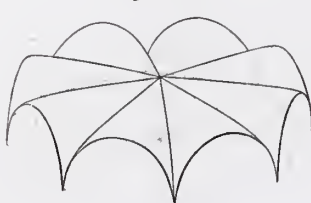
GROINOID.

Fig. 9.



GROINOID.

Fig. 10.



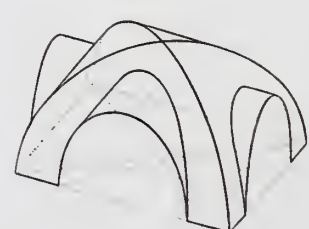
LUNOID.

Fig. 11.



LUNETTED DOMOID.

Fig. 12.



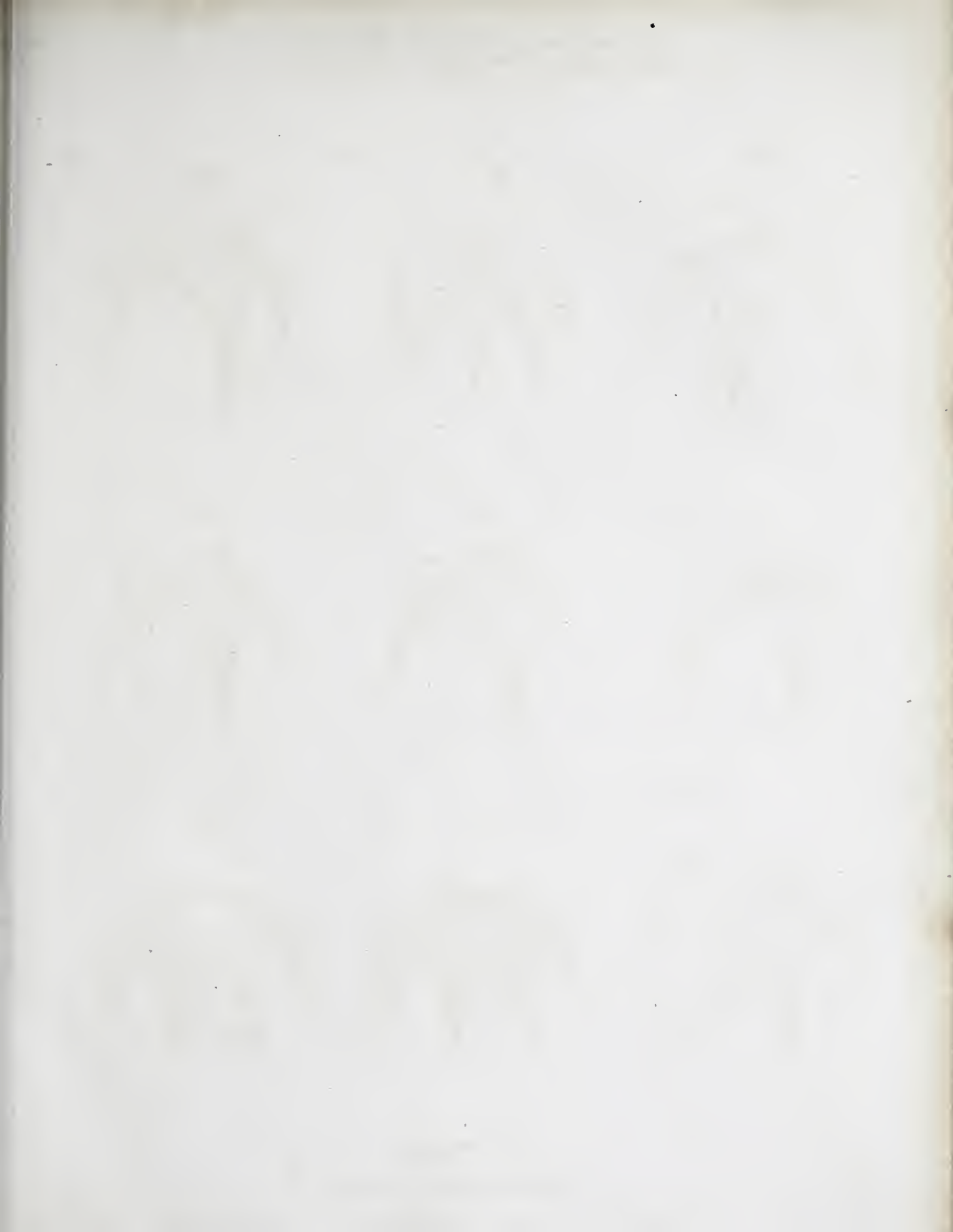


Fig. 1.



Fig. 2.

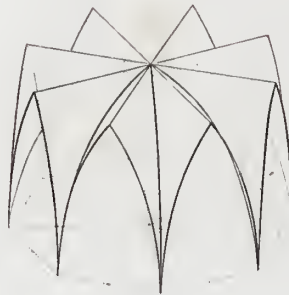


Fig. 3.

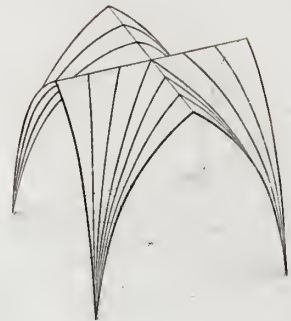


Fig. 4.



Fig. 5.

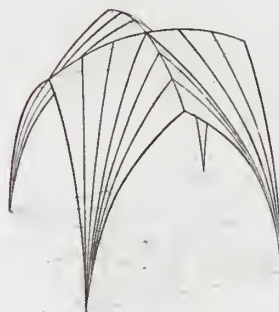


Fig. 6.

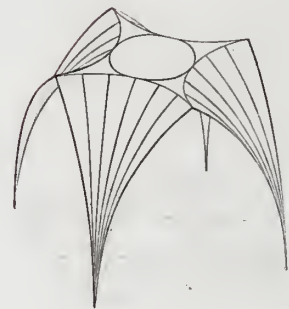


Fig. 7.



Fig. 8.

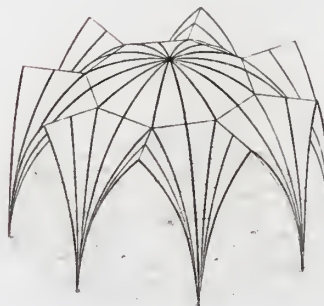
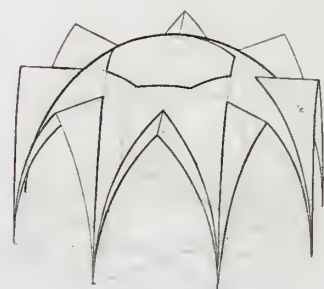


Fig. 9.



Engraved by Armstrong.

London, Published by John Day & Co. Oct. 20, 1822.

PLATE II.

Fig. 1 is a quadrilateral pointed groined vault, such as may be seen in most of our antient churches and cathedrals.

Fig. 2, an octagonal pointed groined vault, such as may be seen in chapter houses belonging to antient British cathedrals.

Fig. 3, a pointed groined rib vault upon a rectangular plan, as practised by our ancestors in the pointed or Gothic style of architecture.

Fig. 4, a pointed groined rib vault upon a rectangular plan, where the apex of each vault is a concave line in a vertical plane.

Fig. 5 is a pendent rib-ridged vault upon a square plan. The construction of the pendants is as follows:—

Suppose the lower point of a simple curve to coincide with the upper extremity of the axis of a vertical pillar, which axis produced is a tangent to the curve in that point; and if the curve be revolved about the tangent as an axis, it will describe a solid, the horizontal of which are circles, having the sections of the axis for a centre.

Fig. 6, rib truncated vault upon a square plan, the upper part being cut off by a plane parallel to the base through the vertices of the pointed arcs: the sections form a quadrilateral with four curved sides, of which each is the quadrant of a circle, and concave towards the interior of the figure. This quadrilateral is generally filled up with curious devices of ornamented polyfoils.

Fig. 7, a pendent rib truncated vault upon an oblong plan. The vaults represented in these elementary figures may be seen by those who have the opportunity, in the Chapel of King Henry VII at Westminster, as also in King's College, Cambridge.

Fig. 8, an octagonal dome groined vault with pointed arches.

Fig. 9, a circular dome groined rib vault with pointed arches. Here the sides of the arches are in vertical planes, in which the ribs are also disposed.

PLANE TRIGONOMETRY.

PLANE TRIGONOMETRY is the Art of Computing the Sides and Angles of a Plane Triangle from certain given data.

The sides and angles of a plane triangle are called its *parts*; and as every triangle has three sides, and three angles, it may be said to consist of six parts.

In every triangle, when any three of the parts are given except the three angles, the other three can be found.

In every right angled plane triangle, two parts besides the right angle are given to find the other three.

In trigonometry, the measure of an angle is ascertained by the number of parts which the arc of a circle, described from the vertex between the two legs, contains of the whole circumference, which is always divided into the same number of parts.

The antient Mathematicians divided the circumference into 360 equal parts, and this practice is still followed by the moderns, except the French, who have divided the circumference into 400 equal parts.

Therefore, admitting the circumference of a circle to be divided into 360 equal parts, the quadrant will contain 90 of these parts.

Each of the 360 equal parts into which the circumference of every circle is supposed to be divided, is called a degree; and in order to obtain the mensuration of angles to the greatest nicety, every degree is subdivided into sixty equal parts called minutes, each minute again into sixty equal parts called seconds, and so on till the divisions become imperceptibly small.

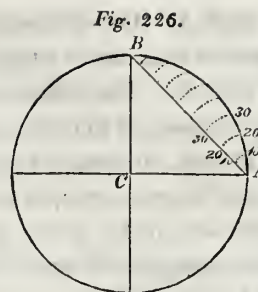
NOTATION.

Degrees are indicated by placing a small cypher on the right a little above the unit figure.

Seconds are denoted by a small accent or dash in the same situation, and so on for the other smaller divisions.

Thus, $35^{\circ} 31' 23''$ is read 35 degrees 31 minutes 23 seconds.

For the convenience of measuring angles on a small scale, describe the quadrant ACB with any convenient radius, and divide the arc AB into 9 equal parts. Draw the chord AB , and from B as a centre, with the distances $A 10$, $A 20$, $A 30$, &c. describe arcs, cutting the chord AB in the points 10, 20, 30, &c. and the distances $A 10$, $A 20$, $A 30$, &c. measured on the chord AB , will be the chord of 10° , 20° , 30° , &c. Such a scale will be found in most cases of mathematical instruments, as well as scales of equal parts.



PROBLEM I.

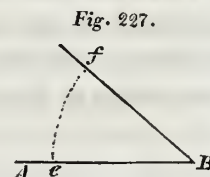
At a given point in a straight line to make an angle that shall contain any proposed number of degrees.

From the given point, with a radius equal to the chord of 60° , describe an arc meeting the straight line; and from the point of intersection with the chord of the proposed number of degrees, as a radius, describe another arc to meet the former; join the point of intersection of the two arcs with the given point, and the angle thus formed will contain the number of degrees required.

EXAMPLES.

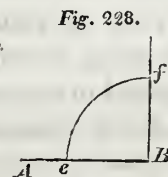
Ex. 1. At the point B in the straight line BA , describe an angle that shall contain 40° .

From the point B with the chord of 60° describe the arc ef , meeting AB in e ; from e , with a radius equal to the chord of 40° , describe another arc meeting the former in f ; join Bf ; then ABf is the angle required.



Ex. 2. At the point B in the straight line BA , to make an angle which with BA shall contain 90° .

From B , with a radius equal to the chord of 60° , describe the arc ef , meeting AB in e , and from e , with a radius equal to the chord of 90° , describe another arc, meeting the former in f . Join Bf , and ABf will contain 90° .



Note. As the three angles of every plane triangle taken together are equal to $90^\circ \times 2$, or 180° , which is the number of degrees in the semi-circle, we infer that when one angle contains 90° , the other two contain 90° between them, and such is called a *right angled plane triangle*.

Def. 1. The difference of an arc or angle from 90° , or a quadrant, is called the *complement* of that arc or angle.

Def. 2. The difference of an arc or angle from 180° , or a semicircle, is called the *supplement* of that arc or angle.

Hence, to make at a given point in a straight line an angle which shall contain a number of degrees greater than 90 , we have only to subtract the given number of degrees from 180 , and at the given point make with the remainder an angle in the opposite direction. An example will make it plain.

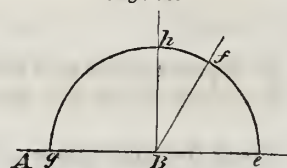
Ex. 3. At the point B in the straight line BA , make an angle which shall contain 130 degrees.

Here $180 - 130 = 50$. Produce AB to e , and from B , with a radius equal to the chord of 60° , describe the arc ef , meeting AB produced in e . From e , with a radius equal to the chord of 50° , describe another arc, meeting the former in f . Join Bf , and ABf will contain 130 degrees.

Or thus,—

From B , with the chord of 60° , describe the arc ghf , meeting BA in g . From g , with the chord of 90° , set off gh , and from h , with the chord of what the given angle exceeds 90° , set off hf . Join Bf , and ABf will contain the proposed number of degrees.

Fig. 229.



PROBLEM II.

An angle being given, to find the number of degrees which that angle contains.

From the point of concourse, with a radius equal to the chord of 60° , describe an arc meeting both lines which contain the angle; then apply the chord of this arc to a scale of chords, observing to place one foot of the compass in zero, and the other will point out the number of degrees that angle contains.

EXAMPLES.

Ex. 1. Suppose the angle ABf were given, and it were required to find how many degrees it contains.

From the point B , with a radius equal to the chord of 60° , describe the arc ef , meeting AB and Bf in the points e and f : the chord ef applied to the scale will show the angle to contain 40 degrees.

Ex. 2. Suppose the angle ABf to be given, and it is required to find how many degrees it contains.

From B , with a radius equal to the chord of 60° , describe the arc ef , meeting the straight lines AB and Bf in e and f : apply the chord ef to a scale as before directed, and the angle will be found to contain 90 degrees.

Fig. 230.

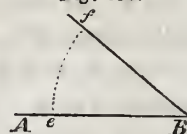
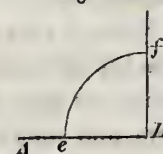


Fig. 231.



Ex. 3. Suppose the angle ABf (*fig. 229*, p. 164) were given, and it were required to ascertain how many degrees it contains. Produce AB to e , and from B , with the chord of 60° for a radius, describe the arc ef , meeting AB in e , and Bf in f : apply the chord ef to the scale as before, and the angle eBf will be found to contain 50° , which must be taken from 180° , and the remainder, 130 , is the number of degrees in the angle ABf .

Or thus,—

From B , with the chord of 60° , describe the arc ghf , meeting AB and Bf in the points g and f : apply the chord of 90° to gh , and the chord of hf to the scale, which will give 40° for the angle hBf , and this added to 90° , gives 130° for the angle ABf as before.

Having given these examples on the construction and measuring of angles, we will now proceed to the construction and computation of the different cases that may occur in the practice of Trigonometry; and as the terms *sine*, *cosine*, *tangent*, *secant*, &c. are defined in the Appendix, we need not detain the student to discuss them in this place.

RIGHT ANGLED TRIANGLES.

DEFINITIONS.

1. IN a right angled plane triangle, the sides containing the right angle are called the *legs*.

2. The side opposite to the right angle is called the *hypotenuse*.

In a right angled triangle only five of the parts are varied, in consequence of the right angle being a constant quantity. In order, then, to ascertain the number of ways the data may be varied, taken two and two at a time, let A, B be the two acute angles, a, b the legs opposite them respectively, and c the hypotenuse. Now the number of ways which two things can be selected out of five is known, by the theory of combinations, to be 10. Let the above representatives of the angles and sides be combined accordingly, and they will stand as below:—

$$\left\{ \begin{array}{l} Aa, Ba, BC, ac, ab \\ Bb, Ab, Ac, bc, BA \end{array} \right\}$$

where we may observe that the number of ways in which a leg and an opposite angle can be given is two, a leg and adjacent angle two, the hypotenuse and adjacent angle two, the hypotenuse and a side two, the two sides one, and the two angles one.

Let the case in which the two angles are given be set aside, as these data are not sufficient to limit the Problem; and of the other combinations, we will have the five following cases; viz.:—

a leg and an opposite angle ;
 a leg and an adjacent angle ;
 the hypotenuse and an adjacent angle ;
 the hypotenuse and a leg ;
 the two legs.

But because, when an acute angle is given, the other is found by subtracting the one given from 90° , the two cases in which a leg and an angle are given may be reduced to one, and consequently the five cases given above will be reduced to the following four ; viz. :—

a leg and an angle ;
 the hypotenuse and an angle ;
 the hypotenuse and a leg ;
 the two legs.

Hence, the construction will admit of four different problems, previously to which we give the two following Theorems for the calculation.

THEOREM 1. The sides of a plane triangle are to one another as the sines of their opposite angles.

THEOREM 2. As the sum of any two sides of a plane triangle is to the difference of these sides, so is the tangent of half the sum of their opposite angles to the tangent of half the difference of these angles ; and this half difference being added to and subtracted from the half sum, will give the greater and lesser angle respectively.

PROBLEM III.

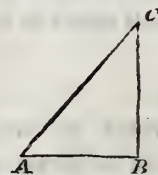
Given one of the legs and an opposite angle, to construct the triangle and find the remaining parts.

Ex. Given the leg BC and its opposite angle BAC . Required AB and AC .

Subtract the given angle from 90° , and the remainder will be an angle adjacent to the given side.

Construction. Draw the line AB of any convenient length, and at the point B in AB draw BC perpendicularly ; and from a scale of equal parts lay off BC equal to the given length. At the point C , make with CB an angle BCA equal to the complement of the given angle. Join CA , and measure AB and AC on the same scale whence BC was taken, and, as the scale indicates, their lengths will be,

Fig. 232.



$$\text{Calculation. } \sin BAC : \sin ACB :: BC : AB = \frac{BC \sin ACB}{\sin BAC}$$

$$\sin BAC : \sin ABC(90^\circ) :: BC : AC = \frac{BC \sin ABC}{\sin BAC}$$

PROBLEM IV.

The hypotenuse and the adjacent angle being given, to construct the triangle and find the remaining parts.

Ex. Given the hypotenuse AC , and the angle BAC , to find the sides AB and BC .

Construction. Draw AC (fig. 232, p. 166) of the given length, taken from a scale of equal parts, and at the point A in AC make the angle BAC equal to the given angle; and from the point C draw CB perpendicular to AB ; then AB and BC , measured from the same scale with AC , will have their lengths determined.

Calculation. Find the angle ACB as before;

$$\text{then, } \sin ABC (90^\circ) : \sin BAC :: AC : BC = \frac{AC \sin BAC}{\sin 90^\circ}$$

$$\sin ABC (90^\circ) : \sin ACB :: AC : AB = \frac{AC \sin ACB}{\sin 90^\circ}$$

PROBLEM V.

The hypotenuse and one of the legs being given, to construct the triangle and find the other parts.

Ex. Given the hypotenuse AC and the base AB , to find the angles and the other side.

Construction. Draw AB (fig. 232, p. 166) of the given length, measured from a scale of equal parts, and at the point B in AB draw BC perpendicular to AB ; and from the point A , with a radius equal to the given hypotenuse, describe an arc cutting BC in C . Join CA ; then the side BC , measured from the same scale as AB and AC , will have its length determined.

Calculation. $AC : AB :: \sin ABC (90^\circ) : \sin ACB = \frac{AB \sin 90^\circ}{AC}$; $\therefore BAC = 90^\circ$
— ACB .

To find BC :—

$$\sin ABC (90^\circ) : \sin BAC :: AC : BC = \frac{AC \sin BAC}{\sin 90^\circ}$$

Note. If we suppose the sine of $90^\circ =$ the radius of a circle to be 1 (the radius in the tables), each of the preceding formulæ, where the sine of 90° enters, will be greatly simplified.

PROBLEM VI.

Given the two legs, to construct the triangle and find the remaining parts.

Ex. Given the base AB , and the perpendicular BC , of a right angled plane triangle. Required the angles and the hypotenuse.

Construction. Draw AB (*fig.* 232, p. 166) of the given length, and at the point B draw BC perpendicular to AB , making BC also of the given length. Join AC ; then AC , applied to the same scale whence AB and BC were taken, will have its length determined, and the angles can be measured as before directed.

The solution of this example will be obtained by Theorem 2. Thus,

$$AB + BC : AB \sin BC :: \tan \frac{1}{2}(180^\circ - ABC) : \tan \frac{1}{2}(BAC \sin BCA) = \frac{(AB \sin BC) \times \tan \frac{1}{2}(180^\circ - ABC)}{AB + BC}$$

= half the difference of the angles at the base, and the half sum is known; therefore the angles themselves are known, and the sides can be found as before.

From what has been done above, it is evident that the two Theorems given are sufficient to solve every case of right angled triangles, without embarrassing the judgment, and loading the memory with a multiplicity of useless varieties.

It may, however, be said, that another case may occur which has not been alluded to, viz. when the three sides are given to find the angles; but when it is considered that the right angle in this case also is known, the other angles may be found by an inversion of the first Theorem.

OBLIQUE ANGLED TRIGONOMETRY.

IN order to ascertain the number of cases that may occur in the construction of oblique angled triangles, let A , B , and C denote the three angles, and a , b , c their respective opposite sides: now, as we must always have three of these six things given to determine the rest, we have to enquire how many ways three things can be selected out of six. The theory of combinations gives twenty, which may be arranged as below:—

No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.
ABC	ABa	BCa	Aab	Cab	abc
	ABb	ACb	Bab	Bac	
	BCb	ABc	Aac	Abc	
	ACc		Cac		
	BCc		Bbc		
	ACa		Cbc		

But although, in the same triangle, the number of ways which three parts can be given in order to find the rest is 20, yet, as six of these selections, No. 2, have this in common, viz. two angles and an opposite side; and three, No. 3, have two angles and a contained side; the nine combinations of data may be reduced to one case, because, when two angles of a plane triangle are given, the third is also given, being the supplement of the other

two; therefore, setting aside No. 1 where the three angles are given (which data only determine the ratio of the sides), we may draw this conclusion,—that the construction will fall under one of the following cases, in which may be given,

two angles and a side	-	-	-	-	-	No. 2 and 3;
two sides and an opposite angle	-	-	-	-	-	No. 4;
two sides and the contained angle	-	-	-	-	-	No. 5;
three sides	-	-	-	-	-	No. 6.

Though the construction will admit these four cases, yet, with regard to the arithmetical solution, there are only three for Nos. 2 and 3, where two angles and a side are given; and No. 4, where two sides and an opposite angle are given, may be reduced to one case, from the property given in Theorem 1, Right Angled Plane Trigonometry.

PROBLEM VII.

Given the three sides of an oblique angled plane triangle, to construct the triangle.

An example is not necessary, since it has already been done, Prob. xxxii, p. 37, Plane Geometry.

As we thought it unnecessary to give a Theorem for this case in Right Angled Trigonometry, we shall give it here, being indispensable in Oblique Triangles.

THEOREM.

As the base or longest side is to the sum of the other two sides, so is the difference of these sides to the difference of the segments of the base made by a perpendicular from the vertical angle.

We will leave the application of this Theorem to the student, and proceed to the construction of the remaining cases, observing at the same time, that, as the Theorems given in Right Angled Trigonometry apply equally to the same cases in Oblique Triangles, we will take no further notice of the calculations.

PROBLEM VIII.

Given two angles and a side, to describe the triangle and find the remaining parts.

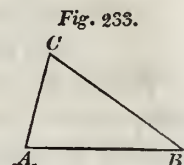
Note.—If the given parts are not contiguous, take the supplement of the two given angles, which will be adjacent to the side given; consequently the side will be situated between two angles.

Draw a straight line equal to the given side, and at each of its extremities make angles as before directed, the one of which is a given angle, and the other the supplement,

found as above; the point of concurrence of the two lines forming the angles with the given line is the vertex of the triangle required, the distance of which from each extremity of the given line may be found as before directed.

Ex. In the oblique angled plane triangle ABC are given the side AB , 34 feet, the angles ABC , BCA , 35° and 62° respectively, to find BC and AC .

Construction. Draw the straight line AB , and from a scale of equal parts take off 34, which apply from A to B . At the point B in the straight line AB , make the angle ABC equal to 35° ; and at the point A make an angle equal to 83° , the supplement of the sum of both given angles. Apply BC and AC to the same scale whence AB was taken, and they will measure 38 and 22 feet respectively.



PROBLEM IX.

Given two sides and the contained angle, to construct the triangle and measure the other parts.

Draw a straight line of any convenient length, and at any point in the same make an angle equal to the given angle; and from a scale of equal parts, lay off from the vertex of the angle, on the legs containing it, the respective lengths of these legs; join their remote extremities, and the line thus joining them will be the third side of the triangle, which must be measured on the same scale with the given sides and the angles.

Ex. Given the two sides AB , BC , 34 and 38 feet respectively, and the angle ABC 35° , to find the other angles and the third side.

Construction. Make $AB = 34$ feet (*fig. 233*, p. 170) as before, and at the point B in AB make the angle $ABC = 35^\circ$; and from the same scale whence AB was taken, make $BC = 38$ feet. Join AC , which being applied to the same scale with AB , BC , will be found to measure 22 feet. The angles BAC , ACB , being measured as before, are 83° and 62° respectively.

PROBLEM X.

Given two sides and an angle opposite to one of them, to construct the triangle, and measure the remaining parts.

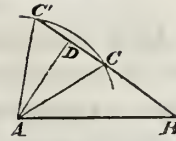
Draw a straight line equal to one of the given sides, at one extremity of which make an angle equal to the given angle; at the other extremity, with a distance equal to the other given side, describe an arc, which, if the data be possible, will either touch the unknown side in one point, or cut it in two; hence this case will, in certain instances, admit of two solutions.

Note.—The unknown parts may be measured as before.

Ex. Given the two sides AB , AC , equal to 34 and 22 feet respectively, and the angle ABC opposite the side AC equal to 35° , to find the third side and the other angles.

Construction. Draw AB , 34 feet, as before directed, and at the point B in AB make an angle equal to 35° ; then from the other extremity A of AB , with a distance equal to 22 feet, taken from the same scale with AB , describe an arc which will either touch the unknown side in D , or cut it in the points C, C' . Join AD if it touch in D , or $CA, C'A$ if it cut in the points C and C' . The line BD, BC , or BC' , will in either case be the third side, that is, $BC = 20$, $BD = 28.4$, and $BC' = 37$ respectively; and the angles being measured as before directed, will, according to the respective sides, be $BCA 113^\circ$, $BAC 32^\circ$, $BDA 90^\circ$, $BA 55^\circ$, $BC'A 66^\circ$, and $BAC' 79^\circ$.

Fig. 234.



OF THE RIGHT ANGLED TREHEDRAL.

DEFINITIONS.

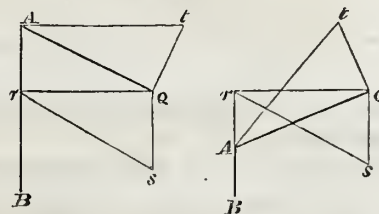
1. A TREHEDRAL is a solid formed by three planes meeting one another in a point.
2. A right angled trehedral is that in which two of the planes are perpendicular to each other.
3. The angle at the vertex of the figure in each of the perpendicular planes is called a *leg*.
4. The angle at the vertex of the figure in the plane opposite the right angle, is called the *hypothénuse*.
5. The angles formed by the plane opposite the right angle, meeting each of the perpendicular planes, are called the angles of the solid.

PROBLEM XI*.

Given one leg and the adjacent angle of the solid, to find the other leg.

Construction. Let BAQ be the given leg; and from any point Q in AQ draw Qr , meeting AB or AB produced perpendicularly in the point r ; make the angle Qrs equal to the adjacent angle of the solid; draw Qs perpendicular to Qr , meeting rs in the point s ; draw also Qt perpendicular to AQ , and make Qt equal to Qs . Join At , and QAt will be the required leg.

Fig. 235.

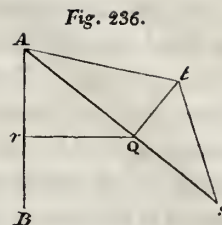


* This is the same as if one leg and the adjacent angle of a right angled spherical triangle were given, to find the other leg.

Formula $\tan \text{ req. leg} = \tan \text{ adj. angle} \times \sin \text{ given leg}$. This is the formula of Napier; but we shall

Or thus,—

From any point Q in AQ draw Qr , meeting AB or AB produced, perpendicularly in the point r . Produce AQ to s , and make Qs equal to Qr ; draw Qt perpendicular to AQ ; make the angle Qst equal to the adjacent angle of the solid. Join At , and QAt will be the required leg.

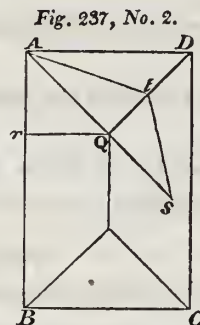
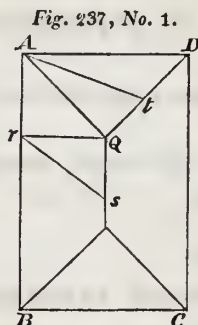


EXAMPLES.

Ex. 1. Given the angle formed by the wall plate of a roof, and the seat of a hip rafter, together with the angle which the roof makes with the wall plate, to find the angle which the hip rafter makes with its seat.

This is the same as if the angle BAQ of one leg and the adjacent angle of the solid were given, to find the other leg.

The construction of *fig. 237, No. 1*, is according to Method 1, and the construction of *fig. 237, No. 2*, according to Method 2.



show how the same may be derived from the figure itself, without referring to the principles of spherical trigonometry.

For this purpose, let AQ be considered as given = the radius of a certain circle.

Then, per Trig., $\sin QrA : \sin QAr :: AQ : Qr = \frac{AQ \sin QAr}{\sin QrA}$; $\text{rad}(1) : \tan Qrs :: \frac{AQ \sin QAr}{\sin QrA} : Qs = \frac{AQ \tan Qrs \sin QAr}{\sin QrA}$. But Qt is by construction = Qs ; $\therefore Qt = \frac{AQ \tan Qrs \sin QAr}{\sin QrA}$; $AQ : \frac{AQ \tan Qrs \sin QAr}{\sin QrA} :: \text{rad}(1) : \tan QAt = \tan Qrs \times \sin QAr$; because AQ in the denominator destroys AQ in the numerator, and $\sin QrA = 1$, the angle QrA being a right angle.

Ex. 2. Given the angle which the style of a horizontal dial makes with the substyle, and the time of the day, to find the angle in the place of the dial which the substyle makes with the style.

Suppose it were required to find the shadow of the style of a horizontal dial four hours after noon, the latitude of the place being given.

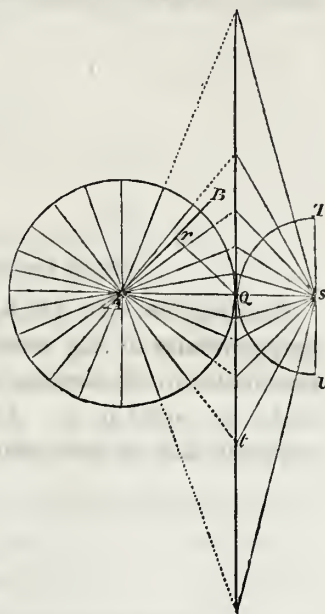
Construction. Let A be the centre of the dial, AQ the substyle or 12 o'clock hour line, and QAB the angle which the style makes with the substyle.

From any point Q in AQ draw Qr , meeting AB , or AB produced, perpendicularly in the point r . Produce AQ to s , and make Qs equal to Qr ; draw Qt perpendicular to AQ . Make the angle Qst equal to 60° (allowing 15° to an hour), and join At ; then QAt is the angle sought.

This is the same in every respect as the second method. In the diagram the whole of the hour lines are exhibited, being all drawn in the same manner as the line At ; but this is more easily effected by dividing the quadrants QT and QU each into six equal parts.

The very same lines apply to the joints of an oblique arch in Masonry, where the lines shown in TQU are for those of the right arch.

Fig. 238.

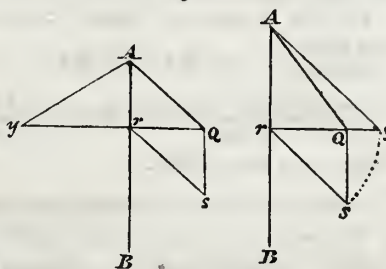


PROBLEM XII.

Given one leg and the adjacent angle of the solid, to find the hypotenuse.*

In AQ take any point Q ; from Q draw Qy , cutting AB or AB produced perpendicularly in r . Make the angle Qrs equal to the adjacent angle of the solid; draw Qs perpendicular to Qr ; make ry equal to rs , and join Ay ; then BAy is the hypotenuse required.

Fig. 239.



* This is the same thing as having given one leg of a right angled spherical triangle and the adjacent angle, to find the hypotenuse.

Formula $\cot \text{ hyp} = \cos \text{ given } \angle \times \cot \text{ given leg}$. The formula here given is that derived from Napier's

The principles of this problem may be successfully applied to the development of roofs, the cutting of timbers, &c.

As an Example,—Given the angle of two faces of a piece of timber, and two lines drawn obliquely from the same point of the ridge line (or arris as called by workmen), to find the angle contained by these lines.

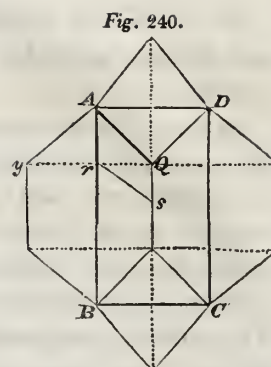


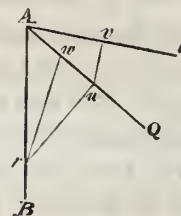
Fig. 240.

PROBLEM XIII.

Given the two legs, to find the adjacent angles of the solid.*

Construction. In *AB*, fig. 241, take any point *r*, and draw *ru* perpendicular to *AQ*, meeting *AQ* in *u*. From *u* draw *uv* perpendicular to *At*, meeting *At* in *v*. From *u* towards *A* on *AQ*, make *uw* equal to *uv*. Join *rw*, and *uwr* is one of the angles required, and the other may be found in the same manner.

Fig. 241.



circular parts; but we shall show how the same may be obtained immediately from the figure, without employing the principles of spherical trigonometry.

Let *AQ* be given as before = the radius of a certain circle; then, $\sin QrA : \sin QAr :: AQ : Qr = \frac{AQ \sin QAr}{\sin QrA}$; $\sin QrA : \cos QAr :: AQ : Ar = \frac{AQ \cos QAr}{\sin QrA}$; $\cos Qrs : \text{rad}(1) :: \frac{AQ \sin QAr}{\sin QrA} : rs = \frac{AQ \sin QAr}{\sin QrA \cos Qrs}$; but $ry = rs$, $\therefore \frac{AQ \cos QAr}{\sin QrA} : \frac{AQ \sin QAr}{\sin QrA \cos Qrs} :: \text{rad}(1) : \tan BAy = \frac{\tan QAr}{\cos Qrs}$, or $\cot BAy = \cos Qrs \cot QAr$.

* This is the same as if the legs of a right angled spherical triangle were given to find the acute angles.

Formula $\cot \text{ req angle} = \sin \text{ adj side} \times \cot \text{ opp side}$, solution from figure. Let *rA* equal the assumed radius; then will $rA \sin rAu = ru$, and $rA \cos rAu = Au$

$$rA \sin uAv \cos rAu = uv = uw$$

$$\tan ruu = \frac{rA \sin rAu}{rA \sin uAv \cos rAu} = \frac{\tan rAu}{\sin uAv}$$

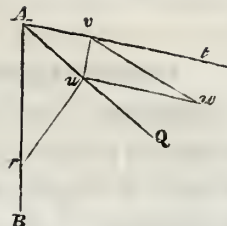
$$\text{or } \cot ruu = \sin uAv \cot rAu.$$

Or thus,—

In AB take any point r ; draw ru perpendicular to AQ , meeting AQ in u ; draw also uv perpendicular to At , meeting At in v , and uw perpendicular to uv ; make uw equal to ur , and join vw ; then will uvw be one of the angles required, and in the same manner the other may be found.

Note.—By either of these methods we may find the backings of hip rafters.

Fig. 242.

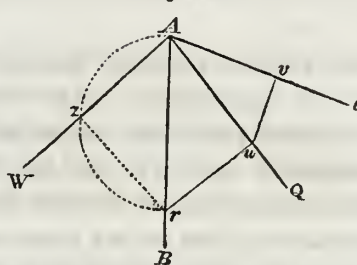


PROBLEM XIV.

Given the legs, to find the hypotenuse.*

Construction. In AB , with any convenient radius, describe a semicircle Azr , meeting AB in r . Draw rw perpendicular to AQ , meeting AQ in u ; draw also uv perpendicular to At , meeting At in v . From A with the radius Av describe an arc, meeting the semicircle in z , and through z draw AW ; then will BAW be the angle required.

Fig. 243.

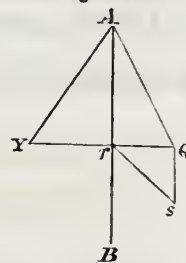


PROBLEM XV.

Given the hypotenuse and either adjacent angle of the solid, to find the adjacent leg†.

Construction. Let BAY be the hypotenuse, and through any point r in AB draw YQ perpendicular to AB . Make the angle Qrs equal to the adjacent angle of the solid, and rs equal to rY . Draw sQ perpendicular to YQ , and join AQ ; then BAQ is the leg required.

Fig. 244.



* This is the same as if the two legs of a right angled spherical triangle were given, to find the hypotenuse.

Formula, $\cos \text{hyp} = \text{rect cosines of legs}.$

Solution from the figure.

Let rA be the assumed radius; then $Au = rA \cos rAu$, and $Av = rA \cos rAu \cos uAv$. But $Az = Av$; $\therefore rA : rA \cos rAu \cos uAv :: 1 : \cos rAz$; that is, $\cos rAz = \cos rAu \times \cos uAv$.

† This is the same as if the hypotenuse and an acute angle of a right angled spherical triangle were given, to find the side adjacent to that angle.

Formula, $\tan \text{req side} = \tan \text{hyp} \times \cos \text{given angle}.$

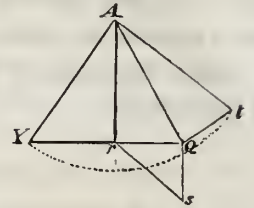
Solution from the figure. Let AY be the assumed radius; then $Yr = rs = AY \sin YAr$, and $Ar = AY \cos YAr$, $rQ = AY \sin YAr \cos Qrs$; $\therefore \tan QAr = \tan YAr \cos QAr$.

PROBLEM XVI.

Given the hypotenuse and either angle of the solid, to find the opposite leg.*

Construction. Find the leg rAQ by the last problem; then through any point r in Ar draw YQ perpendicular to Ar . Make the angle Qrs equal to the given angle of the solid. Draw Qs perpendicular to Qr , and Qt to AQ , making $Qt = Qs$. Join At , and QAt will be the leg required.

Fig. 245.



SCHOLIUM.

Let A and B represent the two acute angles of the solid, a and b their opposite legs, and c the hypotenuse. Now here are altogether five parts; and since the number of ways which two things can be selected out of five is ten, we can have the following combinations of data, $AB, Aa, Ba, Ab, Bb, ab, Ac, Bc, ac, bc$. It therefore appears, that there are ten cases of the right angled Trehedral; we shall, however, dismiss the subject by observing, that the six cases which we have given are sufficient to answer the various examples which appear in the body of the work.

* This is the same as if there were given the hypotenuse and an angle of a right angled spherical triangle, to find the side opposite to that angle.

Formula, $\sin \text{ req side} = \sin \text{ opp ang} \times \sin \text{ hyp}$.

Solution from the figure. Let YA be the assumed radius;

then $YA \sin YAr = Yr = rs$;

$YA \sin YAr \sin Qrs = Qs = Qt$;

but $YA = tA$, $\therefore \sin QAt = \sin YAr \sin Qrs$.

Masonry.

DEFINITIONS.

1. MASONRY is the art of cutting stones and building them into a mass.

2. A wall is a mass of stones generally joined with cement, so that a plumb line from any point of the surface may not fall without the solid.

Stones prepared from the quarry for the use of building, particularly in walling, are generally of a rectangular form.

3. A vault in Masonry is a mass of stones overtopping an area of a given boundary, and supported by one or more walls or pillars placed without the boundary of that area, the stones being so arranged that no one can descend without forcing those that are adjacent from their places.

4. The surface of the vault which is opposite to the area is called the *soffit* or *intrados*, or *vaulted surface*, and is generally concave towards the area, or composed of surfaces that are concave.

5. As it is the proper adjustment of the weight of any part of a vault to the surrounding mass which prevents that part from falling, it therefore becomes necessary in the act of construction to build it upon a mould until the whole is closed; the mould used for this purpose is called a *centre*.

6. The part of the top surface of a wall or pillar on which the first stones of a vault rest, is called the *spring of the vault*, or *bed of the vault*.

7. If the face of a wall or pillar and the soffit of the vault meet together in the bed of that vault, the line of concourse is called the *spring line* of the vault.

8. A complex vault formed by the intersection of several archoids, whether of the same or of different heights, is denominated a *groin vault*.

GENERAL OBSERVATION.

The intrados of every vault used in architecture is the concave surface of some geometrical solid, or the surface of a solid compounded of one or more geometrical solids.

When solids of revolution are employed in vaulting, the axis is generally either perpendicular or parallel to the horizon.

The surface of a cylinder or cylindroid can only be employed in vaulting when the axis is parallel or inclined to the horizon, for when it is perpendicular the concave surface of the cylinder or cylindroid is only the interior surface of a wall upon a circular or elliptic plan.

A conic surface may form the intrados of a vault, whether the axis be perpendicular or parallel to the horizon. Such surfaces are, however, seldom used, except it is in the heads of apertures of doors and windows with the axis horizontal, or in kilns where the axis is vertical.

When the surface of a cylindroid is employed in vaulting, one of the axes of the elliptic ends is always horizontal, and the other in a vertical plane. The position of the axes of a cylinder or cylindroid is most generally horizontal; the inclined position happens but very rarely.

If the surface of a vault be cylindric or cylindroidic, a portion of the surface of the cylinder or cylindroid not exceeding the half is generally employed: the edges which terminate the surface, and which are parallel to the axis, are called the *springing lines* of that surface.

When a spheric surface is employed in vaulting only, a segment not exceeding the half is used.

The surface of every conoid may be used in vaulting, and, when used, the position of the axis is perpendicular to the horizon.

When the surface of a vault is ellipsoidal, the shorter axis has generally a vertical position.

DEFINITION.

In stone-cutting, a narrow surface formed by a chisel or point on the surface of a stone so as to coincide with a straight edge, is called a *draught*.

PROBLEM I.

To form the face of a stone into a plane surface.

Run a draught first along and close to one edge of the stone, and then along and close to the adjacent edge, meeting the former draught. Run another draught along the diagonal so as to meet the other two, and form a triangle; then run a fourth draught along the other diagonal, so as to pass through the meeting of the first two, and through the first diagonal draught. Then reduce the protuberant parts between the draughts so that every part of the surface may coincide with the straight edge and be in a plane with the former draughts.

The reason of this is obvious, since the three sides of a triangle are always in one plane.

STONE CUTTING.

FORMATION OF PLANE, WINDING & SPHERICAL SURFACES.

PLATE I.

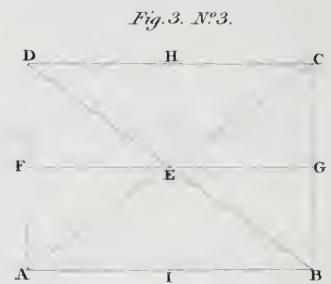
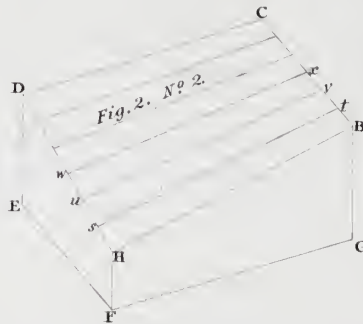
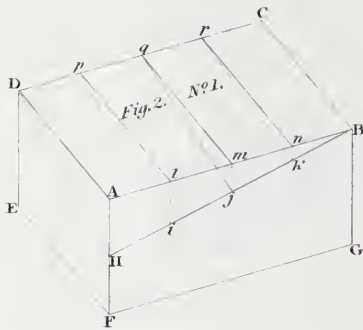
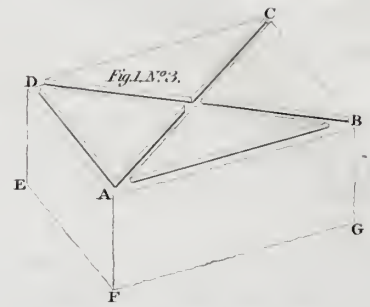
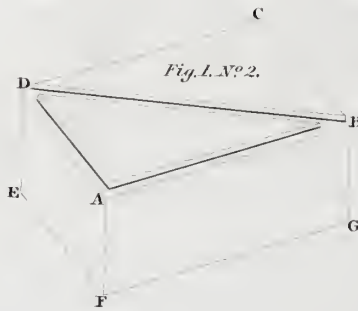
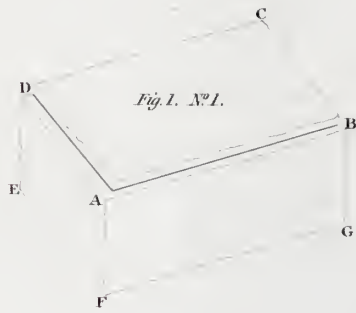


Fig. 3. N° 1.

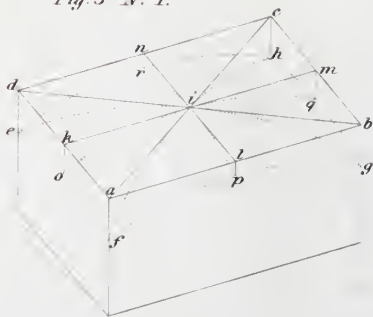


Fig. 3. N° 2.

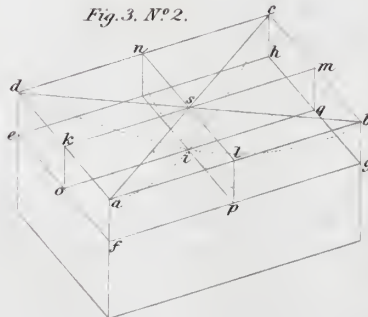


Fig. 3. N° 4.

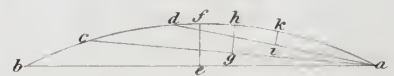


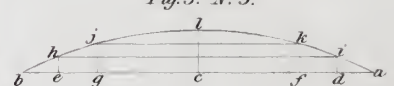
Fig. 3. N° 7.



Fig. 3. N° 6.



Fig. 3. N° 5.



In *Plate 1*, Stone-Cutting, subtitled Formation of Plane, Winding, and Spherical Surfaces,

Fig. 1 exhibits the method of forming the face of a stone into a plane surface. *No. 1* shows the first two draughts along the edges AD and AB; *No. 2* the first two draughts AD, AB, and the first diagonal draught BD, connecting the extremities of the two first draughts; and *No. 3* shows all the four draughts AD, AB, AC, and BD.

It would also be convenient to form two other draughts along the edges CD and CB, in order to reduce the surface to a plane in the easiest manner, and to prevent the edge of the stone from breaking within the surface.

PROBLEM II.

To form a winding surface, fig. 2.

Run four draughts along the edges all in one plane by the preceding Problem, and having brought the other sides to a square, set AH on the perpendicular edge equal to the quantity of winding, and draw HD and HB for the draught lines.

Draw the equidistant lines *li*, *mj*, *nk*, parallel to AH, and *lp*, *mq*, *nr* parallel to AD. Join *ip*, *jq*, *kr*; then HD, *ip*, *jq*, *kr* are the draught lines.

If the surface be now reduced so that a straight edge parallel to the plane AFGH may every where coincide with the surface, and at the same time with the cross draughts, it will be in the form required.

PROBLEM III.

To reduce the face of a stone to a spherical surface.

The first thing is to curve the edge of a rule to the radius of the sphere. Before we proceed further, we may show how this is done; supposing the stone not to exceed four feet in its longest dimension.

Find the versed sine of an arc whose chord is 4 feet, to a circle of which the radius, we shall suppose, is 30 feet; by Prob. ii, p. 61, Primary Problems, this will be found to be very nearly $\frac{1}{3}$ of an inch.

Suppose *fig. 3*, *No. 6* and *7*, to be the rule thus formed; *No. 5* to be a portion of the arc drawn by the edge of the rule; and *No. 3* to be the upper face of the stone squared.

Let the stone of which the surface is to be reduced to a sphere be first squared as in *No. 1*, and let the chord *ab*, *No. 5*, of the arc *alb* be equal in length to the diagonal AC or BD, *No. 3*. Bisect the chord *ab*, *No. 5*, by the perpendicular *cl*.

In *No. 1* let *abcd* be a face of the stone on the perpendicular edges. Make *af*, *bg*, *ch*, *de*, each equal to *cl*, the versed sine of *No. 5*. In *No. 1* bisect the sides *ab*, *bc*, *cd*, *da* in the points *l*, *m*, *n*, *k*. Join *ln*, *km* meeting in *i*, and draw *lp*, *mq*, *nr*, *ko* on the perpendicular faces parallel to the edges *af*, *bg*, *ch*, *de*.

In the chord *ab*, No. 5, make *cg* and *cf* each equal to *il*, or *in*. Draw *gi* and *fk* perpendicular to *ab*, meeting the arc in *j* and *k*.

In No. 1 make *lp* and *nr* each equal to the difference between *gi* or *fk*, and *cl*, No. 5.

Again, in the chord *ab*, No. 5, make *ce* and *cd* each equal to *ik* or *im*. Draw *eh* and *di* parallel to *cl*, meeting the arc in *h* and *i*. In No. 1 make *ho* and *mq* each equal to the difference between *he* or *di*, and *cl*, No. 5.

Then, sinking the draughts in No. 1 from the lines *ac*, *bd*, *km*, *ln*, by the rule No. 6, we shall be enabled to form the convex surface of the stone.

No. 4 is the same as No. 5, except that the chords *ac*, *ad*, are set from the end *a*.

From what has now been described in working the convex face of a stone, No. 1, the process of forming the convex face will be easily understood; we shall therefore omit the description of No. 2.

RAKING MOULDINGS.—PLATE I.

Fig. 1, Stone-Cutting, exhibits the elevation of a pediment with modillions. The construction of the modillion is shown in *fig. 3*.

Fig. 2 shows the method of tracing the section which is perpendicular to the raking or inclined direction of the moulding, the horizontal one being given.

Fig. 4 shows the same for a Grecian ovolo. Draw *AB* perpendicular to the horizontal moulding, and *AP* perpendicular to *AB*. Draw *PM* parallel to *AB* meeting the curve in *M*, *Mm* parallel to the raking line, and *aC* perpendicular to it. Make *ap* equal to *AP*, and draw *pm* parallel to *aC*; then will *m* be a point in the section of the moulding. In the same manner as we have found the point *m*, we may find as many more points as we please.

This same description applies to *figs. 2, 3*, and *4*.

Fig. 5, method of tracing the angle munion of a gothic window, one of the intermediate munions in the front being given.

Let *AB* be the horizontal line of front, and *ABC* the angle, and let *AD* perpendicular to *AB* be the central line of the section of the given munion, and *BE* bisect the angle *ABC*. In *BE* or *BE* produced take any point *a*, and draw *ap* perpendicular to *aE*.

In the curve No. 1 take any point *M*, and draw *Mm* parallel and *MP* perpendicular to *AB*, meeting *AB* in *P*. Make *ap* equal to *AP*, and draw *pm* parallel to *BE*, and *m* will be a point in the section of the angle munion. In the same manner as the point *m* has been found, we may find as many more points in the section No. 2 as we please.

STONE CUTTING.

RAKING MOULDINGS.

PLATE I.

Fig. 1.

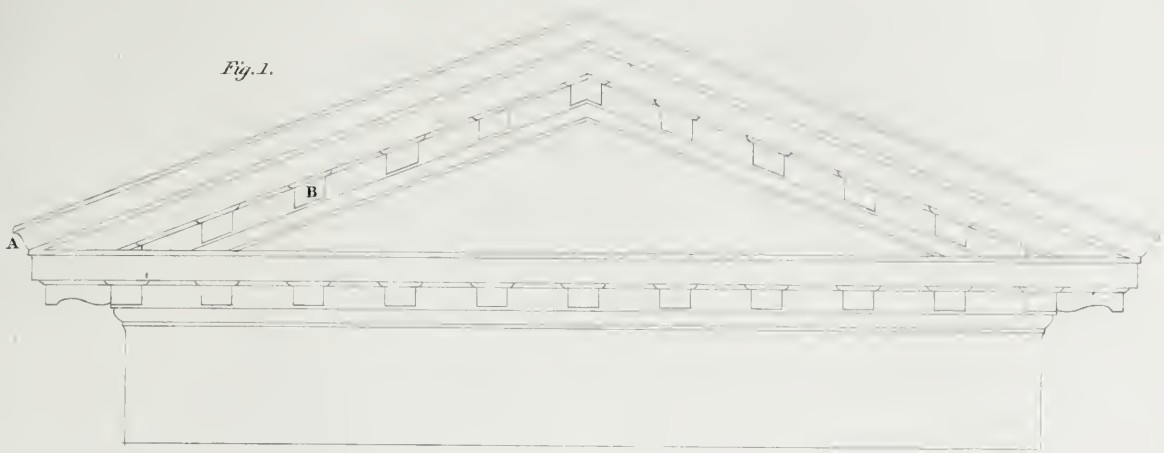


Fig. 4.

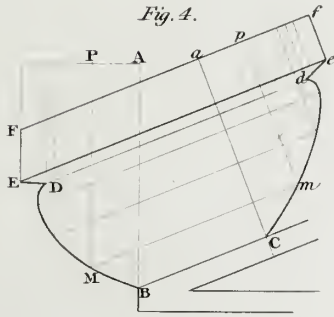


Fig. 3.

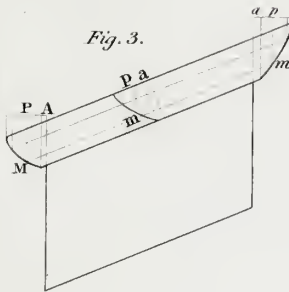


Fig. 2.

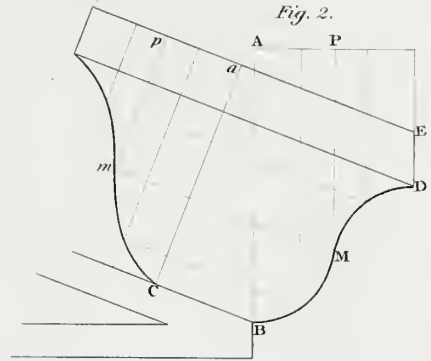
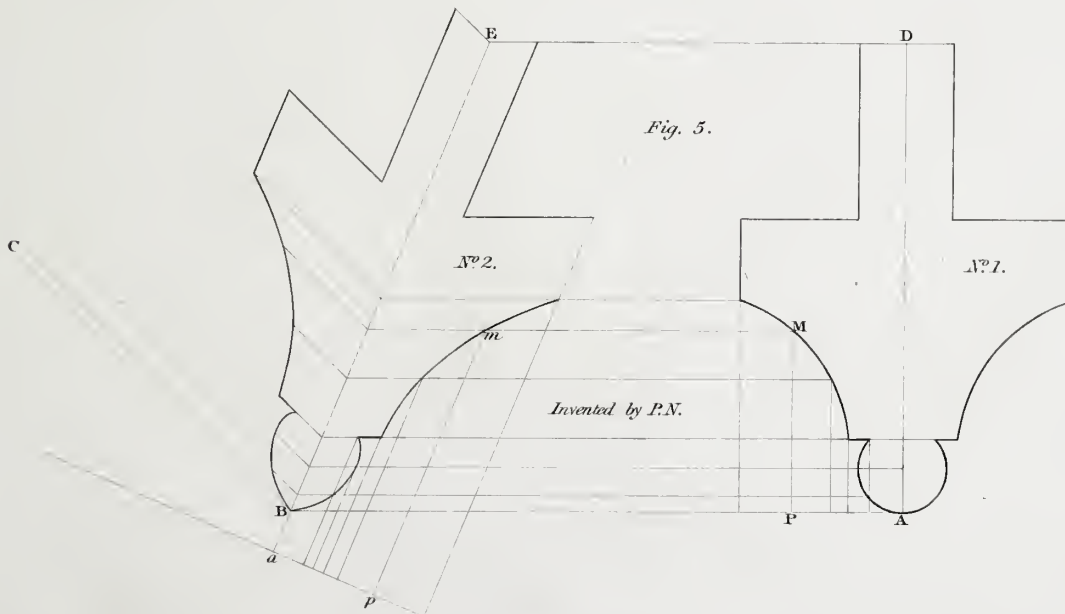


Fig. 5.



Drawn by P. Nicholson.

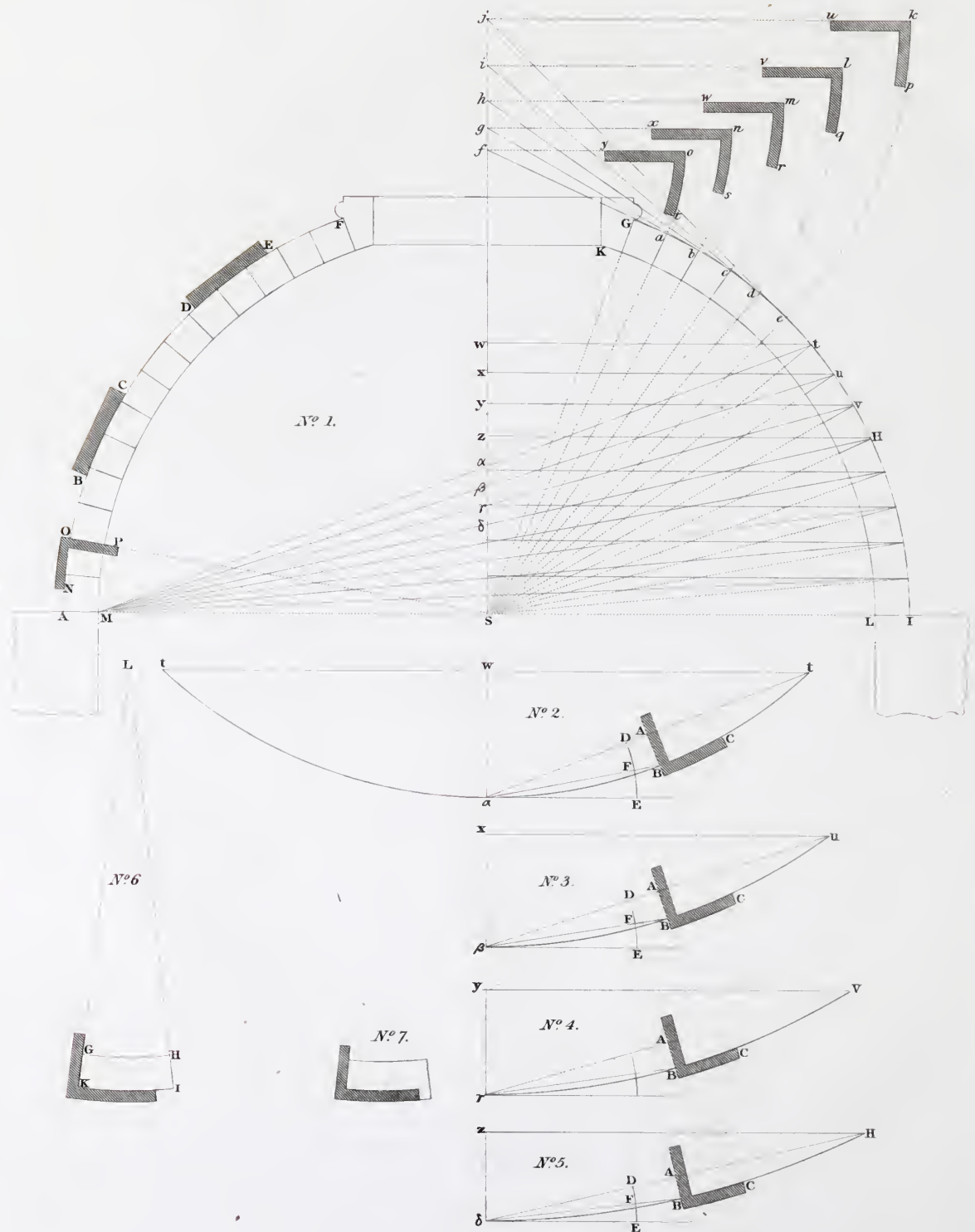




STONE CUTTING.

CONSTRUCTION OF SPHERICAL DOMES.

PLATE 1.

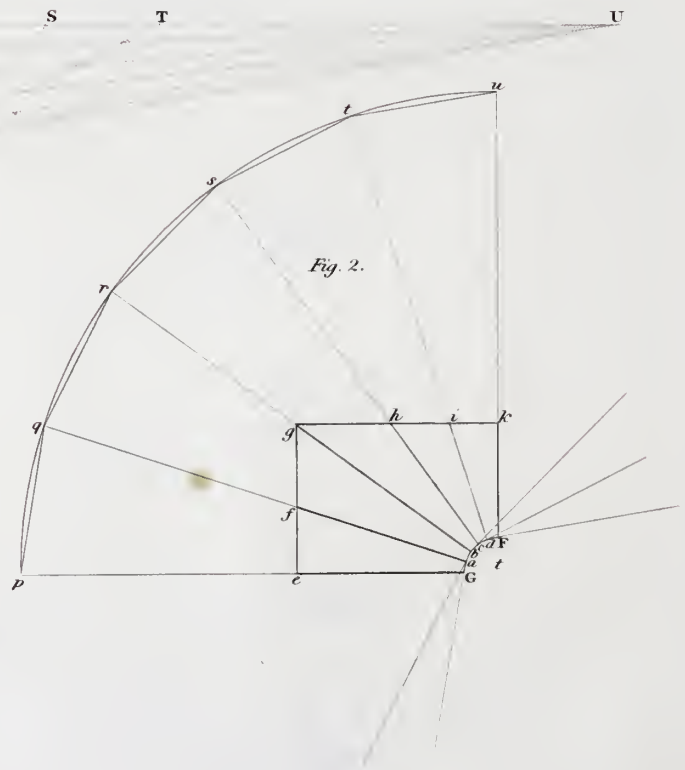
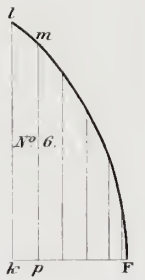
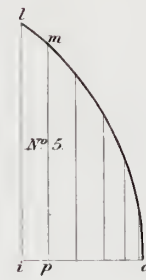
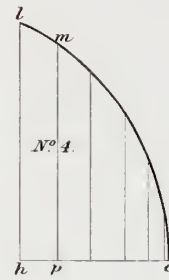
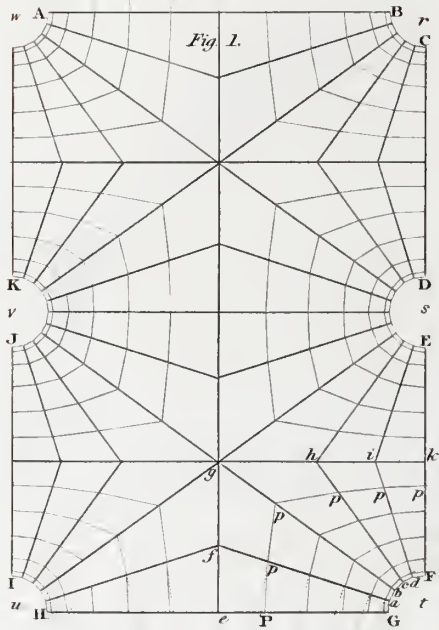
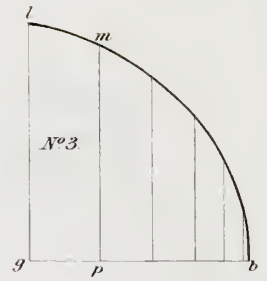
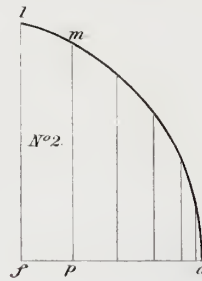
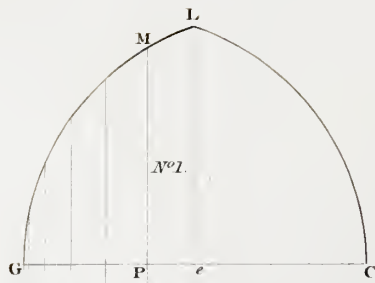




STONE CUTTING.

GROINS.

PLATE VI.



Q Q Q

Introduced by M. Nicholson.

London, Published by John Day & Co. Oct. 20, 1822

CONSTRUCTION OF SPHERICAL DOMES.—PLATE I.

Here the stones are first supposed to be wrought to a conical surface; then the bevels *yot*, *xns*, *vlq*, &c. apply on the convex surface to the horizontal joints, the circular leg being bent upon the surface.

ABC, ABC, &c. No. 2, 3, &c. are the bevels that apply on the convex surface to the vertical joints. No. 6 and 7 exhibit the bevels for working the conical joints.

The versed sines *Wα*, *Xβ*, *Yγ*, &c. of the arcs Nos. 2, 3, 4, &c. are taken from No. 1, between the similar letters and the half chords *wt*, *xu*, *yv*, &c. Nos. 2, 3, 4 are also taken from the distances between the similar letters No. 1. The straight leg AB of the bevel bisects the chord of the half arc of No. 1, 2, 3, &c. perpendicularly.

GROINS.—PLATE VI.

This Plate shows the plan of a groined rib vault formed of cuniconoidal surfaces:—See the article, p. 106, Primary Problems. No. 1 is the section across AB or GH in a vertical plane passing through the axis of the pillars; No. 2 is the rib which has *af* for its seat; No. 3 one of the diagonal ribs insisting upon its seat *bg*; No. 4 the rib which has *ch* for its seat; No. 5 the rib which has *di* for its seat; and No. 6 the rib which has *Fk* for its seat.

Suppose the pillar to be in the circumference of a circle of which the centre is *t*, and let the arc GF be the quadrant of the circumference. Divide the arc GF into as many equal parts as the number of parts between the ribs, and let *a*, *b*, *c*, *d*, be the points of division.

Draw the chord *aG*, and produce *aG* to meet the seat of the apex of the ribs in Q. Through P, any point in *eG*, draw *Qp* meeting *af* in *p*.

Join *ba*, and produce it to meet the seat of the apex line R. Through *p* draw *Rp* meeting *ag* in *p*.

Join *bc*, and produce *bc* to meet the apex line of the transverse vault in S. From *p* in *bg*, draw *pS*, meeting *ch* in *p*.

Join *cd*, and produce it to meet the seat of the apex line of the transverse vault in T. From *p* in *ch*, draw *pT* meeting *di* in *p*.

Join *dF*, and produce *dF* to meet the seat of the apex of the transverse vault in U. From *p* in *di*, draw *pU* meeting *Fk* in *p*.

In GC, No. 1, make GP equal to GP on the plan, *fig. 1*; in *af*, No. 2, make *ap* equal to *ap* in *af* on the plan; in *bg*, No. 3, make *bp* equal to *bp* in *bg* on the plan, *fig. 1*, and so on.

Then supposing the rib GML, No. 1, to be a given rib: draw PM perpendicular to BC. In Nos. 2, 3, 4, &c. draw *pm* perpendicular to the base, and make *pm* in each equal

to PM, No. 1; then *m* will be a point in the curve of each rib. In the same manner we may find as many points as we please for drawing the curve of each rib. *Fig. 2* shows one quarter at equal angles.

CONSTRUCTION OF GROINS.—PLATE II.

THE first preparation is to lay down the arch at full size (or half of it will do) on a straight floor, or on a piece of oil cloth, which is a very good thing; then divide out the number of courses that is intended to be in the arch: draw all the joints, as 1, 2, 3, 4, 5, 6, 7, the half of the arch; draw a line from each of these joints, level or parallel from the base; make a correct bevel to these lines; prepare a straight face of the stone, square it; then run a chisel draft round where the point of the mould cuts it in No. 1 section. At 2 and 3, or second joint from the keystone, apply the bevel as shown; at letter I, No. 2, with the arch mould applied from the bevel, No. 2 section, the bevels are found in the same manner; it will be two bevels for each quoin. ABCD on section shows the size of the quoin stones, *hijklm* on the plan of the ceiling shows the quoins when fixed, *opqrstuvw* shows the face and beds, 3 letter E shows the size of the stone with the bevel and mould applied; 3 and F, 3 and G, the same as letter E; 3 at H single mould.

ARCHITRAVE OVER COLUMNS.—PLATE I.

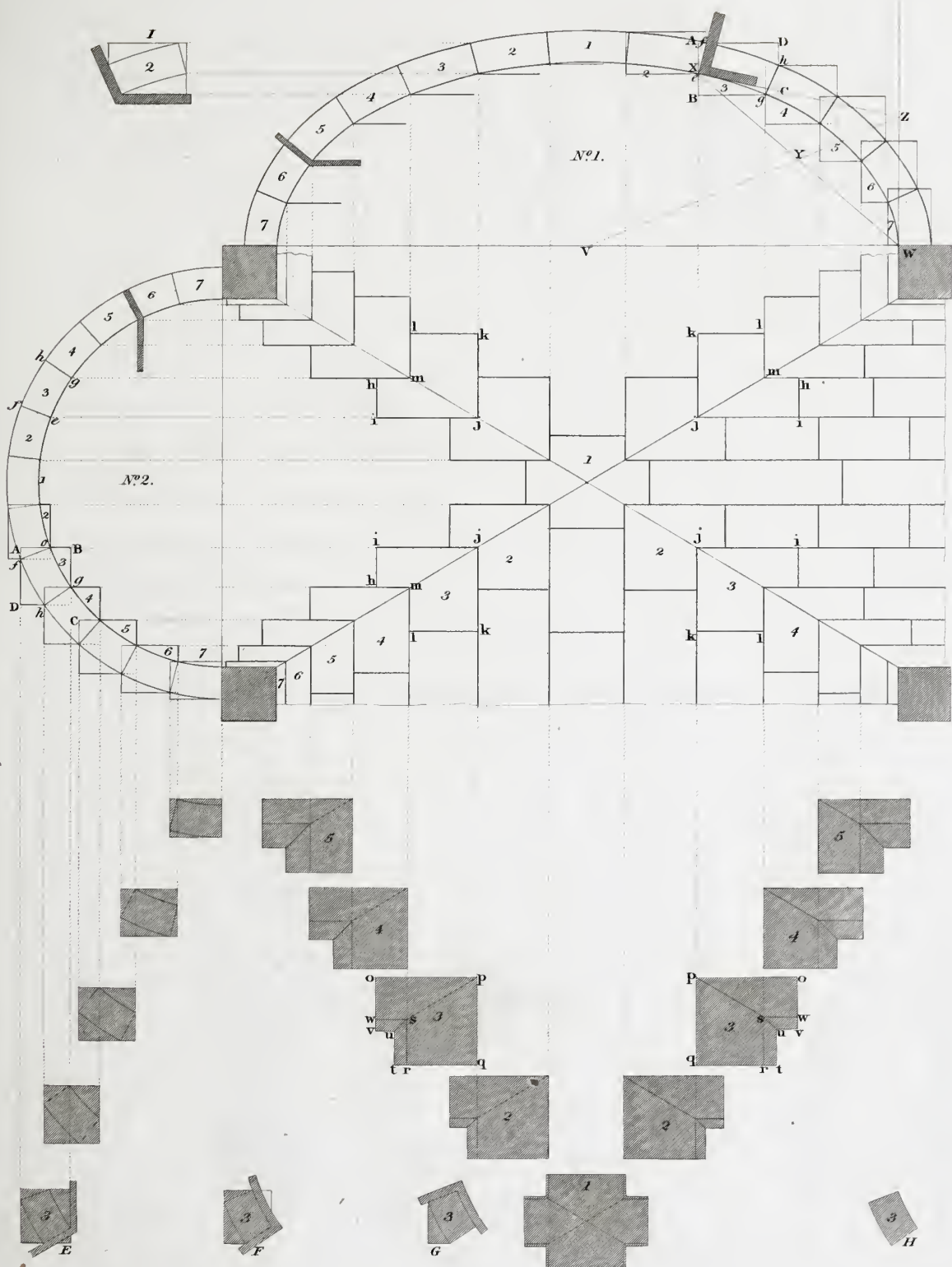
Fig. 1, No. 1, Plan, Elevation, and Section of Architrave over the Column, ABCDE plan of the top bed of architrave, with the chain bar of wrought iron, and collars let in flush with the top bed, and run with lead; ABCD elevation of architrave, *efgh* brick arch over architrave, *xyz* section through the middle at *f* showing the tail of the cornice on the crown of the brick arch to take part of the weight off the stone, *x* shows the springer for the brick arch both ways, *i* end of frieze, &c. *k* a piece of iron let into both stones to prevent the springer moving, as it has no abutment, No. 2 end elevation of C over column, and D to join C, *lmnop* dotted line goggle joint, and *qr* upright joint, *uuu* at No. 2 upright joint; *vw* corresponds to *mnp*, No. 1 dotted line, *t* rope with the lewis let into the stone.

Fig. 2, No. 1, elevation of the lewis, No. 2 section, No. 3 bolt, No. 4 ring, No. 5 outside piece, No. 6 middle piece, No. 7 coteral. In No. 5, when the two are put together, they should be three-eighths of an inch wider at the bottom than the three when put together at the neck under the bolt, and the two outside pieces should always be made straight: some masons make them hollow, but I think it is a bad plan, as, when they are made so, they bear too much at the point: when made straight, they bear more regular all the way up.

STONE CUTTING.

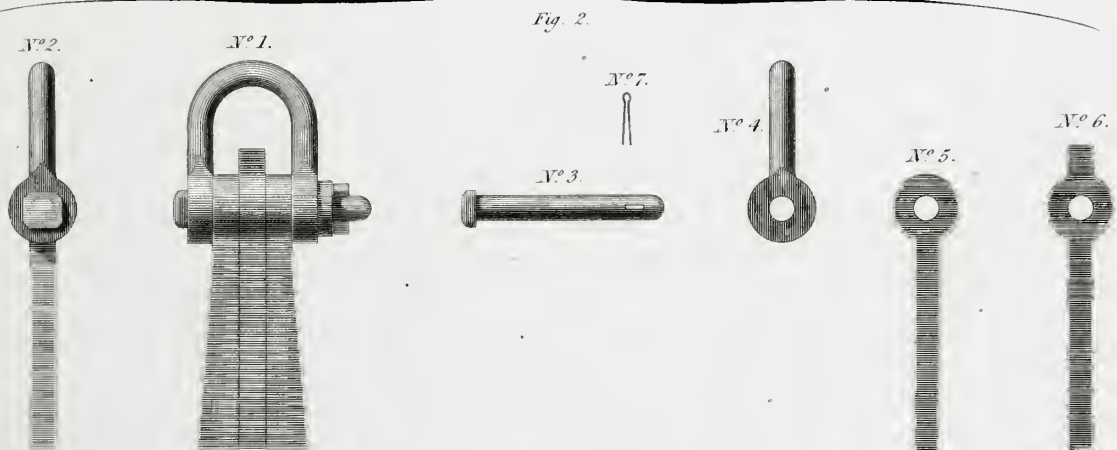
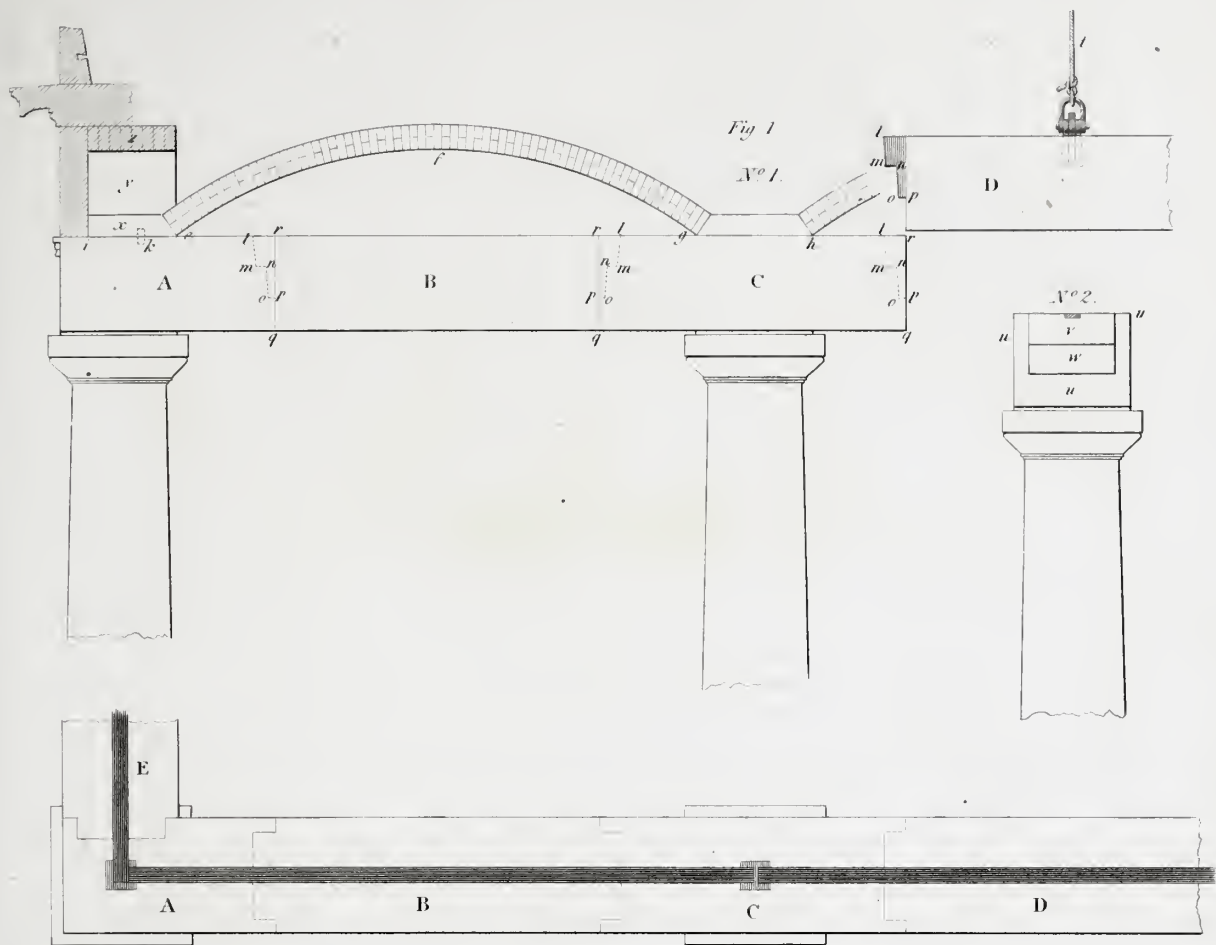
CONSTRUCTION OF GROINS.

PLATE II.



MASONRY,
ARCHITRAVE OVER COLUMNS.

PLATE I.

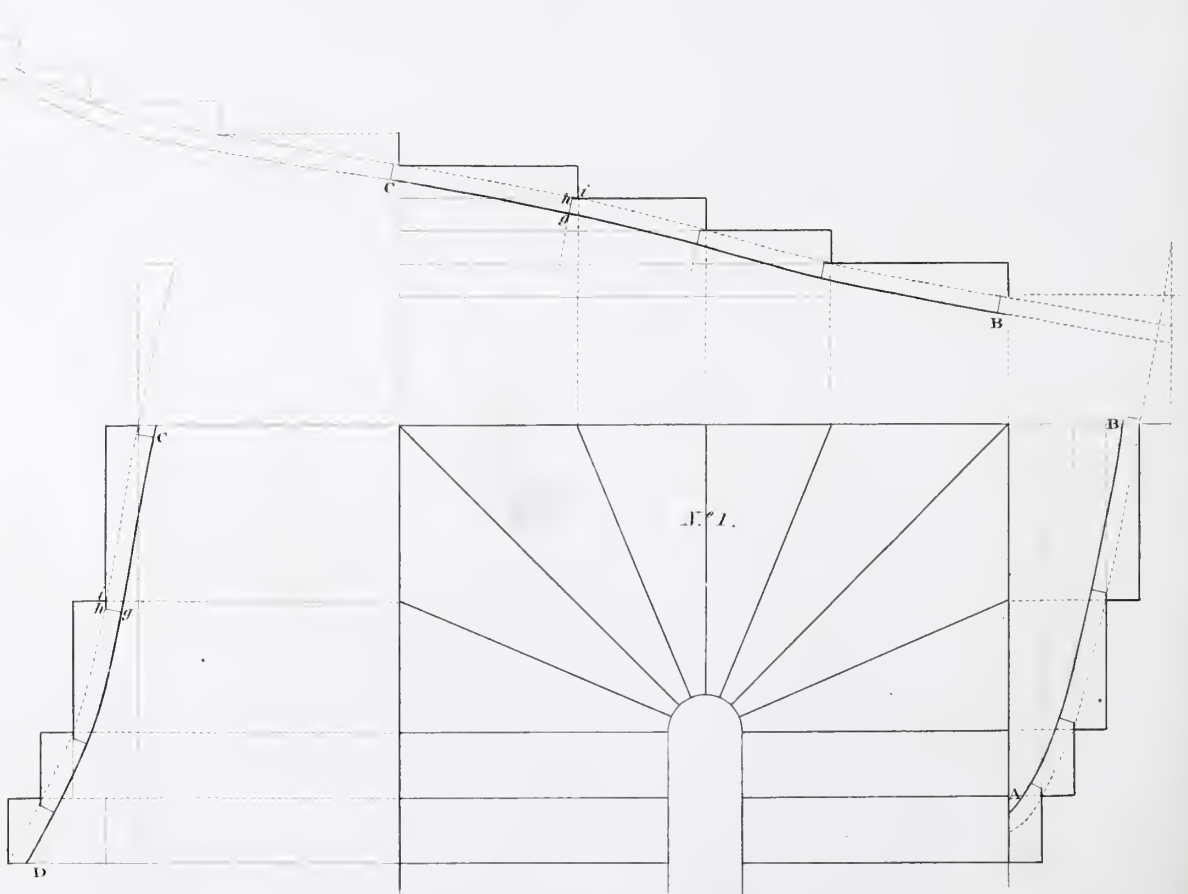
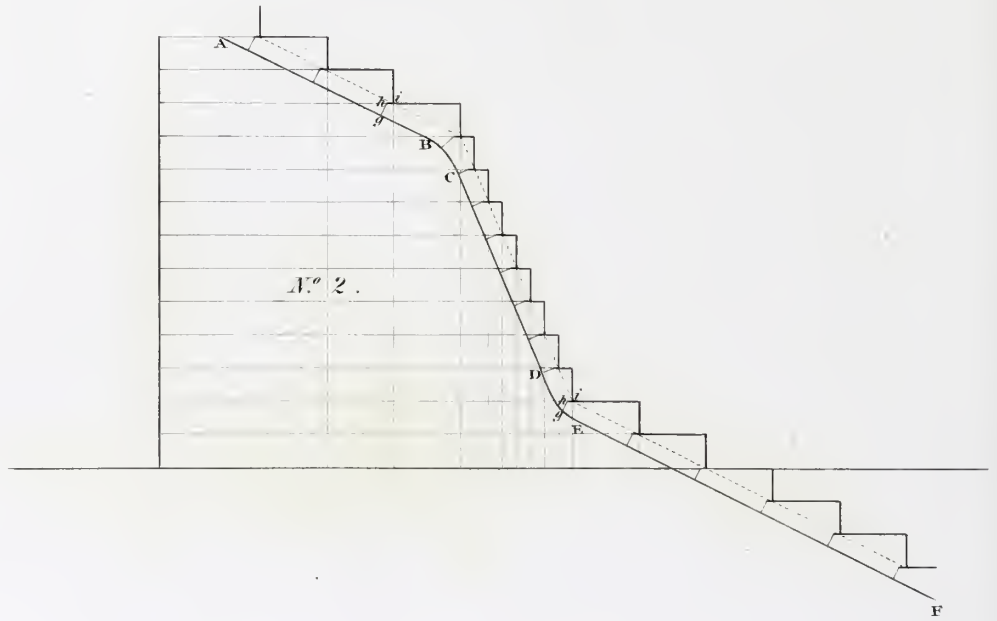




STONE CUTTING.

STAIRS.

PLATE I.



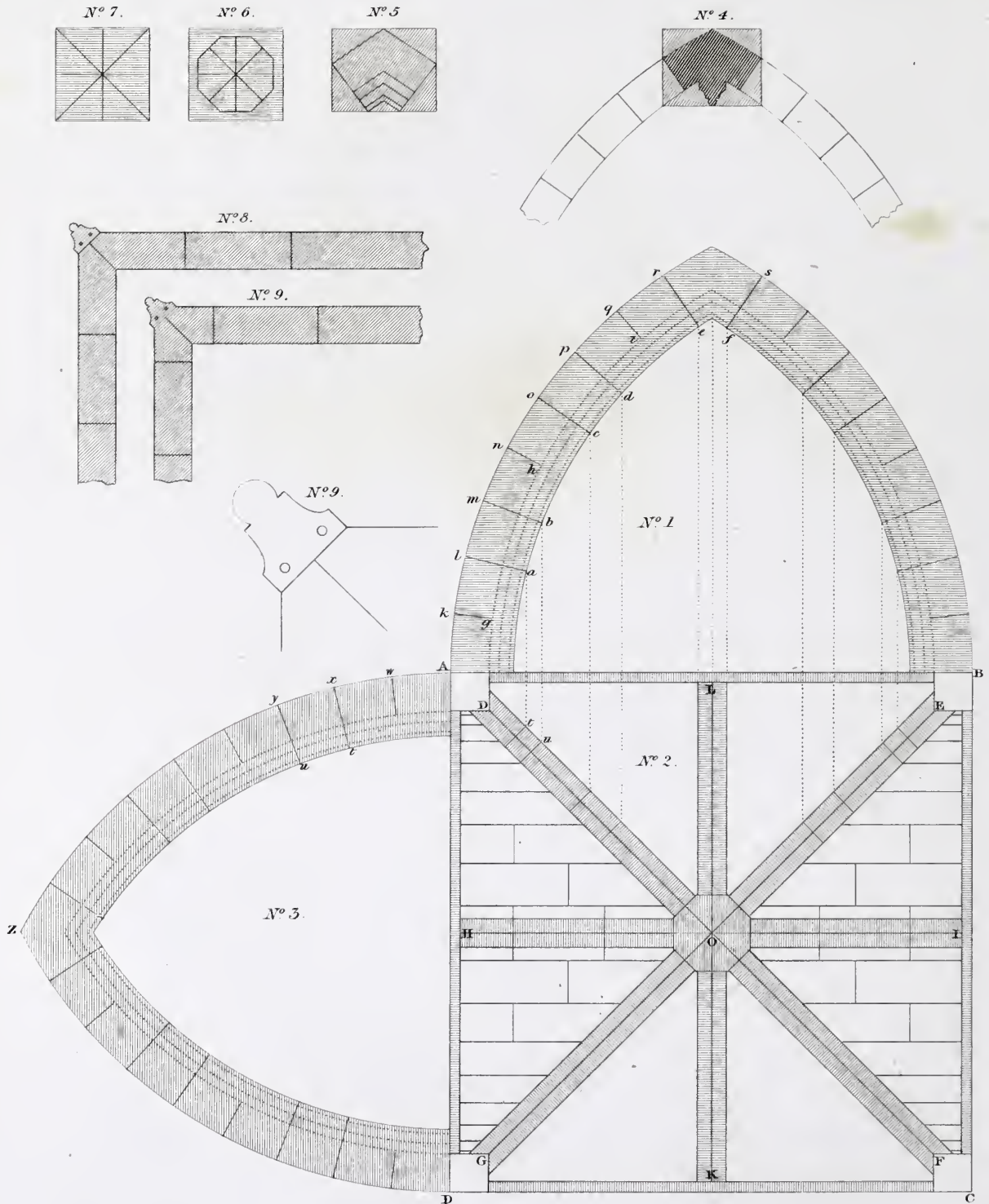
Invented & Drawn by F. Nicholson.



STONE CUTTING,

GOthic GROIN.

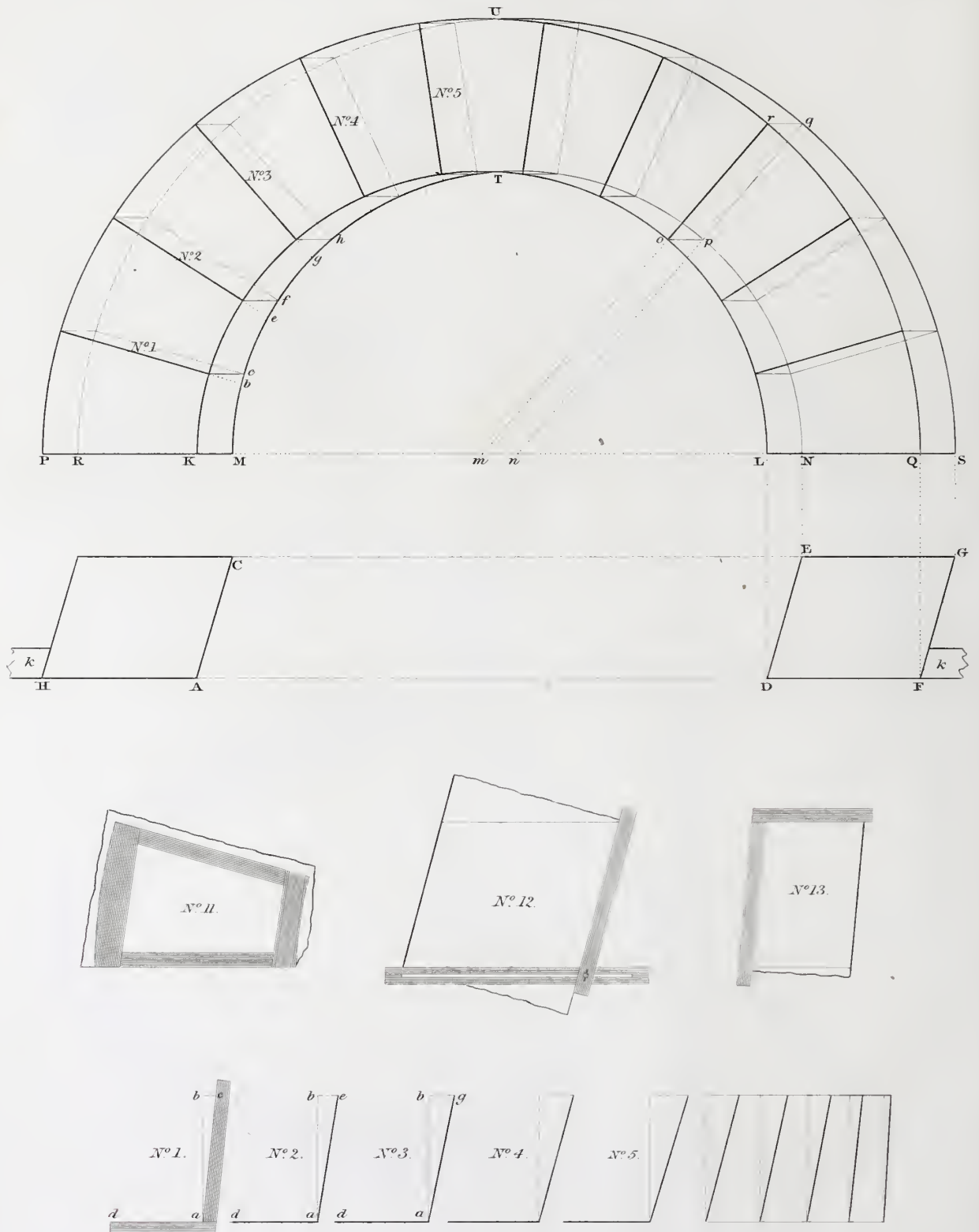
PLATE .I.



STONE CUTTING.

OBLIQUE ARCH.

PLATE I.



STAIRS.—PLATE I.

No. 1 plan of the staircase, No. 2 section at the quoin end laid down at full length; ABCDEF line of the soffit, showing the quoin ends of winders and flyers, with the rabbet drawn square from the soffit at quoin ends of the winders, as *ghi*; from B to E, by laying the section down at full size, it shows where the stone wants to hollow or round at BCDE, and AB section of two flyers and two winders at the wall line, BC section across the end, and CD two winders and two flyers: by laying both sections down at full size, you see where the soffit will be crippled. The steps can be made easy to the eye, and by making all the joints of the winders square from the rake, the stone is stronger; it is not liable to flush, and the back rabbet will wind the same as the soffit. When the rabbet of the winders is cut in with the same mould as the flyers, it leaves a very acute angle, and liable to get broke.

STONE CUTTING:—GOTHIC GROIN.—PLATE I.

ABCD the plan of the walls, and GDEF plan of the piers; No. 1 section showing the manner of connecting the ribs with the arch stones, as *kg, la, mb, nh, oc, pd, gt, re*, and *sf*. To prepare the rib, the height of two courses of the arch stones as No. 9 large scale, and every third course to work the rib on quoin, as shown on the section from *a* to *b*, or from *t* to *u*. HLIK plan of the straight ribs at the crown of the arch, and O intersection of the keystone; No. 2 plan of the ceiling, No. 3 section showing the manner of connecting the quoins, as *yuxtw*, &c. No. 4 keystone with the mould applied, No. 5 section through the keystone, No. 6 plan of the keystone, showing intersections of the angles; No. 7 the plain surface with lines drawn the first process, No. 8 plan of the course at *a* on the section, No. 9 plan of the course from *b* to *m* on section; No. 10* section of single rib with pugs. It is much stronger to do the arch in this way than running the ribs all single, as they are generally so.

OBLIQUE ARCH.—PLATE I.

The given plan HADF and CEG, *kk* ashler joining the arch; PK, LQ is the outer line of the arch, RMNS inner line on the elevation; No. 1, 2, 3, 4, 5 on the elevation gives the bevel of each joint or bed from the face, as *bc, ef, gh* on the elevation; No. 1, 2, 3, 4, 5, *bc, be, bg*, gives the different bevels of each bed from the face, as *da, da, da*, &c.

* The figure under No. 9, which ought to be 10, is also marked by mistake No. 9.

da, bc bevel applied across the bed from the face; No. 11 arch mould applied on the face; No. 12 shift stock applied from the face across the bed, which gives the exact bevel from K to M on elevation; No. 13 same as No. 1; *bc, rg, op, mn* bevel of the joint continued down T and V, will be square from the face at the crown.

NICHE.—PLATE II.

No. 1 elevation of niche and each stone over it, No. 2 plan of the dome and niche at the springing, No. 3 plan of the top of the arch stone of the niche, No. 4 plan of the top bed of the keystone as No. 3 on the plan, No. 5 section of keystone; No. 6 keystone turned upon the top bed, showing the projection from A to B, from B to C, and from C to L. The plan of the top bed and rough back of No. 6 is shown in the section of No. 5. No. 7 section of keystone at the back, GI on elevation, the level bed at the springing, *klmnopqrstu* joints of the niche stones, ABCDEF plan at the springing, IL centre line, GG ashler, HH plan of the springing stone of the niche. By laying down the plan, elevation, and section at full size, there is no difficulty of execution.

ARCH IN A CIRCULAR WALL.—PLATE I.

No. 1 elevation of the arch, No. 2 plan of the bottom bed from A to B on the plan, and from *p* to *o* is what it gains on the circle from the bottom bed to *f* on the joint, and from *n* to *m* is the projection from the bottom bed to *g* on the joint; from *o* to *m* is equal to *fg* at the joint; *abcd* shows what it gains on the circle as it goes round, by dropping a perpendicular line from the bottom bed to *p*, and from *f* to *o*, and from the bottom bed inside to *n*, and from *g* to *m*; then square from those lines where they intersect with the plan, it will give the proper curve for all the different moulds in the same manner. No. 3 and 4 is done in the same way; No. 3, *po* equal to *fh*, and *hi* equal to *om* and *on*; No. 4 equal to *ik*, and *nm* equal to *kl*; No. 5 mould for bottom bed first arch stone, *gs* equal to AB on the plan No. 2, and *rs* equal to *ac*, and *st* equal to Bn, and *tu* equal to *pn*. By laying plan and elevation down at full size, and taking the same methods, it will answer without any difficulty. No. 6 mould for applying on the bed or joint *fg*; No. 7 mould for bed at *hi*; No. 8 mould for bed *kl*, which becomes almost straight and square; No. 9 arch or face mould. Draw two lines on the mould square from the base line; then run two chisel draughts straight, and out of winding; then other two lines across the face parallel from the base line, and run two draughts with the hollow mould as found from the plan at EF; then the whole face may be worked with a straight edge, taking care to apply it square from the base. It may be divided into as many upright lines as may be thought necessary to work the stone correct.

STONE CUTTING.

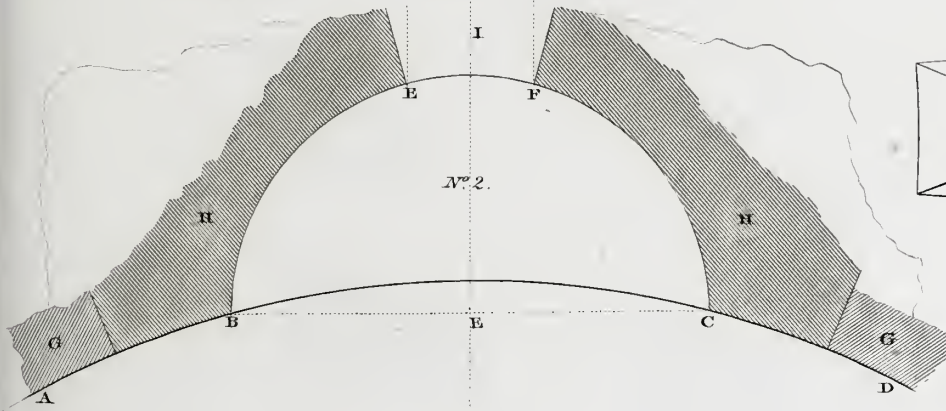
NICHE.

PLATE II.

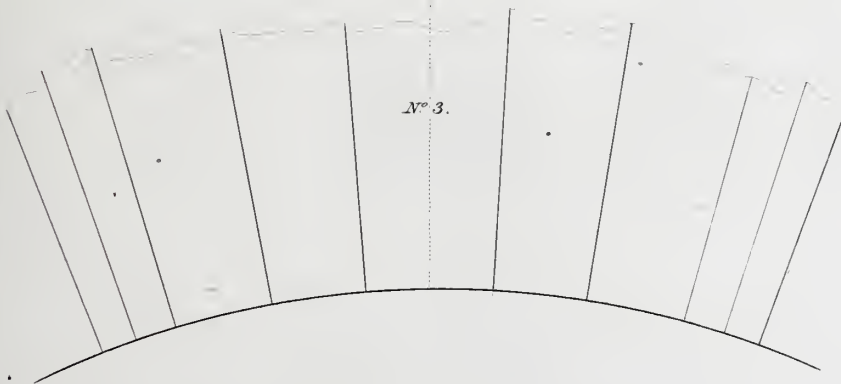
N^o 1.



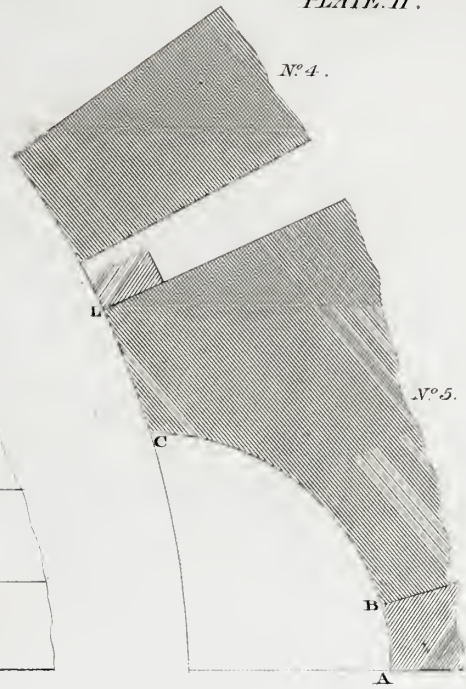
N^o 2.



N^o 3.



N^o 4.

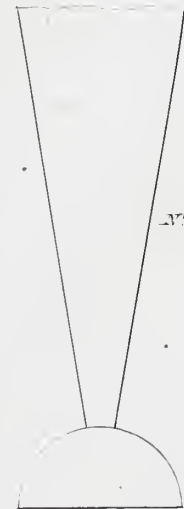


N^o 5.

N^o 6.



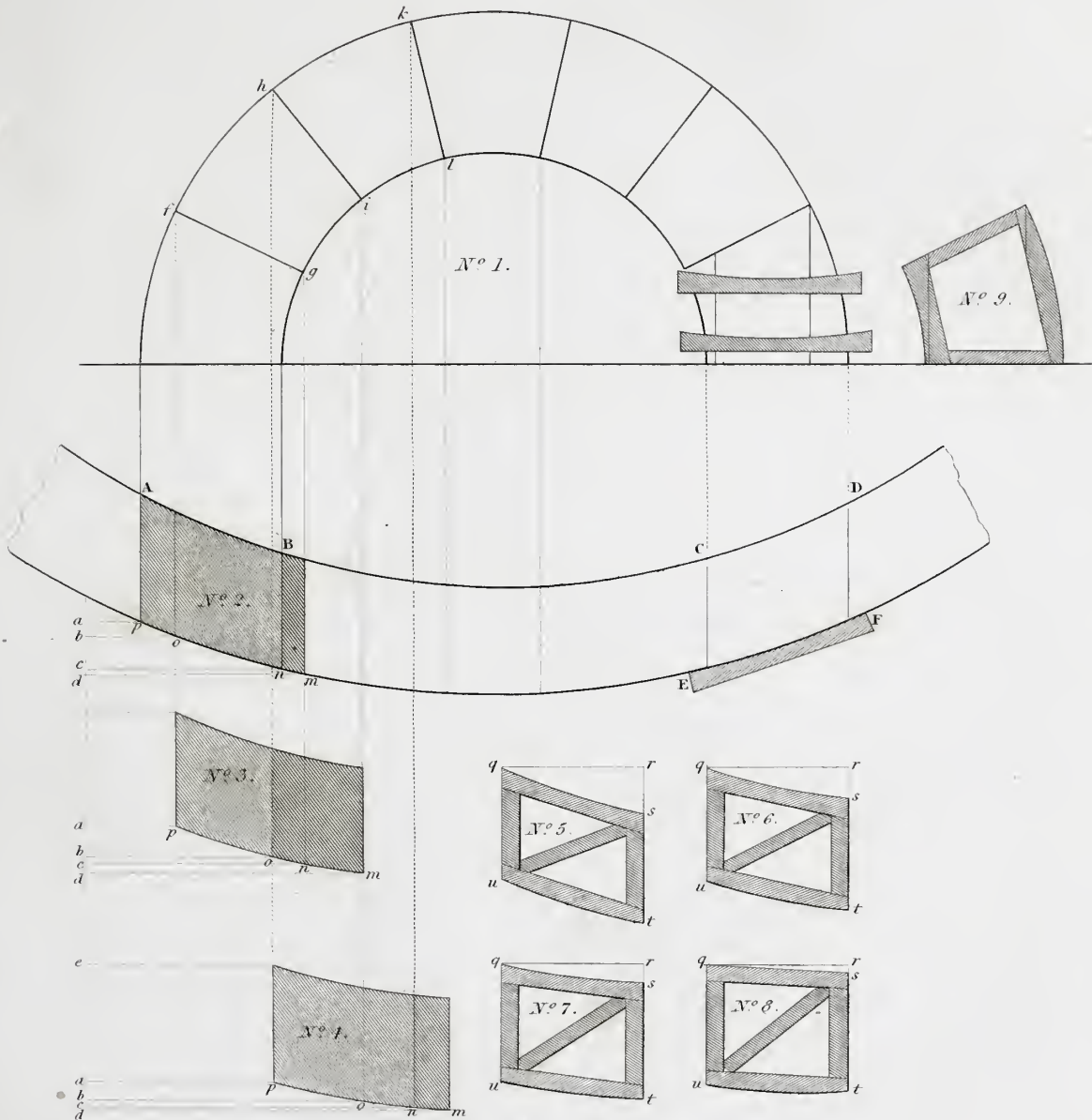
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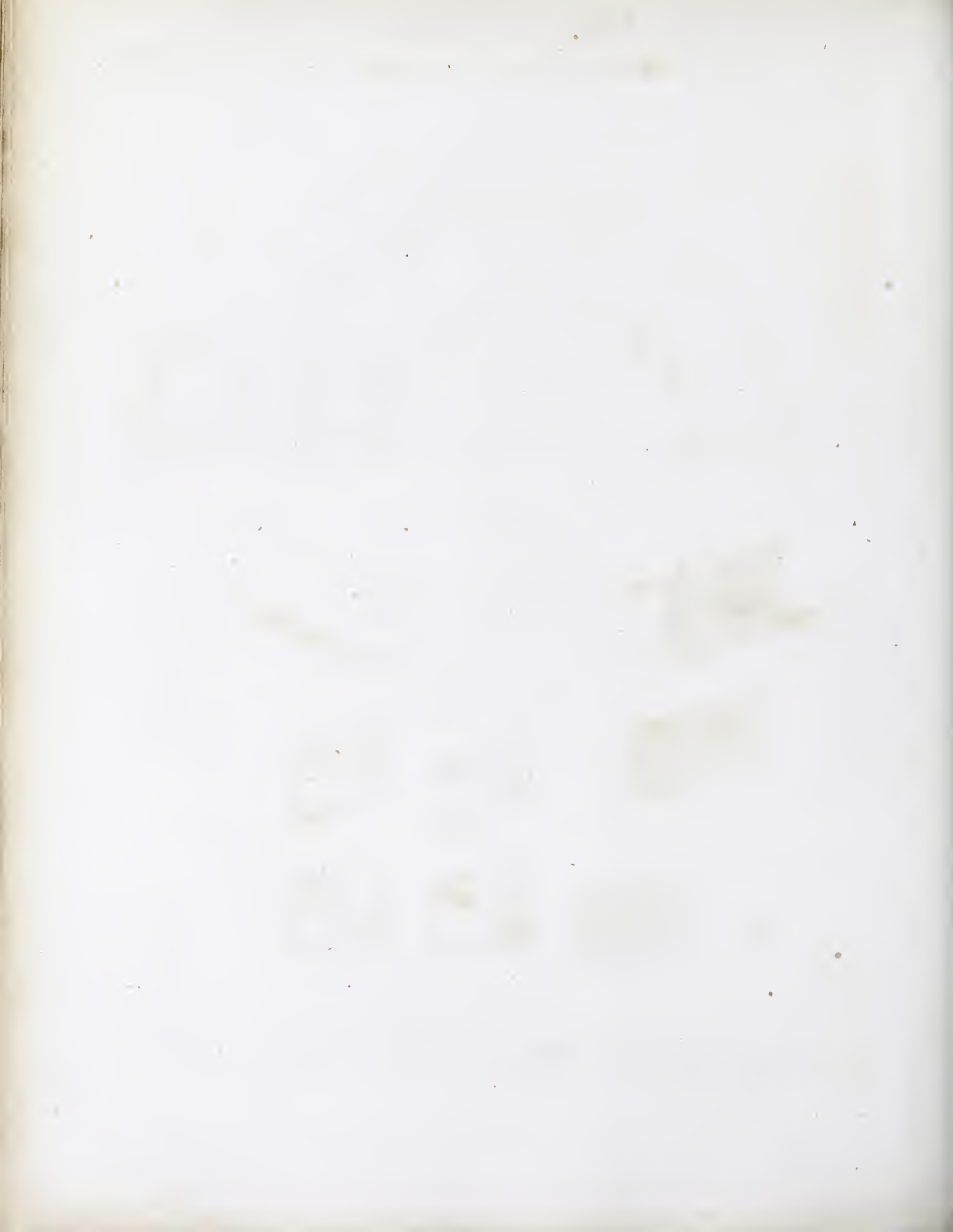


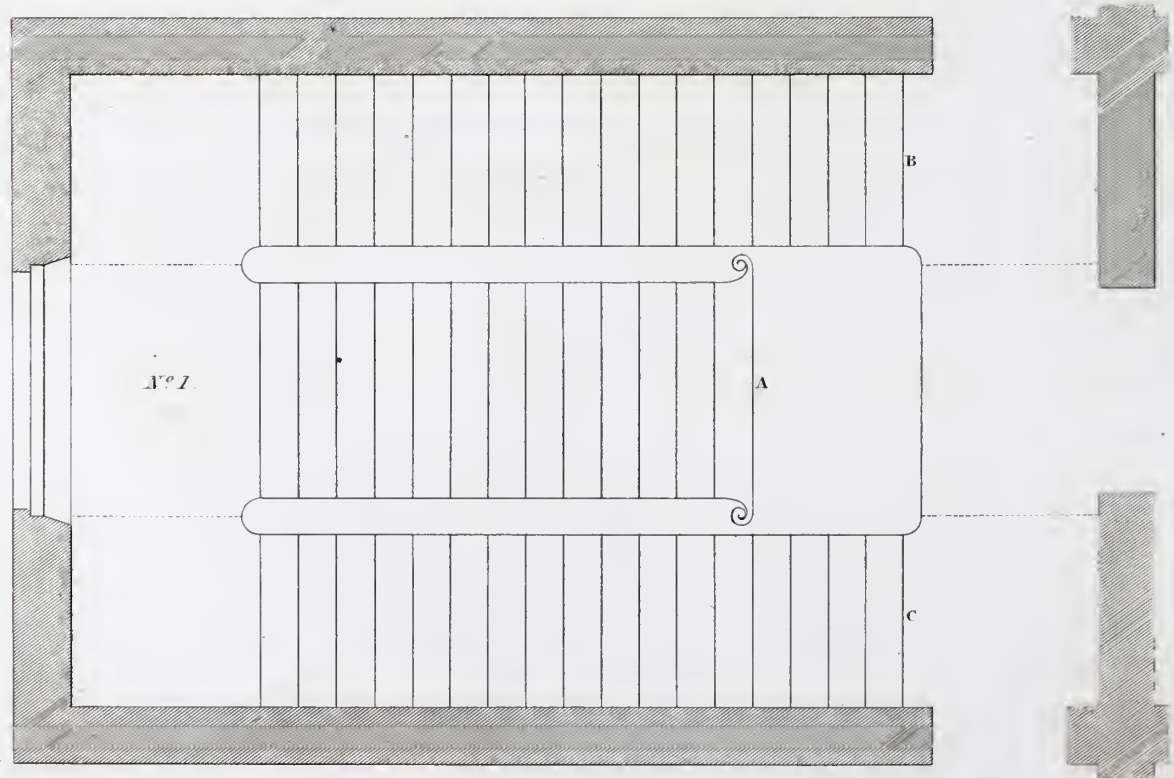
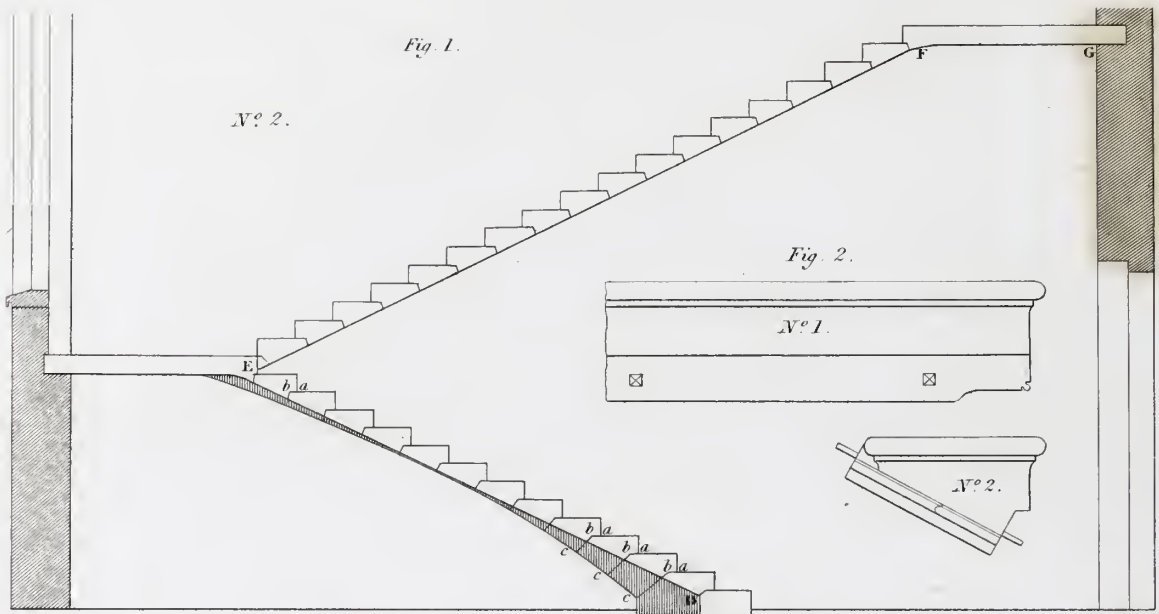
STONE CUTTING,

ARCH IN A CIRCULAR WALL.

PLATE 1.





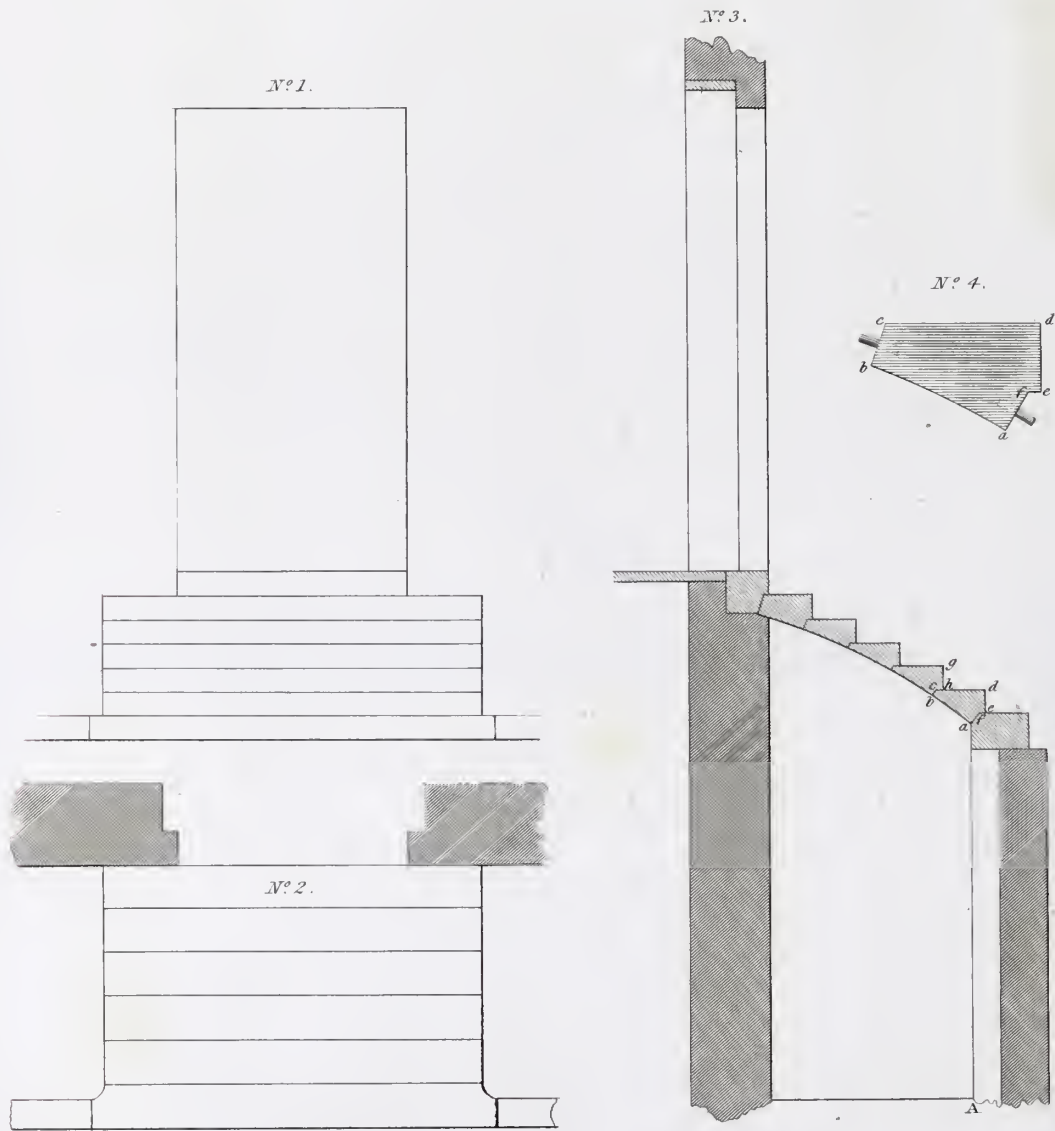




MASONRY,

STEPS OVER AREA TO ENTRANCE DOOR.

PLATE I.



GOTHIC ARCH

The image contains three main architectural drawings. The top drawing is a dome plan labeled 'Nº 1.' with points A through I and various internal labels like 'h, a', 'i, b', 'k, c', 'l, d', 'm, e', 'n, f', 'o, g', 'p, h'. It includes two small detail drawings labeled 'Nº 10.' and 'Nº 11.'. Below the dome is a cross-section labeled 'Nº 2.' showing the dome's profile and supporting walls. At the bottom is a series of vertical sections labeled 'A' through 'M' with corresponding points and labels like 'p, q', 'r, s', 't, u', 'v, w', 'x, y', and 'Nº 3.' through 'Nº 9.'.

ARCHES IN STRAIGHT WALLS.

Fig. 1.

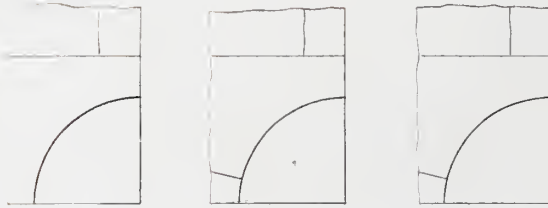


Fig. 2.

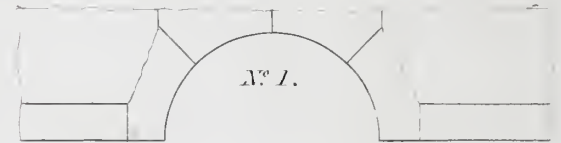
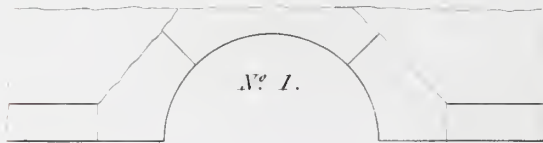
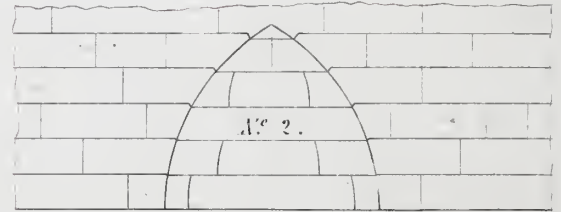
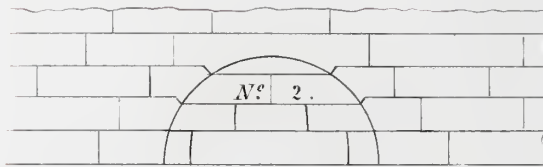
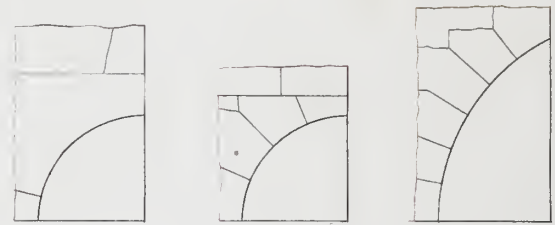


Fig. 3.

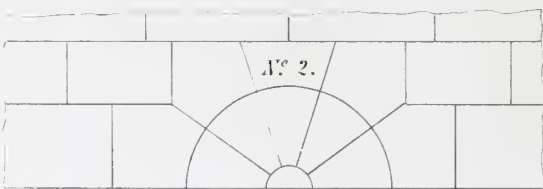


Fig. 4.

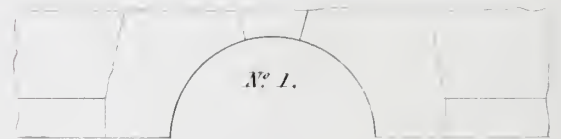
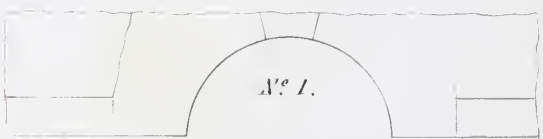
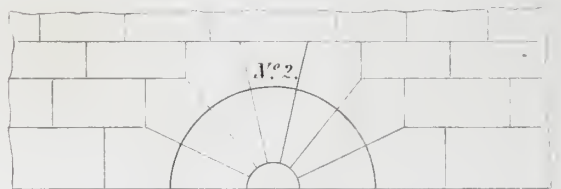


Fig. 5.

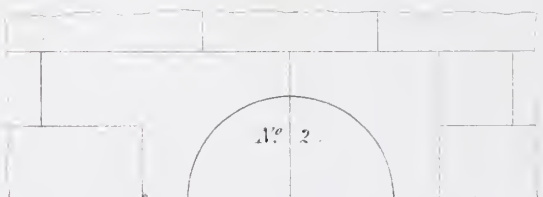
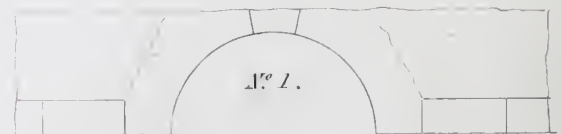
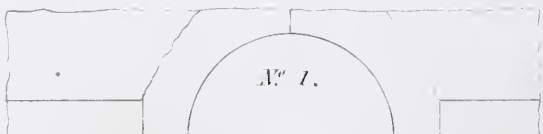
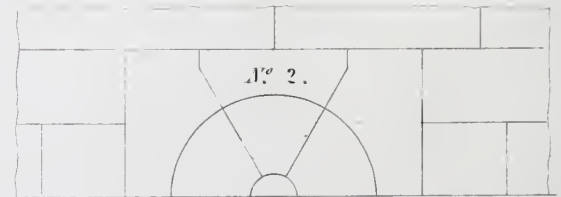


Fig. 6.



STAIRS.—PLATE II.

No. 1 plan of the staircase, A bottom or first step, BC top landings. *Fig. 1*, No. 2, section of the staircase. The first flight from A to No. 1 can be done without any support, by an additional thickness of stone, as shown on the section from E to D shaded dark, and by making all the back rabbets *bc*, *bc*, *bc* arch joints. *Fig. 2*, No. 1 elevation of the sixth step, showing the rabbet with two iron plugs, inch square, let into each stone $2\frac{1}{2}$ inches, and by working off the additional thickness about 12 or 15 inches in from the end with an *ogee*, which makes it go off easy to the eye. No. 2 elevation of the end of one step, showing the plugs the other way at F on the section, turned out with a hollow. Wherever the landings go into a wall, a course of stout Yorkshire paving or thin landings should be laid, and over that three courses of bricks with sand, nine inches in, and over that another course of landings to go through the whole thickness of the wall. When the staircase is ready for fixing, the bricks are taken out with little trouble, and without injuring the wall in the least. The landings in all public buildings should be done so. Those at the New Custom House, where I had the direction of the Masonry for Mr. H. Peto, Builder, were constructed in this manner.

STEPS OVER AREA TO ENTRANCE DOOR.—PLATE I.

No. 1 elevation of steps, No. 2 plan, No. 3 section, No. 4 section of the second step, *af*, *bc* rabbets made arch joints with thicker stones, and each joint plugged, and, by building a brick pier from A on the section, one at each end of the step, the steps will do without any brick arch below them.

GOTHIC ARCH.—PLATE I.

No. 1 elevation of inside of arch, No. 2 plan splayed at the jambs and level at the crown, No. 3 the splayed part of the first arch stone as ABC, No. 4 the second stone DEF, No. 5 the third GHI, and so on; No. 10 plan of the bed with two double plugs, No. 11 the same, No. 12 section of the label.

NICHES IN STRAIGHT WALLS.—PLATE I.

THIS Plate exhibits the construction of niches whether the stones are laid in horizontal courses, or in courses tending to a centre, according to the number of stones; No. 1 being the plan, and No. 2 the elevation in each of the six figures.

Bricklaying.

GROIN VAULTS.

Plate 1 exhibits the plan and elevation of a series of groin vaults.

Fig. 1 is the plan. The cross lines which extend from the angles are the seats of the groins, which are the intersections of the upper surfaces of the boards. The hyperbolic curves within the cross lines are the seats of the intersections of the under sides of the boards of the side arches, with the upper surfaces of the boards of the large arch, and the two side arches parallel to the large arch.

Fig. 2 elevation or section of the principal vault, and the two side vaults.

Fig. 3 transverse elevation or section of the crossing vaults.

Fig. 4, 5, 6 exhibit the moulds for binding over the boarding, in order to find the true intersections agreeable to the seats on the plan.—See the Developements of Solids, and Orthoprojection.

GROIN VAULTS.

Fig. 2.

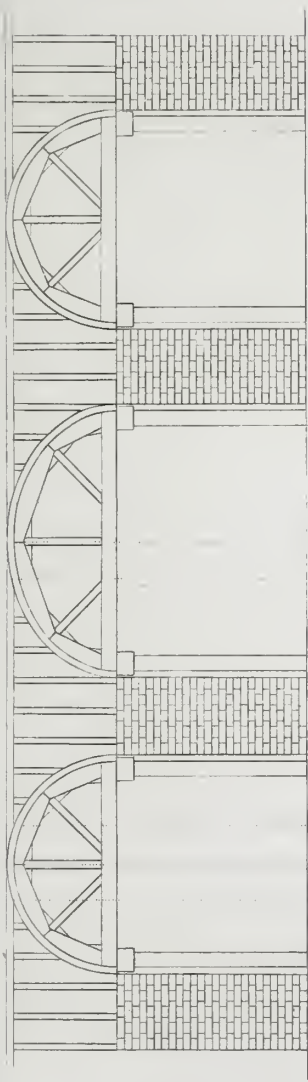


Fig. 3.

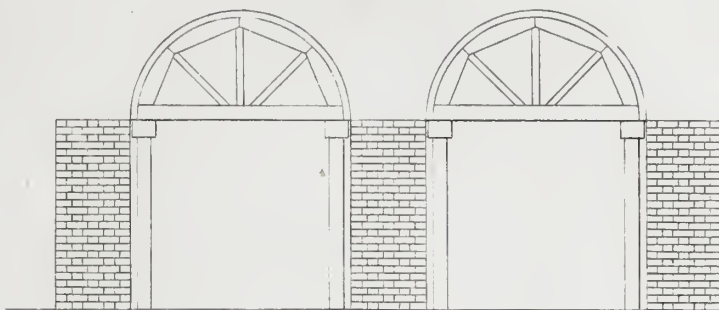


Fig. 1.

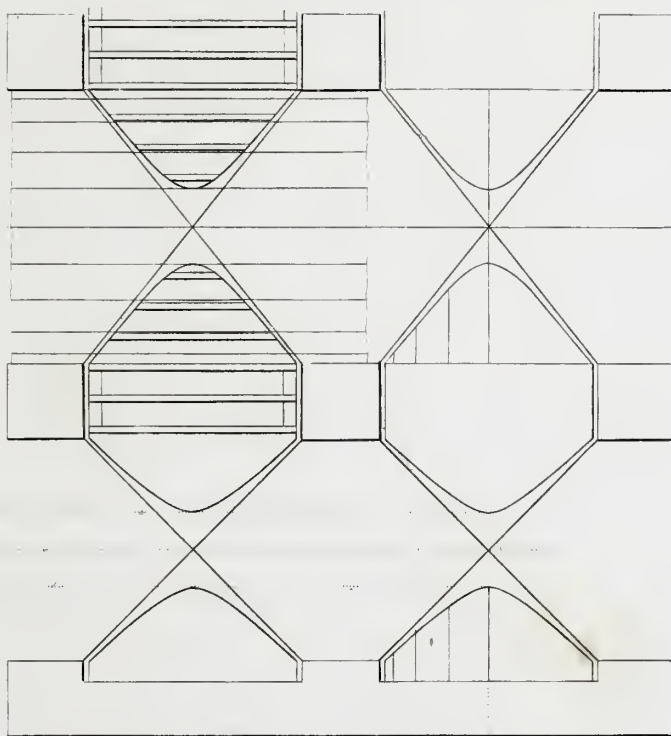


Fig. 4.

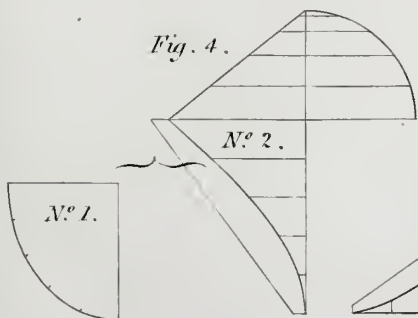
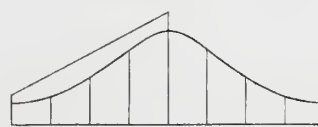
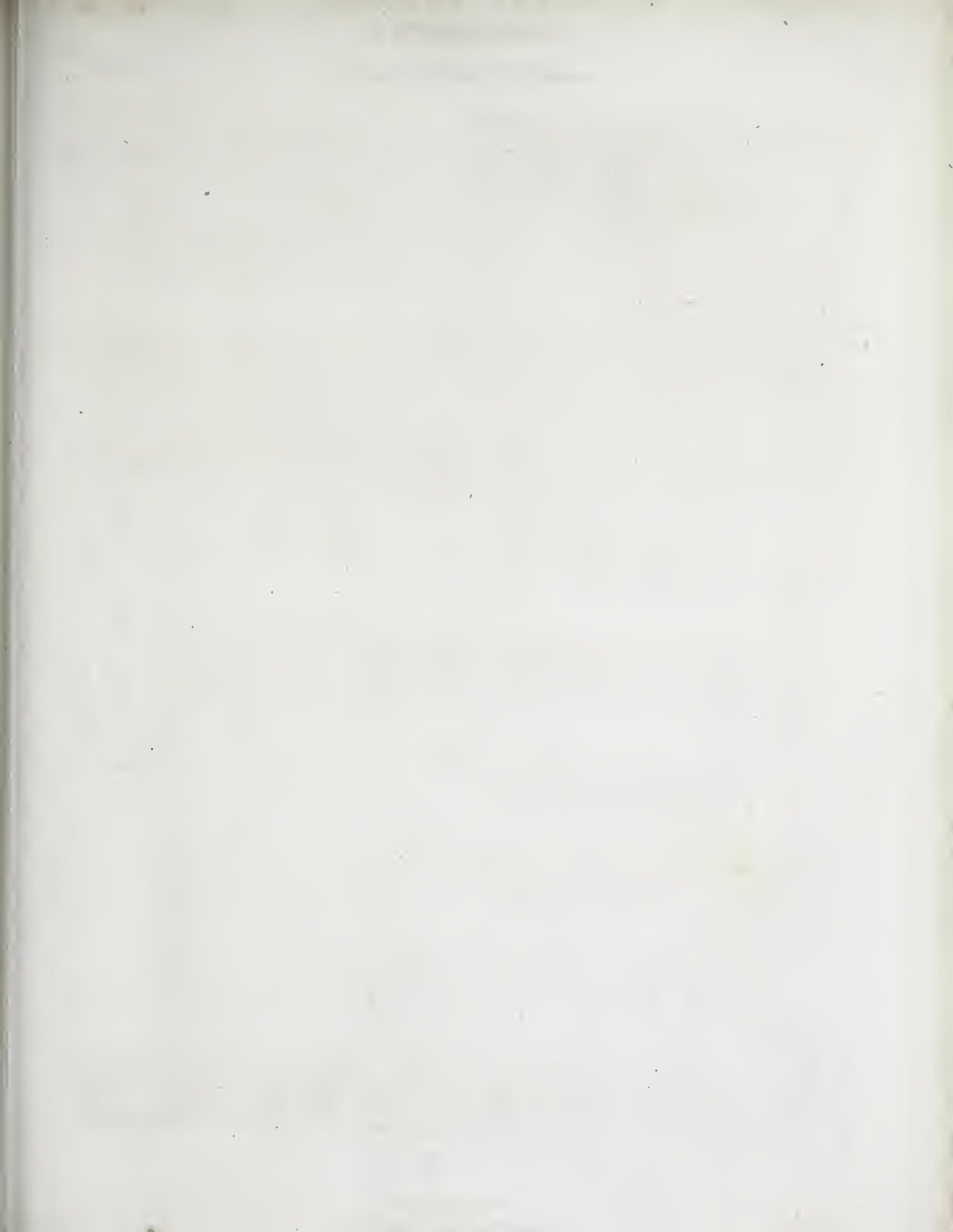


Fig. 5.



Fig. 6.





CARPENTRY,

METHODS OF JOINING TIMBERS.

PLATE 1.

Fig. 1.

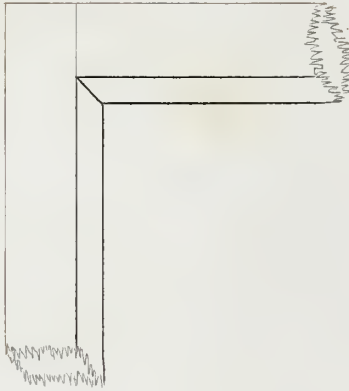


Fig. 1. N° 1.



Fig. 1. N° 2.

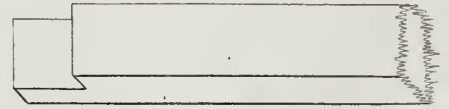


Fig. 1. N° 3.

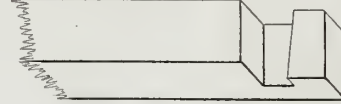


Fig. 1. N° 4.



Fig. 2.

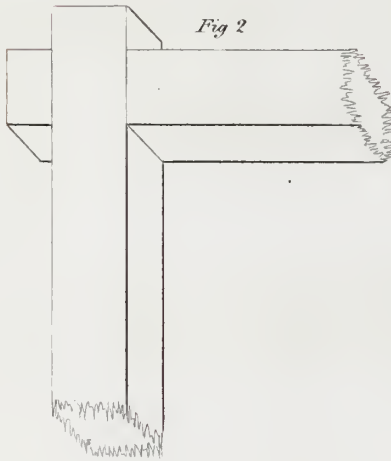


Fig. 2. N° 1.

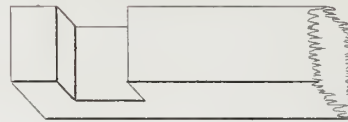


Fig. 2. N° 2.



Fig. 3.

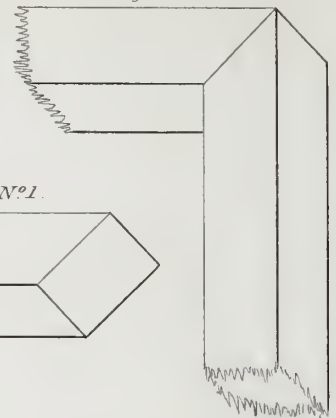


Fig. 3. N° 1.



Fig. 4. N° 2.



Fig. 4. N° 4.



Fig. 4.

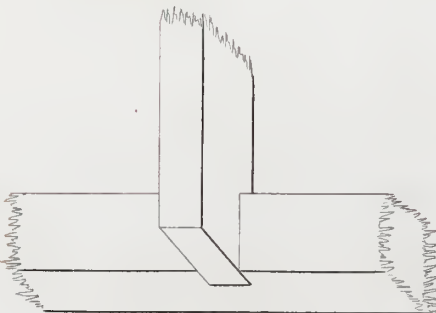
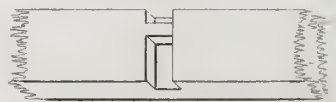


Fig. 4. N° 1.



Fig. 4. N° 3.



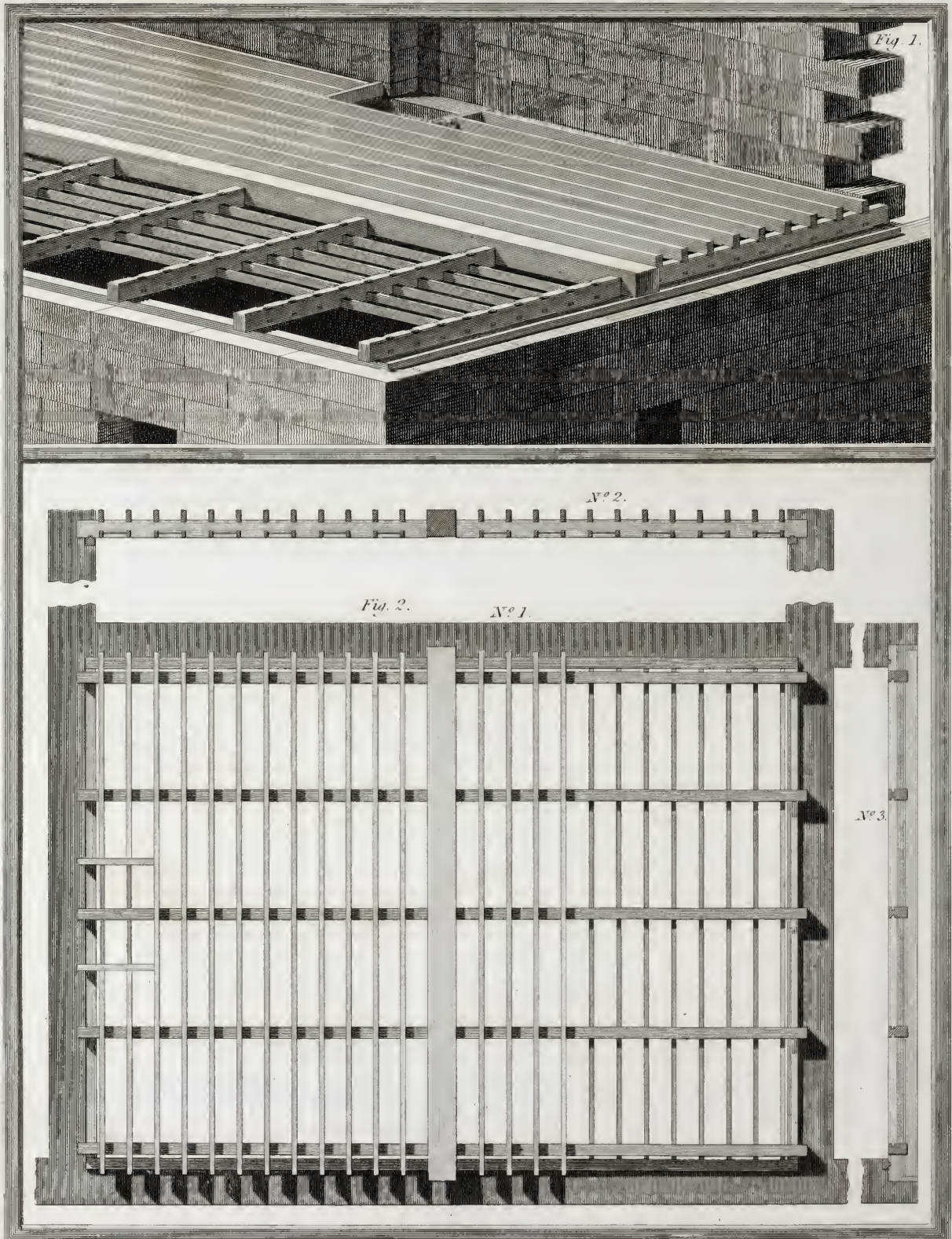
Introduced by E. Nicholson.

London. Published by John Day & Co. Oct. 20, 1870.

CARPENTRY.

NAKED FLOORING.

PLATE I.



Carpentry.

CARPENTRY is the Art of employing Timber or Iron in the Construction of Buildings.

Plate I (subtitled *Methods of Joining Timbers*).

Fig. 1 a common joint where the two pieces are halved upon each other; *No. 1* and *2* the two pieces before the surfaces are brought in contact; *No. 3* and *4* exhibit another method. In this case the end of one piece does not pass the outer surface of the other.

Fig. 2, methods of joining timbers where the end of each one piece passes the end of the other at a small distance.

Fig. 3 shows how two timbers are joined by metering them together. In this case the two pieces ought to be fixed together with a bolt at right angles to the metre joint.

Fig. 4 how one piece of timber is joined to another, when one of the pieces is extended on both sides of the other piece.

NAKED FLOORING.—PLATE I.

THIS Plate exhibits the construction of a double floor.

The large piece of timber in the middle is called a *girder*.

The pieces of timber which are supported by the wall and the girder, and which appear to run in a transverse direction to the girder, are called *binding joists*.

The pieces of timber which are supported by the binding joists, and fixed near the lower edges of them, and which are parallel to the girder, are called *ceiling joists*.

The pieces of timber that appear upon the walls, and which support the binding joists, are called the *wall plates*.

The pieces of timber that run parallel to the girder and over the binding joists, are called *bridging joists*.

The bridging joists are those next to the wall where the chimney is shown, and prevent the binding joists and ceiling joists from being seen. For this reason the binding joists are removed in the other half, in order to show the binding and ceiling joists.

Fig. 2, *No. 1*, shows the plan of the floor, and *No. 2* a section of the same.

NAKED FLOORING.—PLATE II.

Fig. 1 exhibits the manner of joining the binding joists and girders.

No. 1, girder with binding joists on each side of it joined together.

No. 2, binding joist prepared for being joined to the girder, No. 3.

Fig. 2, section of the girder, with one binding joist inserted, and the other ready for insertion, as in *fig. 1*, No. 2 and 3.

Fig. 3, section of the binding joists, showing the notches for the bridging and ceiling joists.

Fig. 4 exhibits in No. 1 the bridging and binding joists joined together.

No. 2, the bridging joist with the lower edge uppermost, in order to show the notch.

No. 3, binding joist, exhibiting the notches.

Fig. 5 exhibits the method of tumbling in a joist between two binding joists or girders, when the joists are to be framed flush at the top.

The method is thus:—lay the top of the joist which is to be framed in, undermost upon the girders or binding beams; then with a sharp pointed instrument draw a line upon the upper side, now undermost, close to the adjacent faces of the girders or of the binding joists; then turn the edge now uppermost underneath, and shift the joist in a direction of its length until the line drawn on the top fall in the plane of the surface of the girder, or that of the binding beam. Place a straight edge so that it will at once coincide with the vertical surface of the girder and that of the joist; in this position draw a straight line, which is the line for cutting the joist. The other end will be found in the same manner.

TIMBERS IN A ROOF DEFINED.—PLATE I.

THE pieces of timber which lie on the tops of walls, are called *wall plates*.

The pieces of timber which extend over the area and rest upon each wall plate, are called *tie beams*.

The piece of timber which rises from the middle of each tie beam perpendicular to it, is called a *king post*.

The two pieces of timber that branch out from the bottom of each king post, are called *struts*.

The piece of timber which rises obliquely from each end of the tie beam, and which meets the king post at the top, and which is supported in the middle by the strut, is called a *principal rafter*.

The piece of timber which is supported by the ends of the tie beams, and overtopping each wall plate, is called a *pole plate*.

The piece of timber on either side which is supported by the principal rafters at the middle, and parallel to the wall and pole plates, is called a *purlin*.

The piece of timber which runs along the tops of the king posts, is called the *ridge piece*.

CARPENTRY,

NAKED FLOORING.

PLATE II.

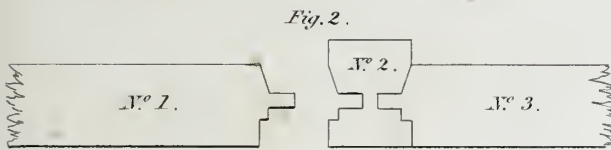
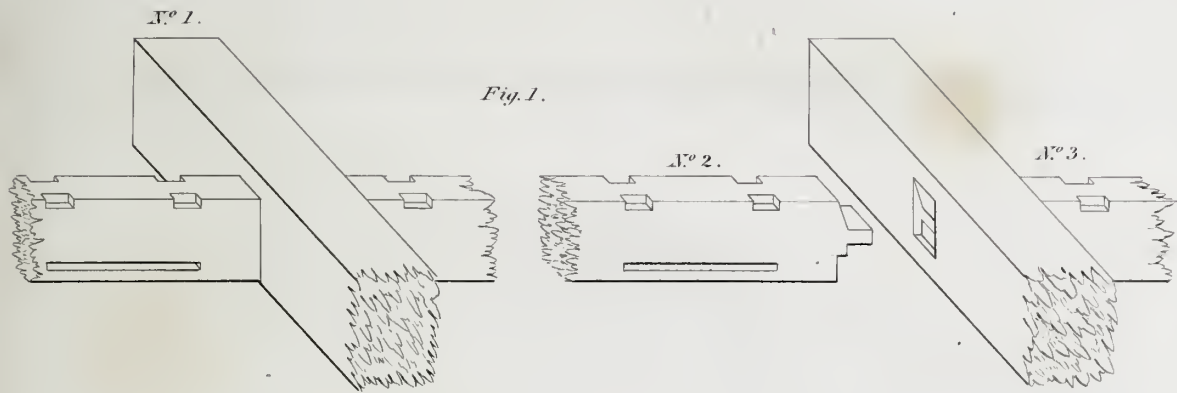
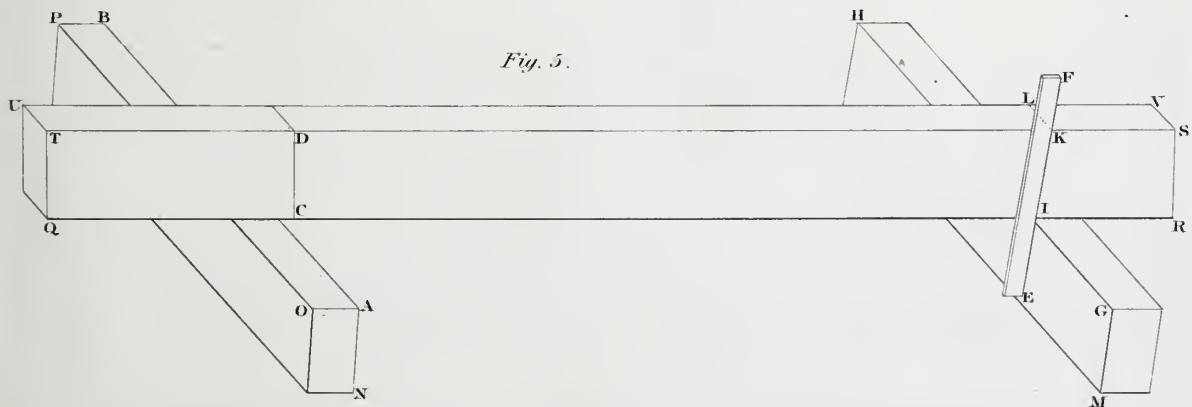
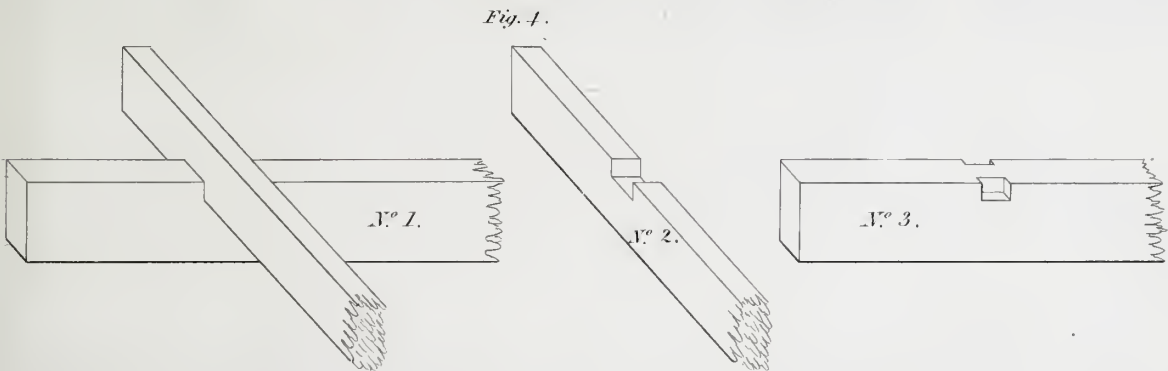


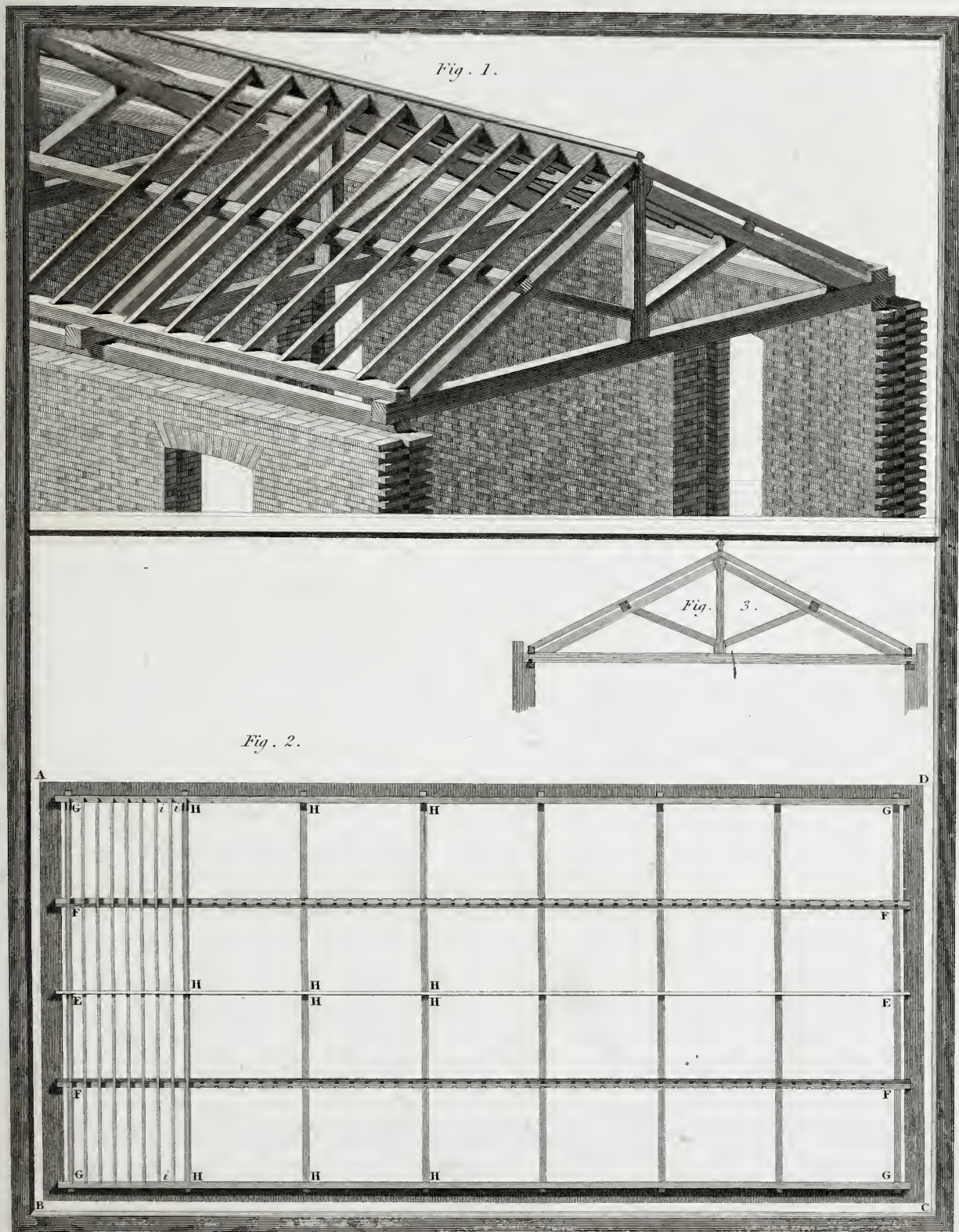
Fig. 3.

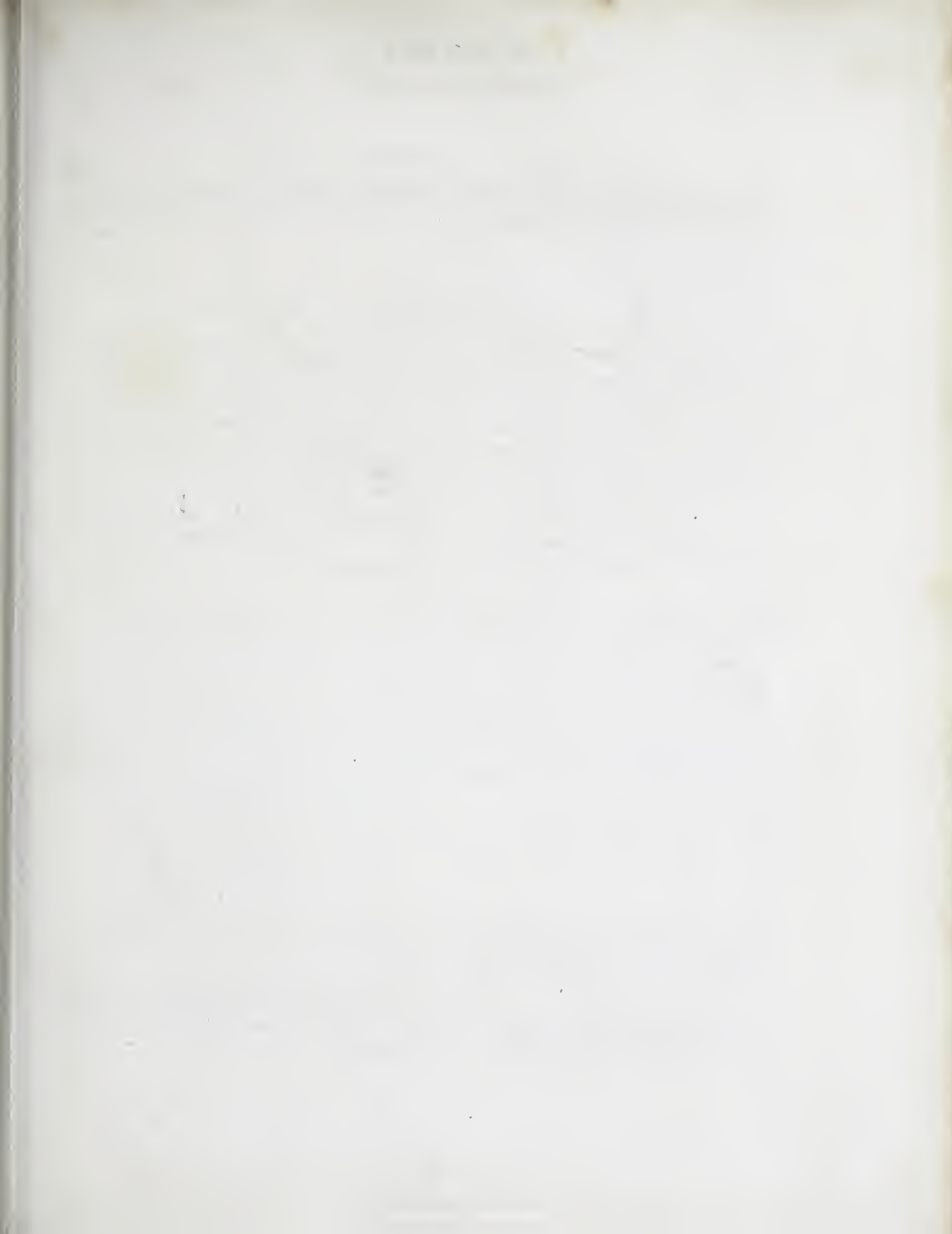




CARPENTRY,
TIMBERS OF A ROOF DEFINED.

PLATE I.

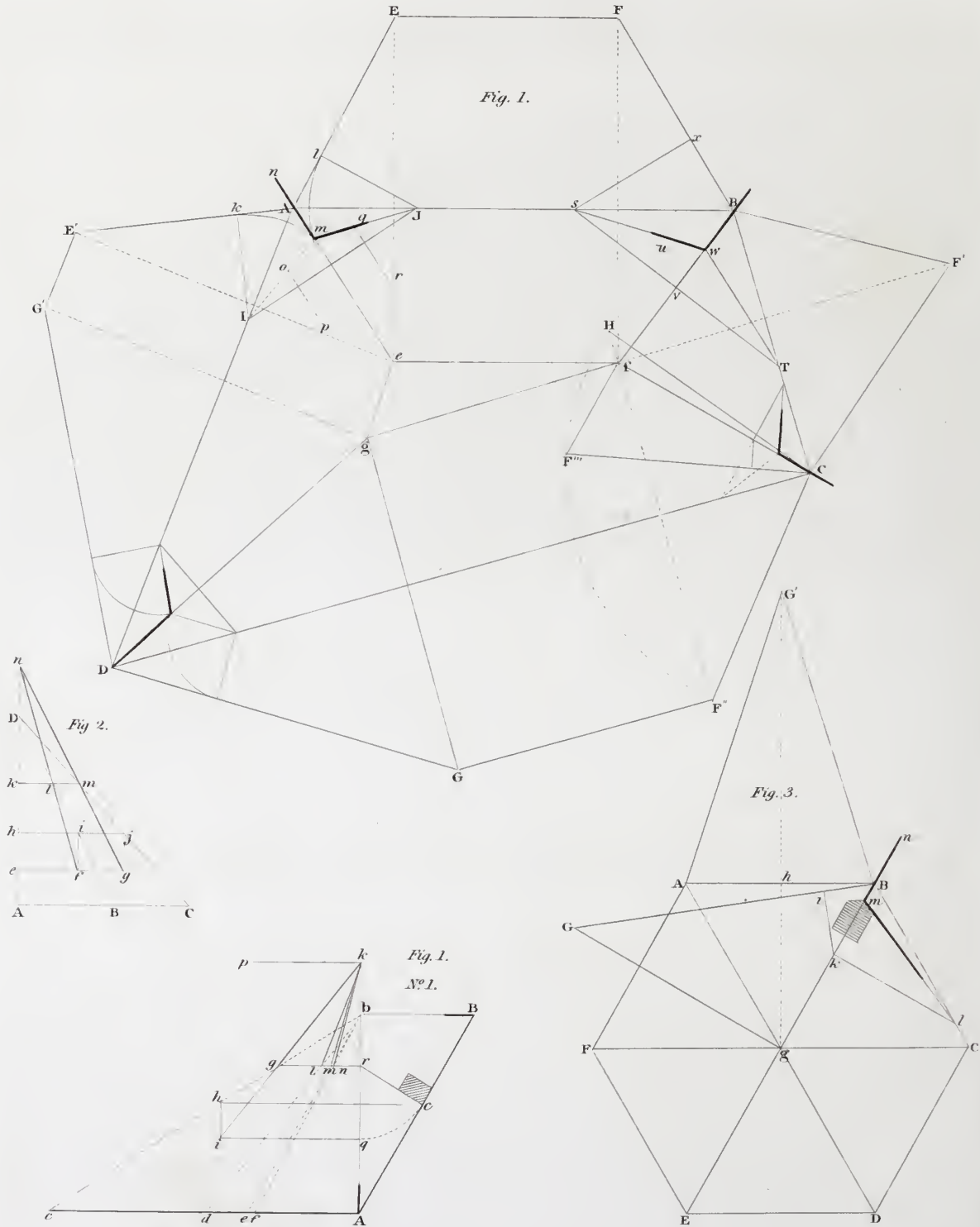




CARPENTRY.

CONSTRUCTION OF HIP ROOFS.

PLATE I.



The pieces of timber on either side which are supported in the middle by the purlin, at the bottom by the pole plate, and at the top by the ridge piece, are called *common rafters* or *spars*.

CONSTRUCTION OF HIP ROOFS.—PLATES I, II, III, IV.

Fig. 2 a plan of the above roof; *Fig. 3* one of the principal frames called a *truss*.

Plates 1, 2, 3, 4 exhibit the construction of hipped roofs. By means of the plan we have the seats of all the lines; therefore if we have the heights of these lines we can easily find their lengths by drawing a right angled triangle, of which one of the legs is the seat of the line on the plan, and the other the difference of the heights of each end of the line from its seat.—See Orthopjection.—We may find all the angles from the principles of projection; but the method employed throughout all the figures exhibited in the four plates is that used in the construction of trehedrals, *fig. 3, Plate 4, No. 1*.

Draw *Hi* parallel to the side *CD*, meeting the seat *ge* of the edge of the slate in *i*. In *ge*, make *il* equal to the height of the roof, and draw *HI* for the length of a common rafter.

Let *k* be the upper angle of the end of a purlin in *HI*; draw *mn* parallel to *HI*; from *k*, with any convenient radius, describe the semicircle *mlon*. Draw *kl* perpendicular to *HI*, and *ms, nu* parallel to *CD*; draw also *lr, kg, and ot* parallel to the height of the roof, and join *AE*. Draw *AB* perpendicular to *Ao*, and *ef* perpendicular to *DC*, *fig. 3*, meeting *CD* in *I*. In *AB*, make *Ad* equal to *Dl, fig. 3*, and join *de*. Let *g* be the place of the purlin in the rafter *AE*. Draw *gf* perpendicular to *Ae*, meeting *Ao* in *f*. From *f* with the radius *fg* describe the arc *gh* meeting *Ao* in *h*. Draw *fn* and *hl* parallel to *AB*, and draw *gk* parallel to *AB*, meeting *de* in *k*, and draw *kl* parallel to *Ao*, and join *il*; then *hil* is the inward bevel as required.

But we may here find the other three inward bevels of the purlin for *fig. 3* in a much more rapid manner than that now described.

For this purpose, draw *g2* perpendicular to *CD*, meeting *CD* in *2*. Also draw lines from *e* and *f* perpendicular to *AB*.

Proceeding now with *No. 1*: In *AB* make *Ac* equal to *C2, fig. 3*. Produce *li* and *Ae* to meet each other in *o*, and join *co*, meeting *fn* in *m*; then *fio* is the inward bevel of the purlin on the hip represented by its seat *De, fig. 3*, to *CD*, meeting *gC* in *r, q, t*. Draw *rs* and *tu* parallel to *Hi*. Join *qs* and *qu*; then *pqs* will be the inward bevel of the purlin, and *pqu* that which is applied in the side of the roof.

Fig. 1, Plate 1, Construction of Roofs, exhibits the developement of a roof upon an irregular quadrangular plan. The principle has been shown under the article Solid

Trigonometry. In such constructions it is much better to terminate the side of the roof in a plane parallel to the horizon, than to make it meet in a line which would be inclined to the horizon.

With regard to the inward bevel of the purlin, the method of finding it is more particularly shown in *fig. 1, No. 1*, which, though apparently different, is the same in principle as *fig. 3, No. 1, Plate 4*.

By *fig. 2, Plate 1*, it was intended to show the geometrical demonstration of this principle of finding the inward bevel of the purlin; we shall, however, pass over the investigation, leaving it to the learner.

It will not be necessary to explain every particular figure in the construction of hip roofs, as the same principle applies to all, whether the plans be regular or irregular; observing that, when a plan has all its angles equal, the finding of one backing of hip, or of one bevel of a purlin for any hip rafter, serves equally for all the other.

By the developement of the inclined sides of a roof, we obtain not only the bevels of the purlins in the sides of the roof, but the bevels at the top and bottom of every rafter and jack rafter in the most natural way.

Plate 2, fig. 1, Nos. 1, 2, 3, show how the bevels are applied in order to back the hip rafter. *No. 1* is a right section of the hip; in *Nos. 2 and 3* the point *a* is in the middle of the thickness at the upper end or back of the rafter; *No. 2* shows how the bevel is applied on one side from *a*; and *No. 3* shows how it is applied on the other side.

When the plan is irregular, as in *fig. 1, No. 1, Plate 3*, the most expeditious method of finding the lengths of all the hips is exhibited at *No. 2*. The construction is this:—Draw an indefinite straight line *At*, and draw *tT* perpendicular to *AT*. Make *tT* equal to the common height of the roof, and make *TA* equal to the seat of any rafter; then *AT* joined will be the length of that rafter. To apply this to the present example, make *tA* equal to *QA, No. 1*; *t2* equal to *QB* or *Q2, No. 1*; *t3* equal to *t3, No. 1*, &c. Join *AT, 2T, 3T*, &c.; then *AT, 2T, 3T*, &c. will be the lengths of the rafters represented by their seats *QA, QB, Q3*, &c. To find the backing of the hip for any one of these rafters, as that whose seat is *AQ, No. 1*:—In *No. 1* make the angle *QAt* equal to *tAT, No. 2*, and proceed as in the former cases.

To find the bevel of a purlin by the principles of projection:—In *fig. 3, Plate 3*, let *FGH* represent the angle of a purlin where *FG* is in the plane of the roof, and draw *Fl*, *Gm*, and *Hk* parallel to *AB*, meeting the seat *Bt* of the hip in *l, m, k*. Draw *ki* perpendicular to *Gm*, meeting *Gm* in *j*, and *Fl* in *i*.

Any where apart:—Draw the straight line *MN*. Through any point *J* in *MN*, draw *KI* perpendicular to *MN*. Make *JI* equal to *GF*, and *JK* equal to *GH*. Draw *IL* parallel to *MN*. Make *IL* equal to *il*, and *JM* equal to *jm*, and join *ML* and *MK*; then the two bevels for cutting the purlin are *NMK* and *NML*.

CONSTRUCTION OF HIP ROOFS.

Fig. 1.

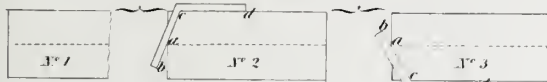


Fig. 2

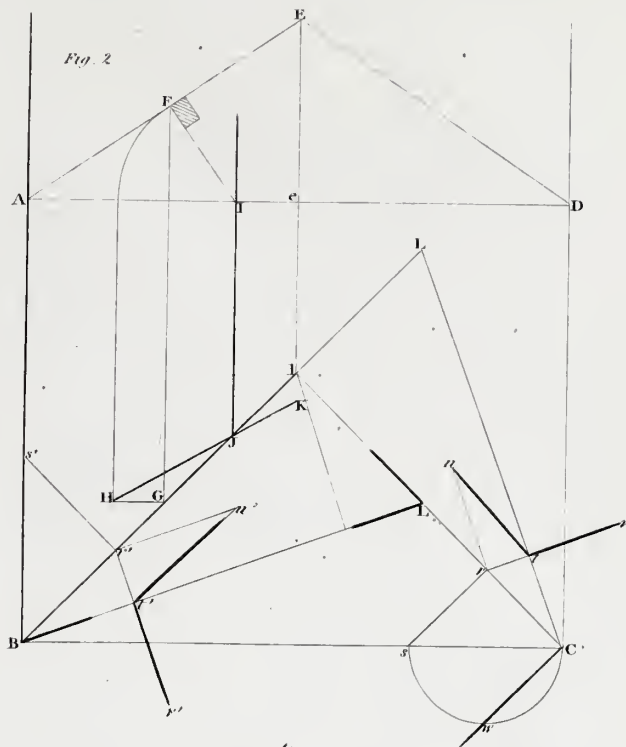


Fig. 4.



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CONSTRUCTION OF HIP ROOFS.

PLATE IV.

Fig. 1.

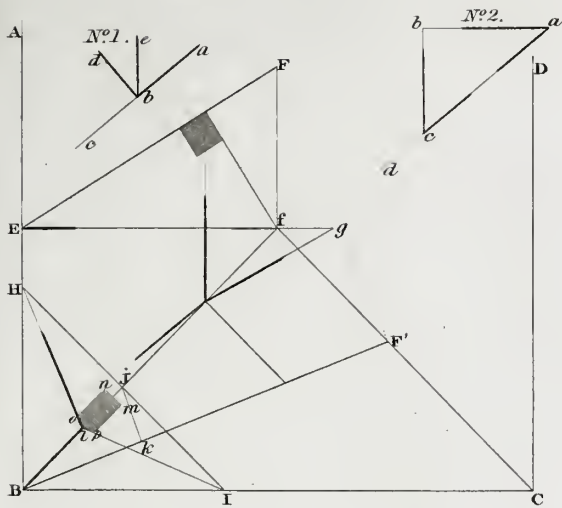


Fig. 2.

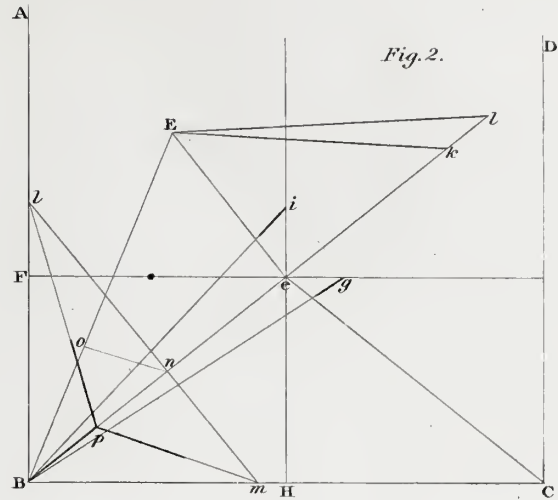


Fig. 3.

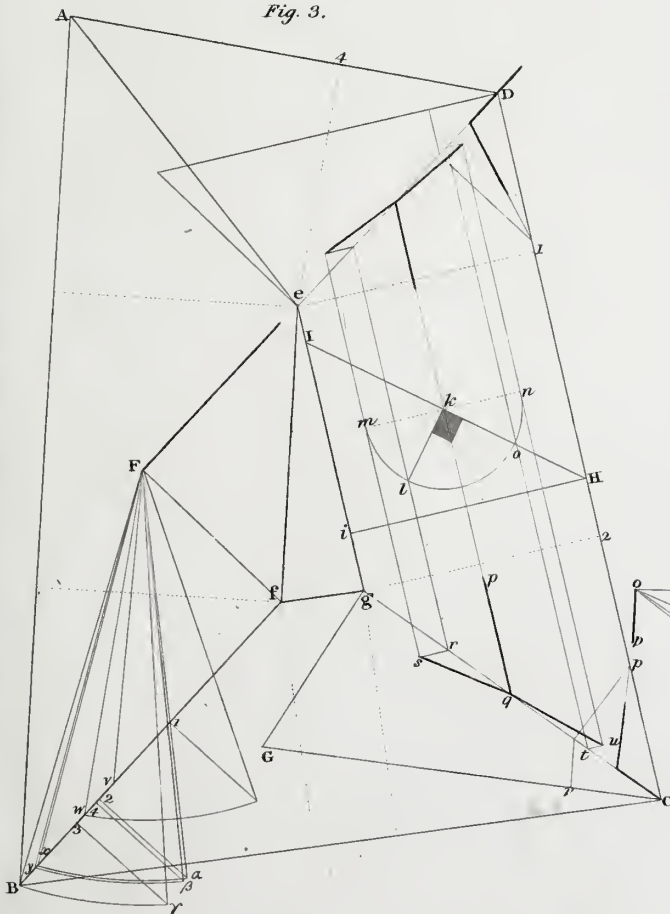


Fig. 2.

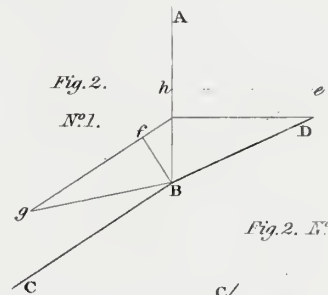


Fig. 2. Nº 2.

Fig. 3. Nº 1.

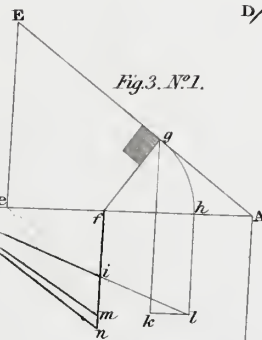
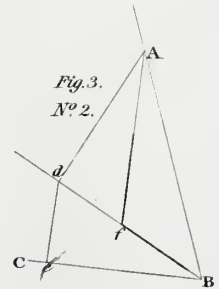


Fig. 3.
Nº 2.





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CONSTRUCTION OF HIP ROOFS.

PLATE III.

Fig. 1.
N^o 1.

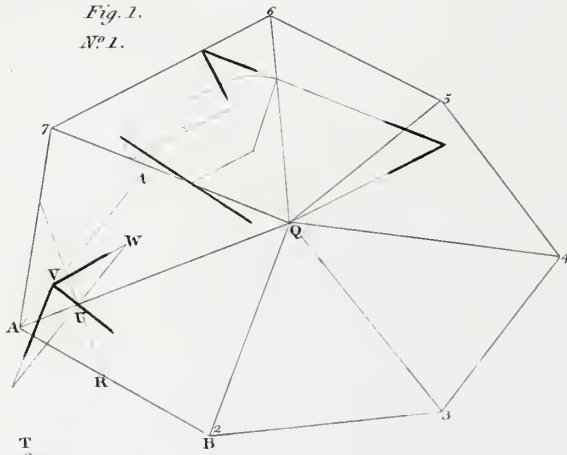


Fig. 2.

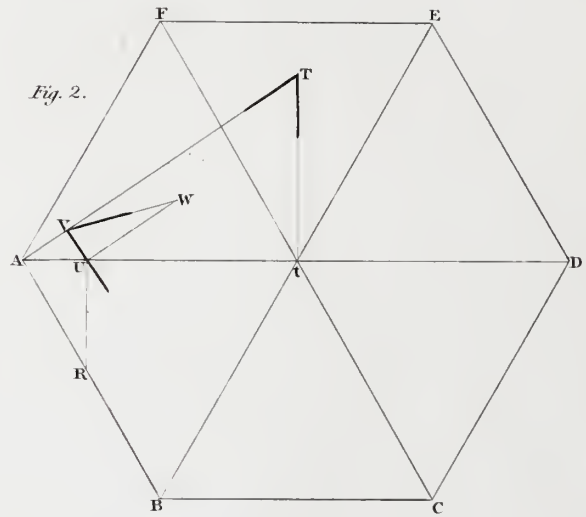


Fig. 1.
N^o 2.

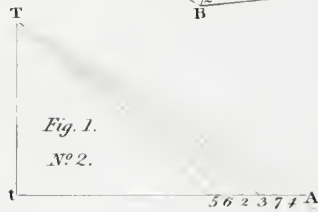
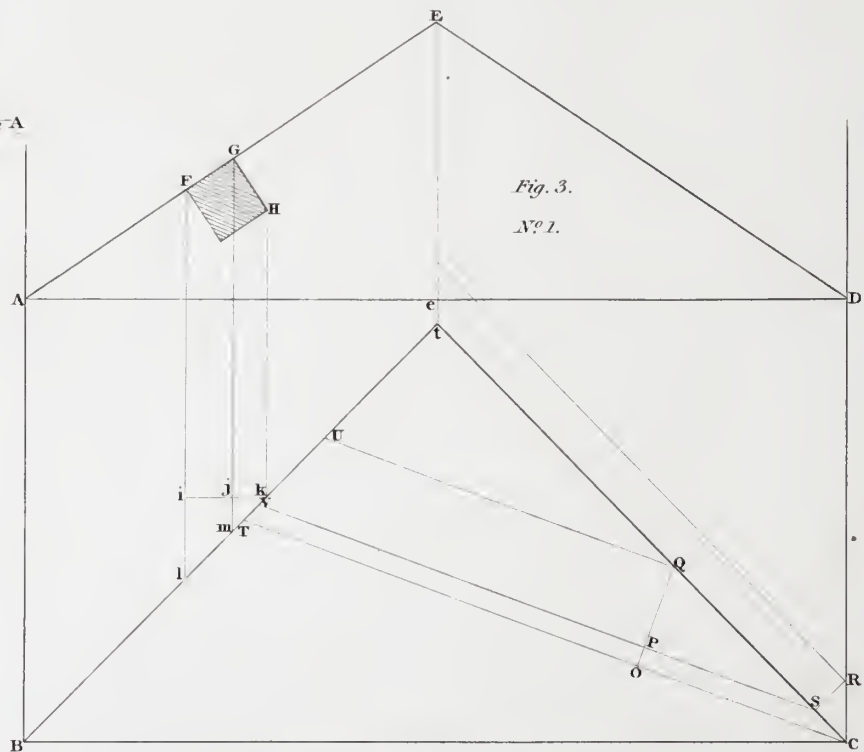
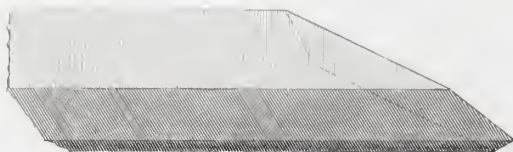


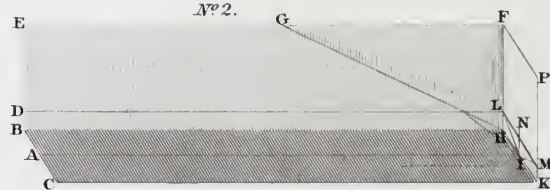
Fig. 3.
N^o 1.



N^o 3.



N^o 2.



A method of finding the backing of the hip without the principles of the trehedral is as follows:—See *fig. 3, Plate 3*.

Let it be required to find the bevel of the hip whose seat is Ct .

Make the angle tCT equal to the angle which the back of the hip makes with its seat, or because the angle CtB is a right angle; in tB make tT equal to the height of the roof, and join CT ; then will tCT be the angle which the hip makes with its seat.

Draw a line parallel to Ct , at a distance equal to half the thickness of the hip, meeting CD in R , and draw RS perpendicular to tC , meeting tC in S . Draw SV parallel to CT , and QU at a distance from CT equal to the breadth of the hip, supposed to be measured in a vertical plane. Draw QO perpendicular to CT , meeting it in O , and SV in P . Having made this preparation, we shall now show how to apply the lines to the hip itself.

Let No. 2 represent a part of the hip rafter at the lower end, made to the breadth and thickness of the hip rafter.

Let $BHFE$ be a rectangular plane representing one face of the hip which is to stand in a plane perpendicular to the horizon, and let $BHKC$ be another rectangular plane at a right angle with the plane $BHFE$, representing the top of the hip before it is backed, and let the plane $FHKP$ be the end of the piece.

In HF , No. 2, make HL equal to OP , and in KP make KM also equal to OP . Bisect HK in I , and join MI and LI ; then LIM is the backing of the hip. Draw IA and LD parallel to HB ; then cutting away the triangular prism $AILD$ on the one side, and the similar one AIM on the other, we shall form the back of the rafter as required.

CONSTRUCTION OF NICHES.

A NICHE is a recess in a wall for the purpose of placing some ornament in it.

The backs of niches are generally formed of cylindric and cylindroidic surfaces, and the heads of spherical, conoidal, or ellipsoidal surfaces.

When a conoidal or spherical surface is employed in the formation of the head of a niche with the axis of the conoid vertical, a cylindric surface, having for its axis the continuation of the axis of the conoid and for radius that of the base of the conoid, is employed.

The principles of spherical niches are the most simple possible.

As every section of a sphere is a circle, the form of the edges of the ribs for sustaining the lath and plaster of a spherical niche must be circular.

With regard to the position of the ribs of a spherical niche, when the sections of a

sphere all intersect each other in a line passing through the centre, the radial sections are all equal; therefore if the head of a niche be spherical, one of the easiest methods of constructing the ribs is to make them all intersect each other in a horizontal line passing through the centre of the sphere.—See Construction of Niches, *Plate 1, fig. 1*, where the ribs of the niches all meet in a horizontal line passing through the centre of the sphere. Here all the ribs have the same chords as the half plan.

In *fig. 2* all the ribs meet in a vertical line which divides the front rib into two equal parts. To find the radius of curvature of any rib, complete the circle of which the inside of the plan is an arc; produce the middle line of the plan of any rib, as of *di*, to meet the opposite side of the circumference in *g*; on the whole line *dg*, or its equal *fk*, as a diameter, describe a semicircle, and from the point *i*, where the ribs intersect, draw a perpendicular *ik* to meet the semicircular arc in *k*; and the portion *fk* of the arc intercepted between the perpendicular and the back of the niche will be the curve of that rib.

In *fig. 3* all the ribs are in vertical planes, and perpendicular to the front. Draw a line *Dg* through the centre *D* of the plan parallel to the seat of the front rib. Continue the line of the seat of any rib, suppose that of *fh* to *g*. Then from *g*, with the radius *gf*, describe the arc *fk* meeting the inside of the front rib in *k*. Draw *De* perpendicular to *fg*, meeting *fg* in *e*. From *g*, with the radius *ge*, describe an arc meeting the seat of the inside of the front rib in *i*; then the arc *fk* is the curve of the one side of the rib represented by the seat *hf*, and *ei* is the curve on the other side of the rib, and the distance between these two curves *fk* and *ei* exhibits the quantity of bevel which the edge of the rib requires.

In *fig. 4* the ribs are all parallel to the front rib, and consequently the diameter of any rib is the intercepted portion of that rib by the inner line of the plan.

Plate 2 exhibits the plan and elevation of a spherical niche.

Here the ribs are all in vertical planes passing through the centre of the sphere.

Let *lg* be the middle line of the seat of a rib, meeting the centre of the sphere in *g* on each side of the line *lg*; draw a line at a distance equal to half the thickness of a rib, and let these lines meet the seat of the inside of the front rib in the two points *o*, *P*. Draw *Oo'*, *Pp'* perpendicular to *lg*, meeting the inner edge of the arc of the plan in two points *o*, *p*; then *lop* is the length of this rib, and the distance between the lines *oo'*, *pp'* shows the quantity of bevel to be taken off the top to fit against the front rib.

Nos. 1, 2, 3 show the ribs by themselves as found by their seats.

In *Plate 6* the plan and elevation are each the segment of a circle, and consequently the two chords must be equal. In order to describe any rib:—Through the centre *H* of the plan draw a straight line, and produce the arc of the inside of the plan to meet this line, and the intercepted distance will be a chord. Draw the line *HZ* perpendicular. Make *HZ* equal to the difference between the radius and versed sine of the front rib. Join the extremity *W* of the chord and *Z*; then, with the radius *ZW*, from *Z* describe an

CARPENTRY.

CONSTRUCTION OF NICHES.

PLATE 1.

Fig. 1.

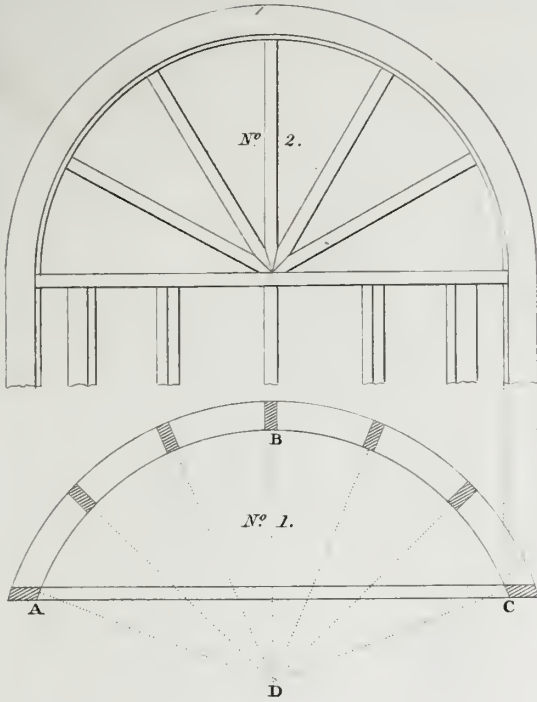


Fig. 2.

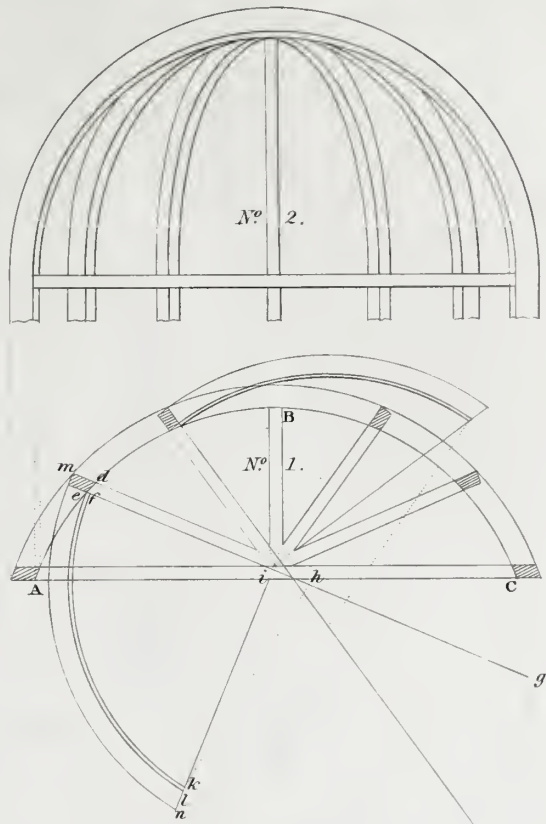


Fig. 3.

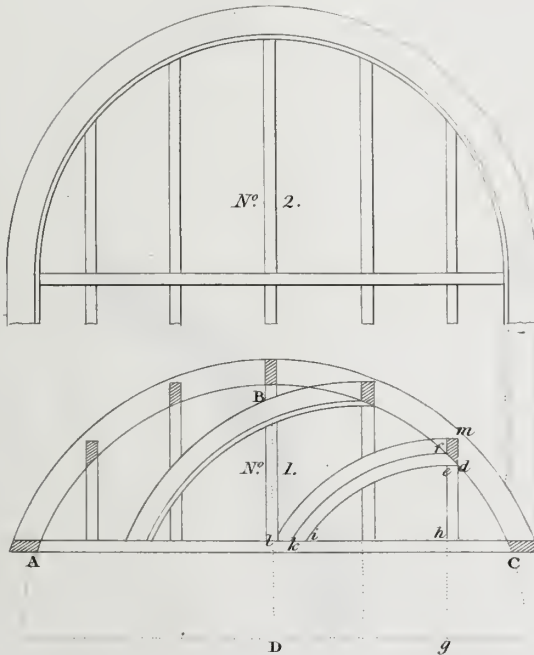
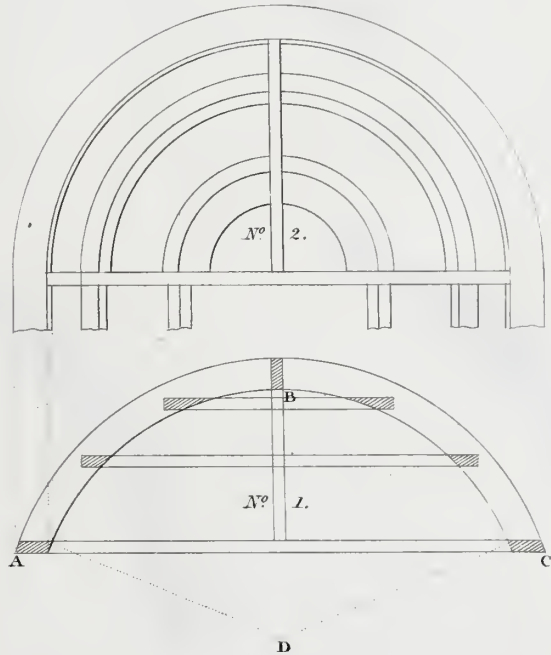


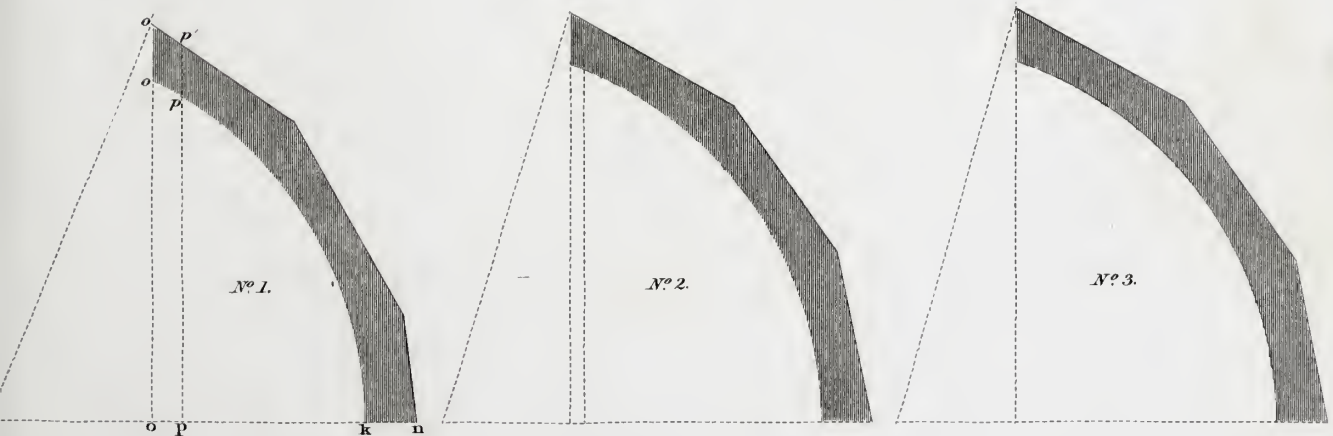
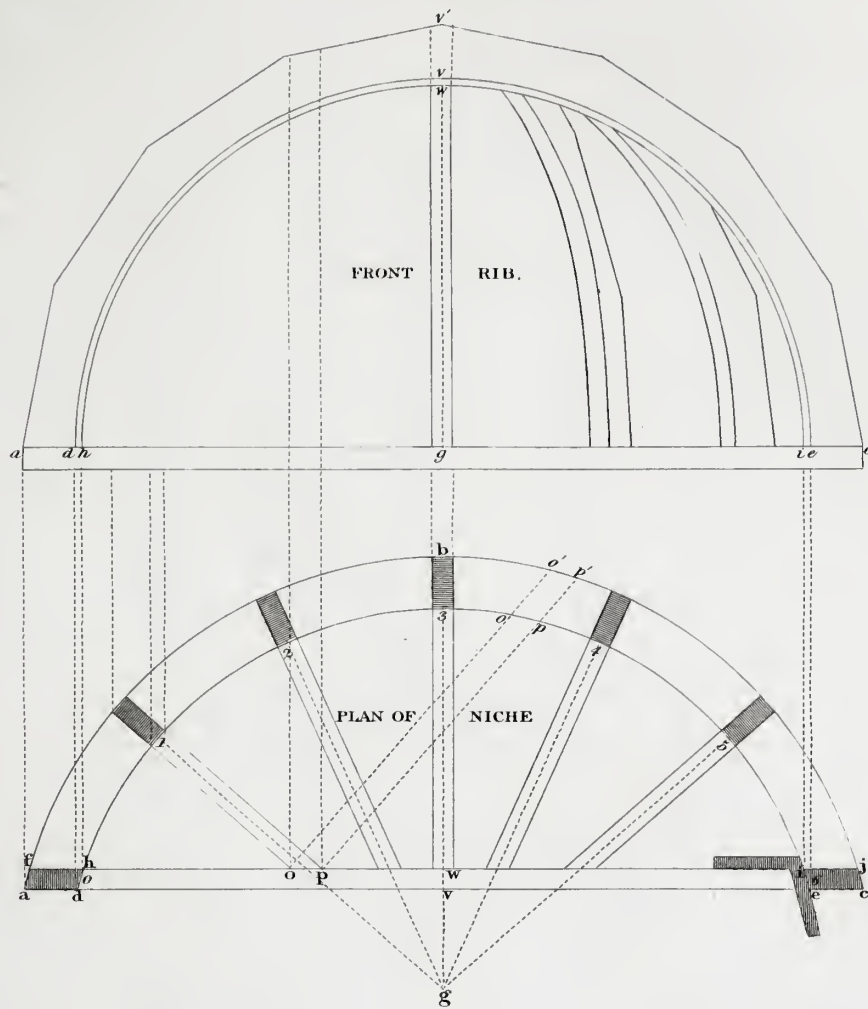
Fig. 4.



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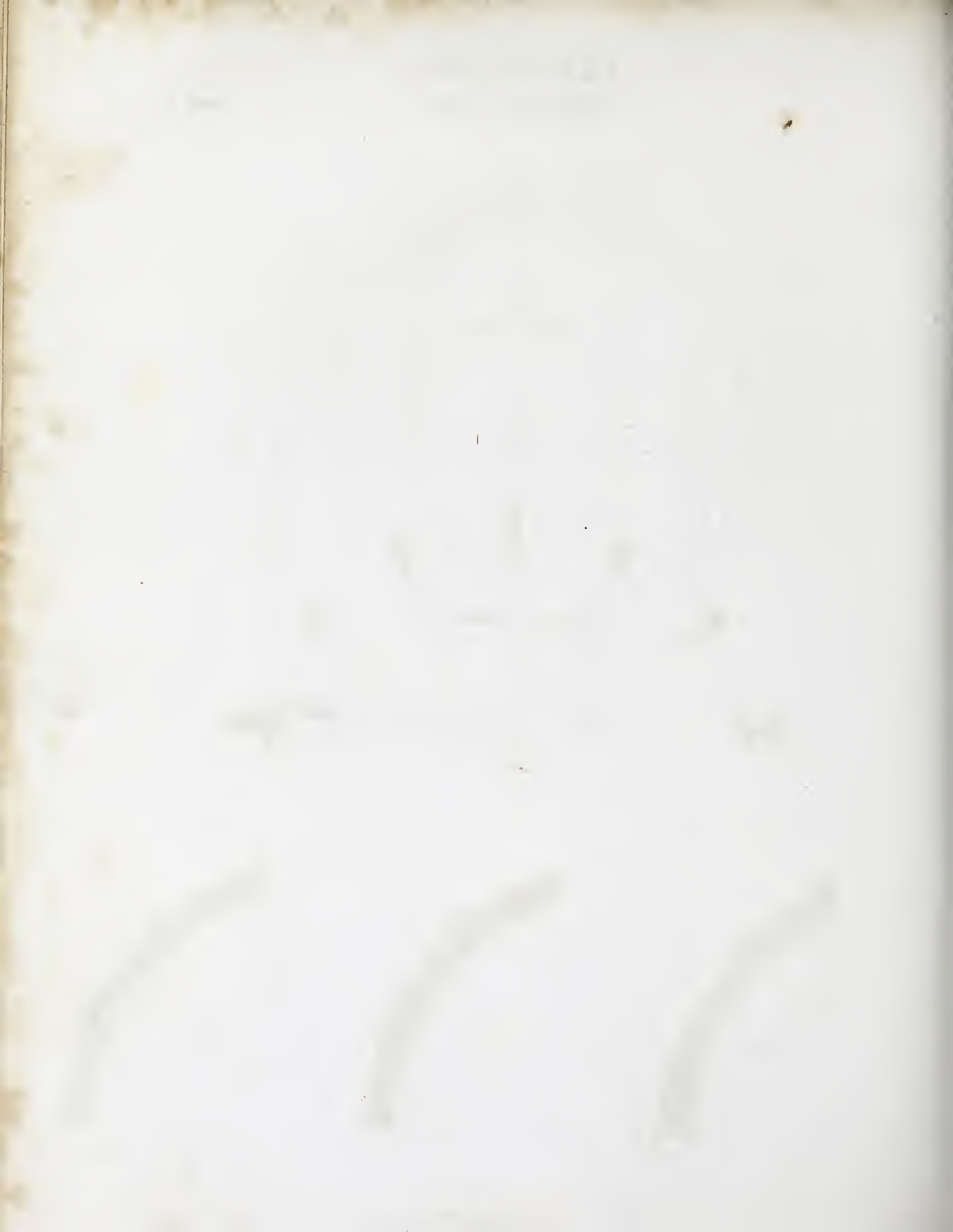
CONSTRUCTION OF NICHE.

PLATE II.



Invented & Drawn by P. Nicholson.

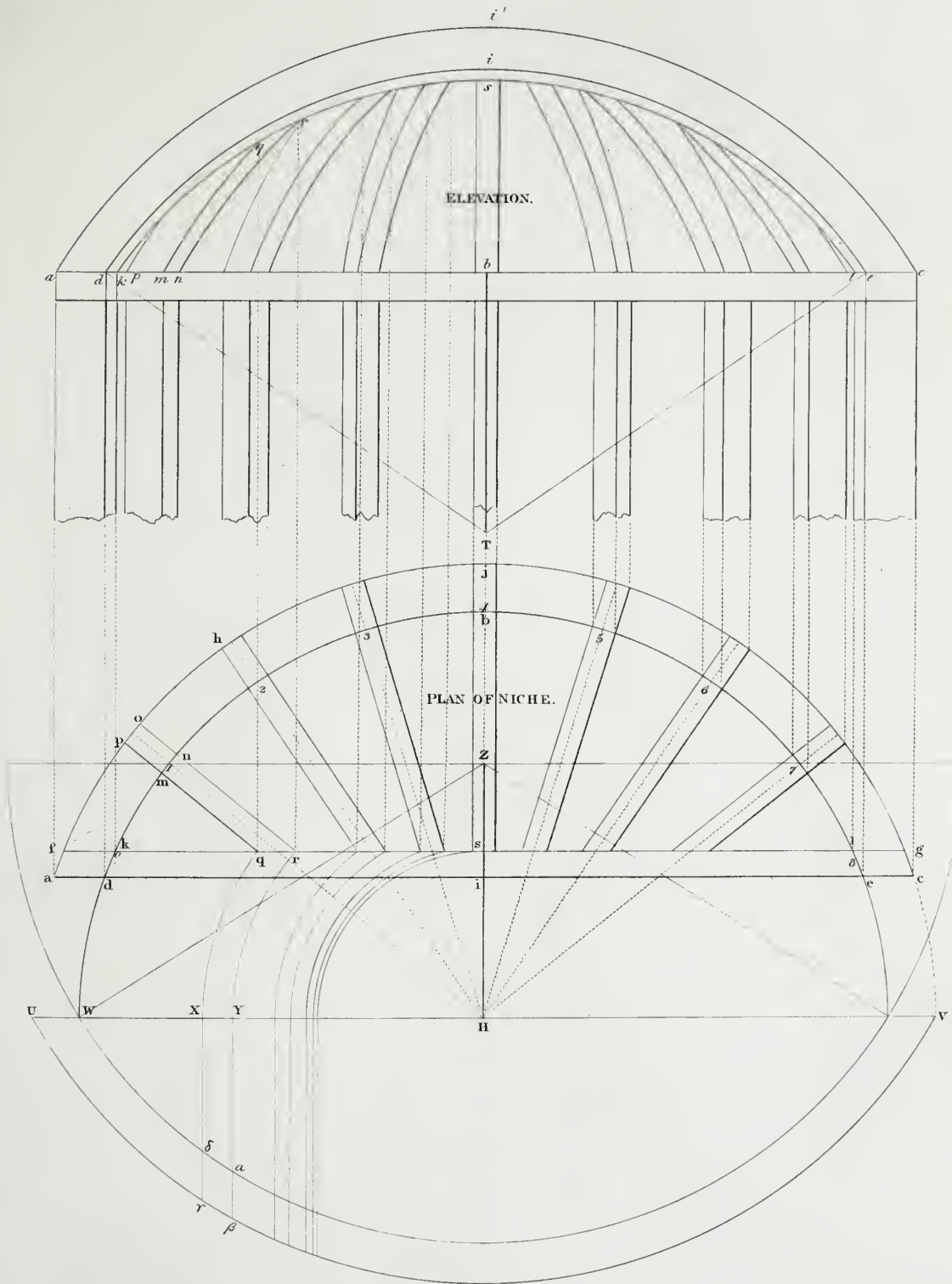
London Published by John Day & Co. June 20 1822.

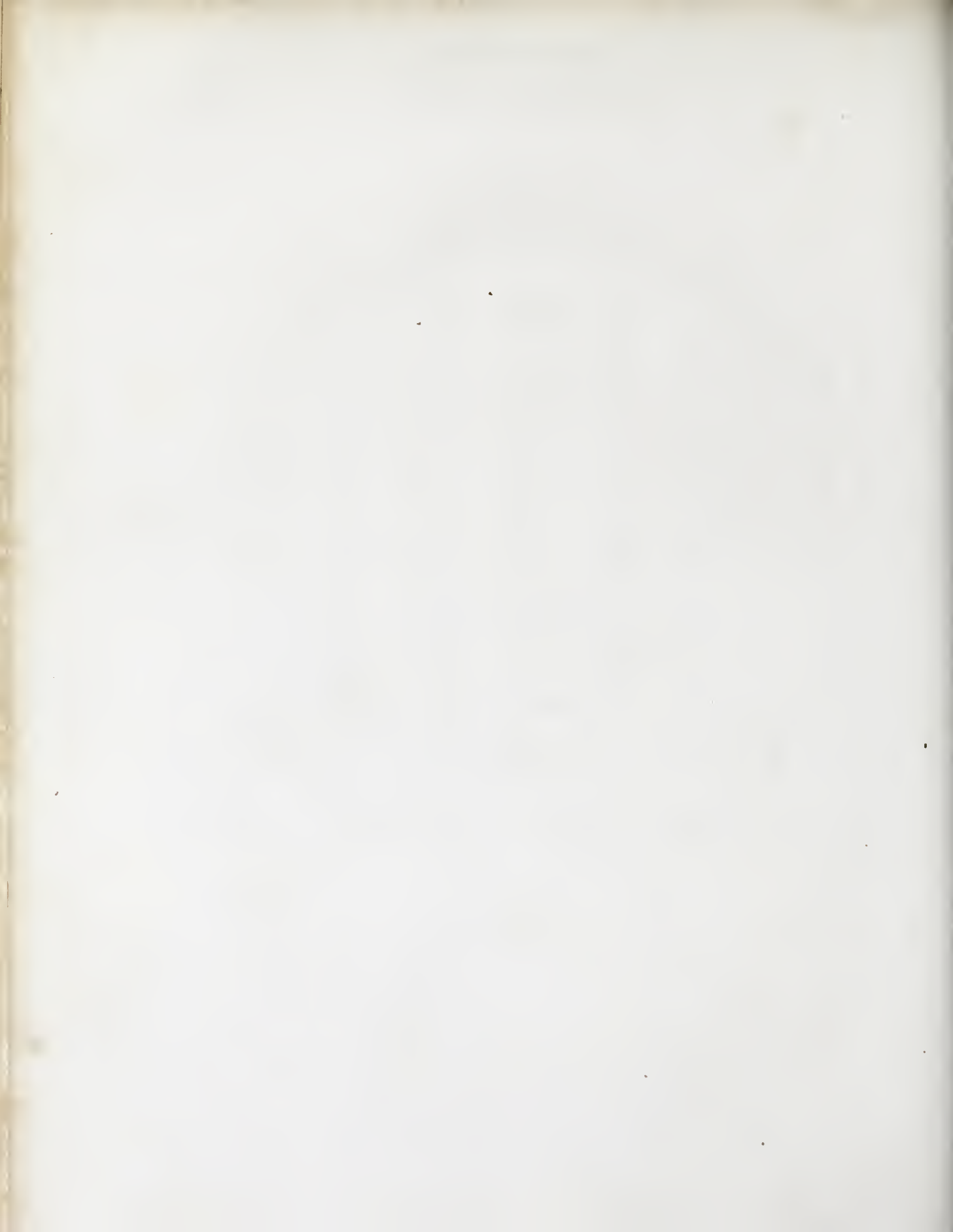


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CONSTRUCTION OF NICHE.

PLATE VI.

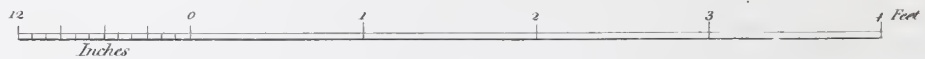
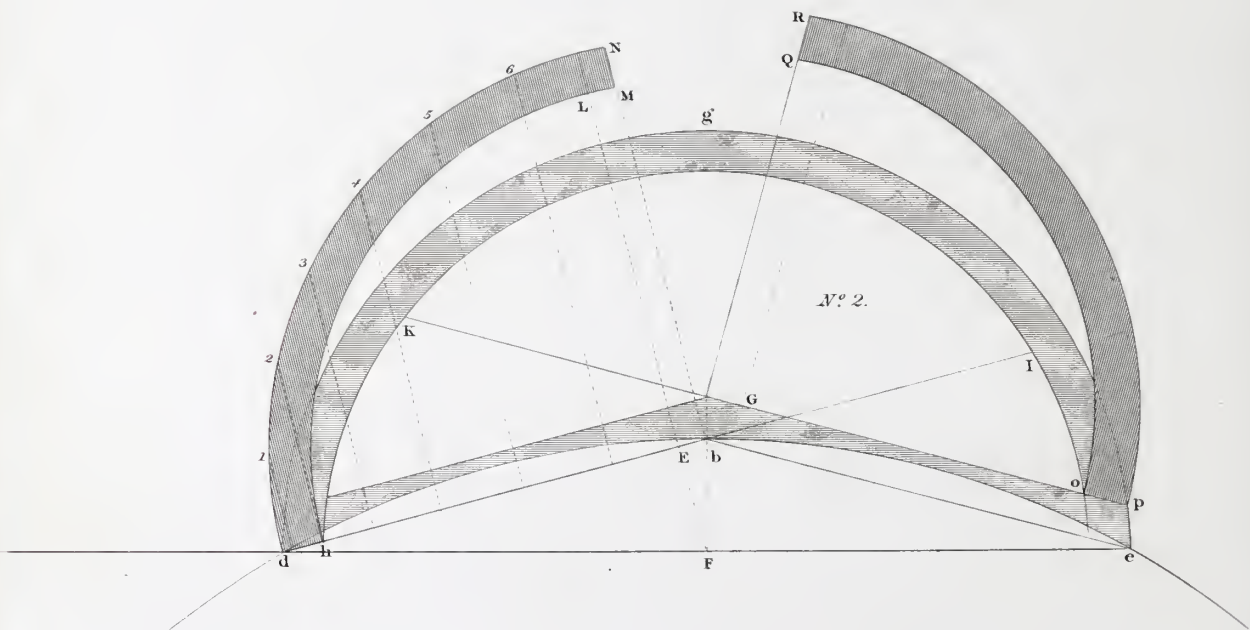
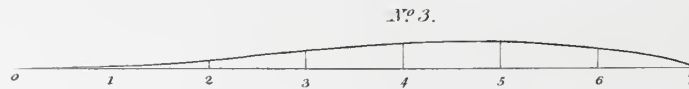
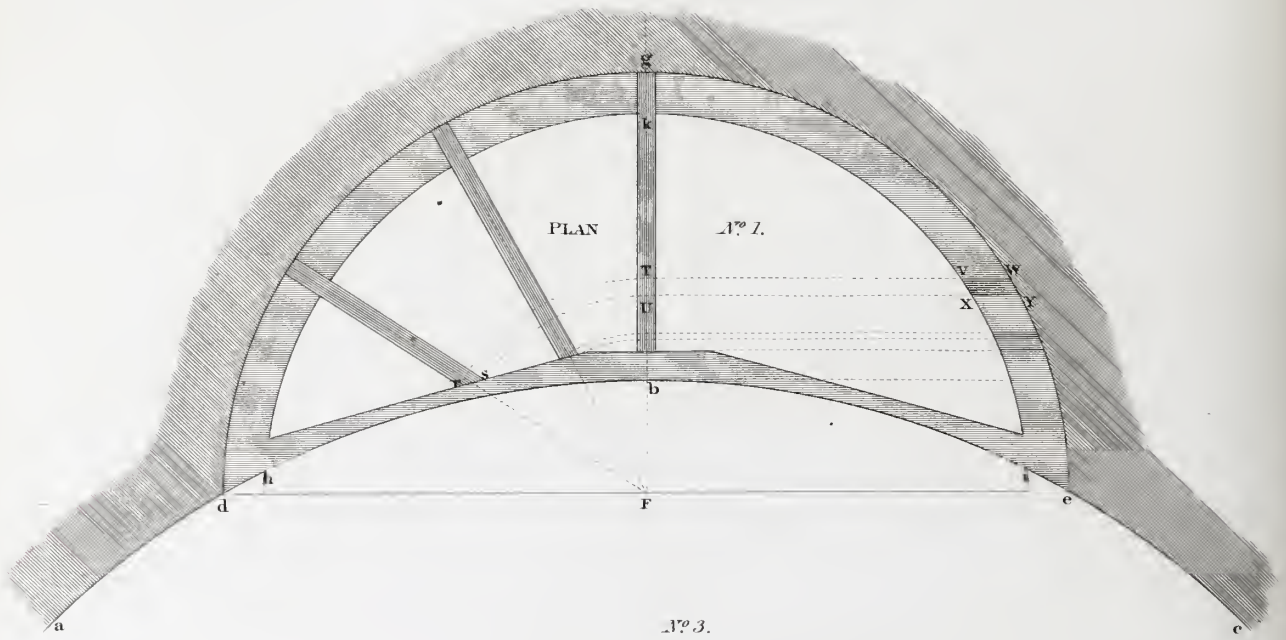




CARPENTRY.

CONSTRUCTION OF NICHES.

PLATE VII.



Imposed & Drawn by T. Prosser

arc which is the curvature of all the ribs; the length of any rib may be found by applying the length of the seat of that rib from the extremity *X* of the chord upon the half chord *WH*, and, drawing a perpendicular to meet the arc, the portion of the arc between *W* and the point where the perpendicular meets it, is the length required.

Plate 7 represents the plan and moulds for cutting out and preparing the ribs of a spherical niche in a circular wall on true principles, and in a more comprehensive and simple manner than any thing of its kind ever before published.

No. 1 represents the plan, and *dge* the back or base rib; *dbe* the plan of the front rib, which should be got out in two or more parts, and jointed perpendicularly over the letter *b*: let the distance between the lines *be* and *GO*, *fig. 2*, represent the thickness of the plank required to cut out the front rib; let *dhMN* represent the mould for marking the front side of the plank, which may be obtained as follows:—

Draw *db*, and continue it to *I*; draw *bM* perpendicular to *dI*; let *F* be the centre, and draw *FL* parallel to *bM*, crossing the line *dI* at *E*; then *E* will be a centre for describing the mould *dhMN*.

The mould for the back side of the rib may be obtained by continuing the line *OG* to *K*; then *G* is the centre required for describing the mould *OPQR*.

Take the mould *dhMN*, and mark each side and end on the face of the plank; then cut it off truly to the line *dh*, and at right angles to the face of the plank: from the line *be* take the bevel at *o*, and apply it to the end of the plank at *h*, which will give one point at the back edge of the plank for applying the mould.

From the line *db* take the bevel of the joint at *b*: cut the end of the rib to the line *MN*, and to the bevel found at *b*.

From *b*, *No. 1*, draw a line at right angles to the line *FG* until it cuts the inside of the plan, which line and the curve of the plan will give the bevel of the under-side of the rib at *M*, which will give another point at the back side of the plank to apply the mould *OPQR*; then work to the lines so obtained, which will give the upper and under side of the rib.

Divide the curve line *dN*, *No. 2*, into any number of equal parts, and from those points draw lines parallel to *EL* across the face of the mould, and continue them to the line *db*; transfer the lines crossing the mould to the face of the rib itself, and take the corresponding distances from between the straight line *db* and the curve line *db*, and transfer them to the upper and under sides of the rib; trace curves through the points so found, which will stand perpendicular over the plan. Or,

Take the stretch out of the curve line *dN*, and lay it out on a line as at *No. 3*; take the corresponding distances from the line at *No. 2*, and trace a curve through them; then *No. 3* will be a mould that will bend over the top of the rib, and give the curve required. Another may be obtained for the under side of the rib by proceeding in the same manner.

Plate 8, fig. 1, a niche upon a circular plan and elliptic elevation.

In this the ribs are set in vertical planes, and, though elliptic, are all of the same curvature.

In *fig. 2* the plan is an ellipse, and the head a semicircle. The ribs are here fixed in planes perpendicular to the plane of the wall; the position of the ribs in this diagram is exactly the reverse of that in *No. 1*. The ribs of this niche are all of the same curvature and the same length, being each a quadrant of an ellipse.

In *fig. 3* both plan and elevation are elliptic. The ribs are vertical and perpendicular to the face of the wall. The method of finding the ribs is shown in the Primary Problems.

In *fig. 4* the ribs are disposed in vertical planes parallel to the front rib; the plan is a semi-ellipse, and the elevation a semicircle.

PENDENTIVE CEILINGS.

PLATE IV.

Let *abcd*, *fig. 1*, be the plan of a room or staircase to be bracked, so as to form the surface of a pendentive ceiling. Let *aSVe* be the section across the diagonal; it is required to find the curvature of the springing ribs.

Draw *iL* perpendicular to *ab* meeting *ab*. Take the distance from *i* to the line *ab*, and set it from *i* on the line *ia*, and from this point draw a perpendicular to meet the curve *aS* of the diagonal rib. Make the versed sine of the segment *aLb* equal to the perpendicular, and describe the segment *aLb*, which is the springing line required. If from the centre *i* an arc be described with a radius equal to the length of the seat of a rib to meet the seat of the diagonal rib *ia*, and if from the point of meeting a perpendicular be drawn to meet the curve *aS*, the portion of the arc of the diagonal rib between *a* and the perpendicular will be the length of the rib corresponding to the seat which was taken.

In *figs. 2* and *3* the diagonal section is a semicircle.

In *fig. 4* the plan is an octagon, and therefore the springing lines on the sides are semicircles.

Plate 6 (subtitled Pendentive Ceilings).

Here the same principle is observed as in *Plate 4*, and the bracket from which all the others are taken is the quadrant of a circle.

The portions of the brackets to be used are exhibited at *No. 1, 2, 3, 4, 5, 6*, found according to the length of their seats.

In *fig. 2* the bracketing is upon an elliptic plan, and the general bracket, from which all the others are portions, is the quadrant of a circle as before. The portions of the brackets to be used over each seat are shown at *No. 1, 2, 3, 4, 5, 6, 7*.

In *Plate 7* the springing lines are in the sides of a square pyramid, of which its vertex is

Fig. 1.

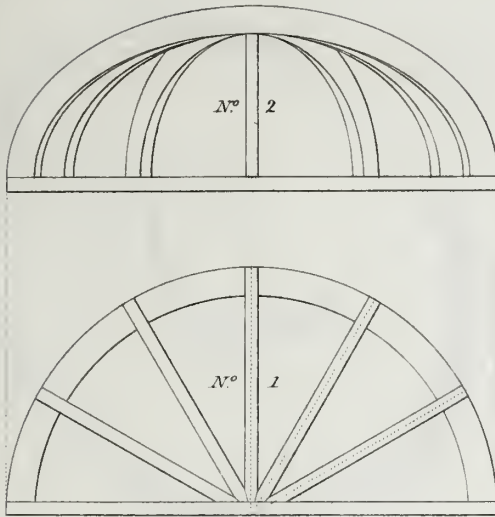


Fig. 2.

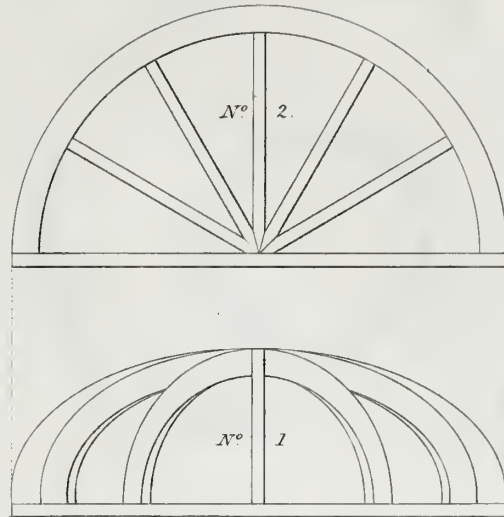


Fig. 3.

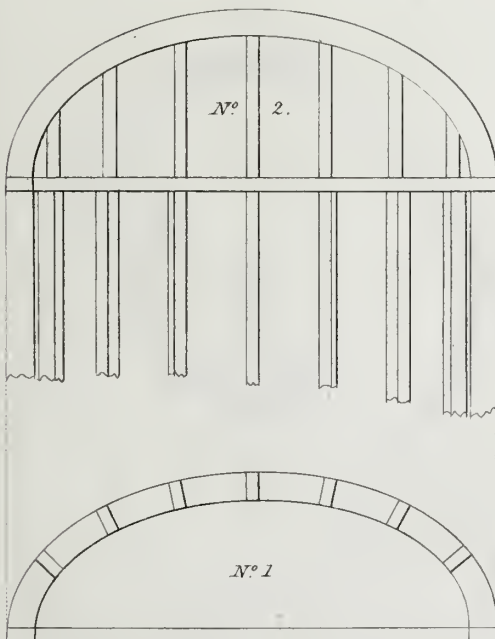
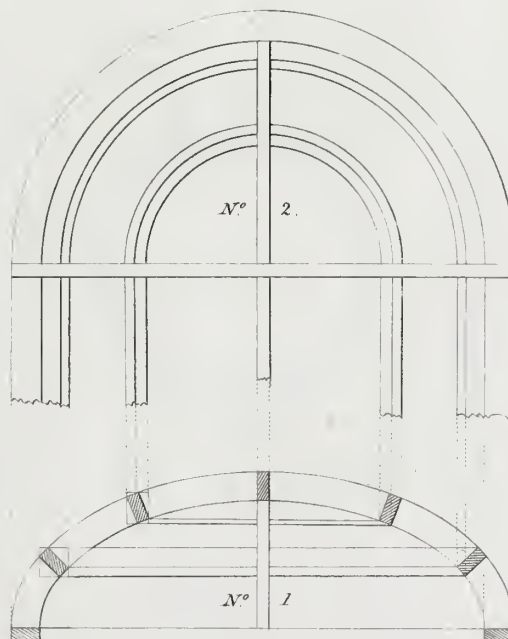


Fig. 4.



CARPENTRY.

PENDENTIVE CEILINGS.

PLATE IV.

Fig. 1.

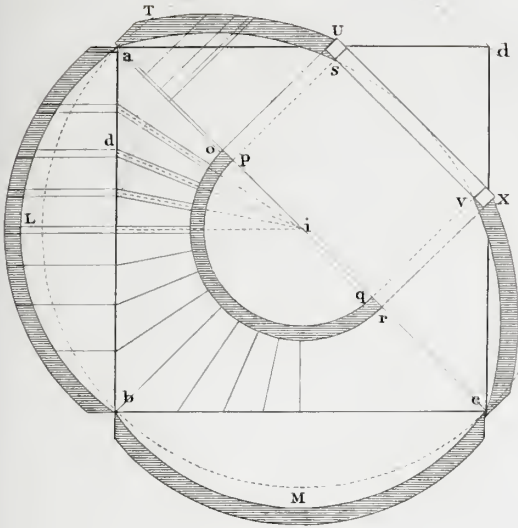


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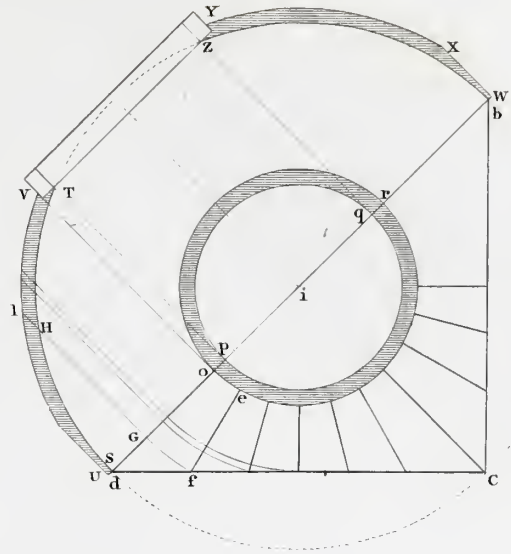


Fig. 3.

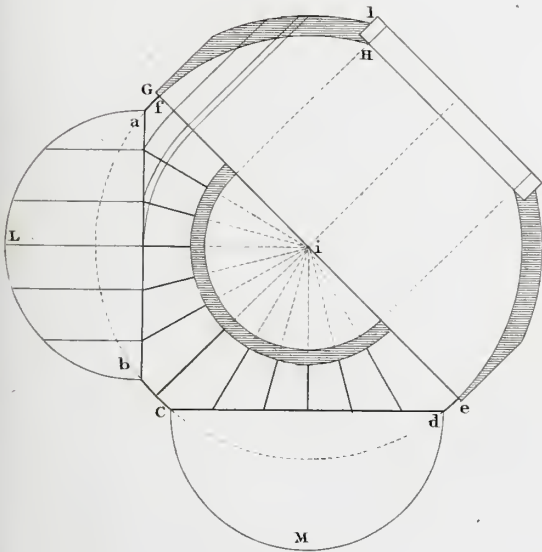
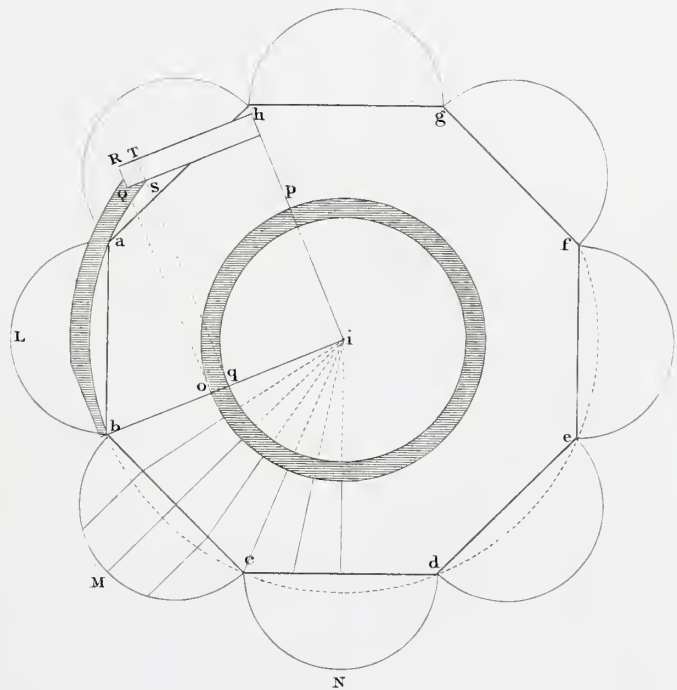


Fig. 4.

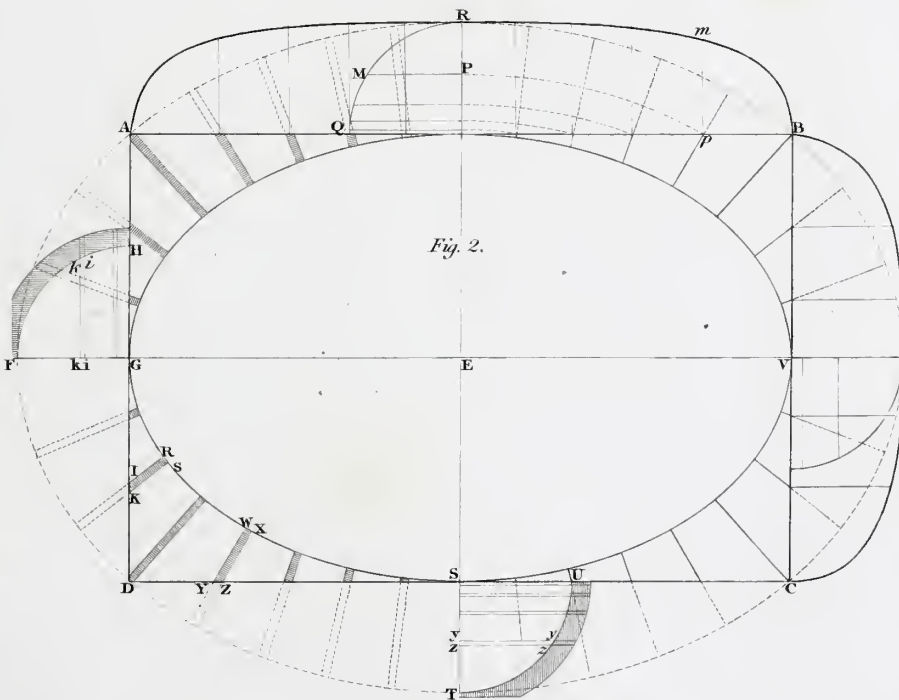
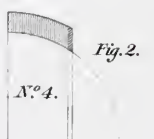
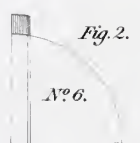
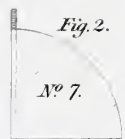
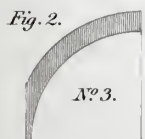
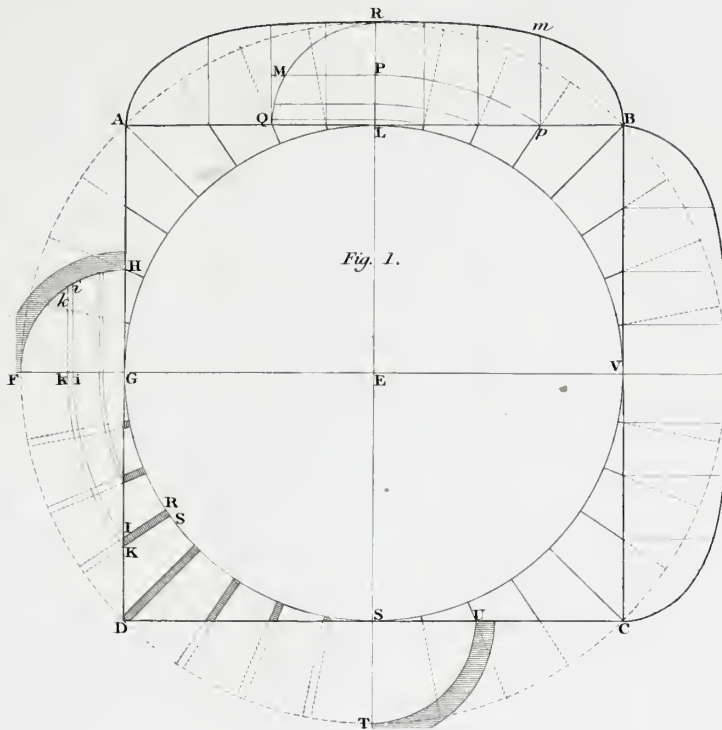
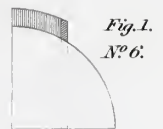
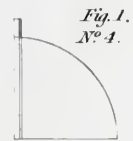


Invented & Drawn by F. Nicholson.

CARPENTRY.

PENDENTIVE CEILINGS.

PLATE VI.



PENDENTIVE CEILINGS.

[illegible]

Engraved by W. Low

London, Published by John Day & Co. Oct. 20, 1822.

above the base $abcd$. The edges of the pyramid are tangents to the springing lines at their lower extremities, which are represented by the seats a, b, c, d .

We shall now suppose that any one of the four angles contained by any face and the base of the pyramid is given, the angles which the edges of the pyramid make with the edges of the base will be readily found by the principles of the 'Trehedral, Primary Problems. One of these angles is daH' , and the angle which the seat aC makes with the adjacent angles is gaH .

In order to find the springing curve upon the line ad , we have only to describe a circle aEd to touch the straight line aH' at the point a .

Draw gE parallel to ab , meeting the arc of the circle in E ; Ei' parallel to da , meeting aH in i' ; $I'i$ parallel to ab , meeting the diagonal ac in i ; and ie parallel to ad , meeting gE in e .

We know that, since the springing curve aEd is the arc of a circle, it will be projected into the arc of an ellipse, of which one of the axes will be in the line Ee , and the vertex at e . If we now bisect the line joining the points ae , and draw a line from i through the point of bisection to meet eE , the point where it meets will be the centre. This operation is shown upon the side cd by the line ko , drawn from k , the meeting of the tangents dk, ke'' . Through O , draw LM parallel to dc for the line of the axis major; then by Prob. iv, p. 70, Primary Problems, describe a semi-ellipse $Ldec$, of which one of the axes is oe'' , and d or c a point in the curve.

TRUSS GIRDERS.

PLATE I.

Fig. 1, iron truss bolted together at the ends.

Fig. 2, girder divided into two lengths; *No. 1* a vertical longitudinal section, *No. 2* plan of the girder from the top. This example consists of two braces uniting a king bolt in the middle. *Fig. 3* a girder divided into three lengths. *Fig. 4* design for a girder extending between two walls at a great distance from each other. In this design the surface of the floor is raised at a greater height above the ceiling, as the span between the walls is greater. *Fig. 5* is a washer seen on the top of *No. 2*. *Fig. 6* bolt at the end of *fig. 2*—*fig. 6* the bolt in the middle of *fig. 2*, shown in *No. 1*, also in the quoins, *fig. 3*, *No. 1*, at the end of the horizontal piece in the middle. *Fig. 6, 7* are the bolts.

PLATE II.

THE truss herein contained is entirely made of iron. *No. 1* perspective elevation, e and f screws in order to tighten the two iron ties between a and b : this is also tightened by means of wedges shown at A and B . The recesses at i and k are sockets for the ends of the binding joists.

No. 2 longitudinal section showing the ends of the binding joists, ceiling joists, and bridging joists longitudinally.

No. 3 plan of the joisting seen from the top.

No. 4 plan of the joisting inverted, showing the tie rods.

No. 5 and 6 transverse views at ends.

TRUSS PARTITION.

PLATE I.

Fig. 1, truss partition with two small doors.

Fig. 2, truss partition with folding doors.

PLATE II.

Truss Partition, with an aperture for folding doors, and another for a common door.

TRUSSES EXECUTED.

PLATE I.

Fig. 1, truss of the roof of the chapel of Greenwich Hospital.

Fig. 2, truss of the roof of St. Paul's, Covent Garden.

Fig. 3, truss of the roof of the late Theatre, Drury Lane.

PLATE II.

Fig. 1, truss of the roof of Covent Garden Theatre.

No. 1, 2, 3, 4 exhibit the double posts and the method of fixing the irons.

Fig. 2, truss of the roof of the present Theatre, Drury Lane.

No. 2, sections at the wall of wall plates, &c.

No. 3, the method of strapping collar beam and queen posts at each end.

PLATE III.

Fig. 1, truss of the roof of Islington Church. This roof has failed, and tie beams have, at last, been introduced.

Fig. 2, truss of the roof of St. Martin's in the Fields.

CARPENTRY,

TRUSS GIRDERS.

PLATE 1.

Fig. 1.



Fig. 2. N°1.



N° 2.



Fig. 3. N°1



N° 2.



Fig. 5.

N°2.

N°1.

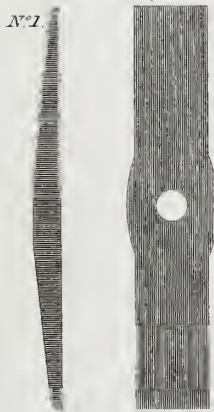


Fig. 6.

N°1.

N°2.

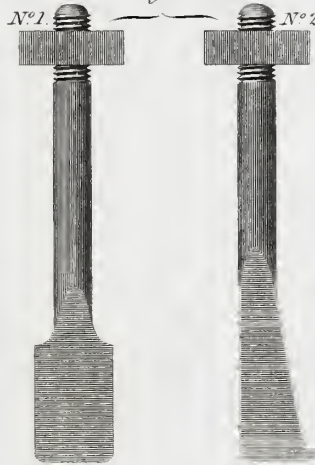


Fig. 7.

N°1.

N°2.

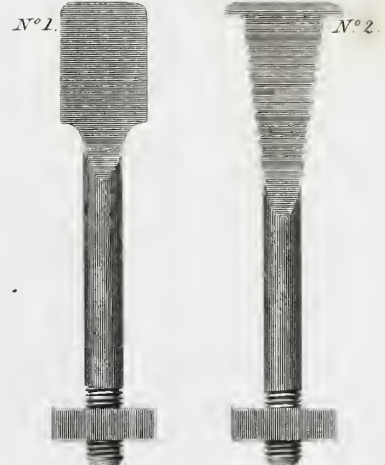
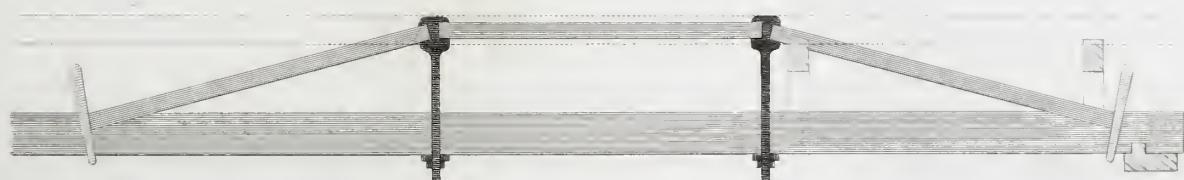


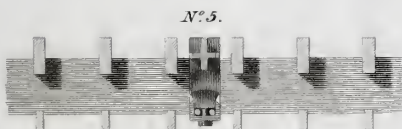
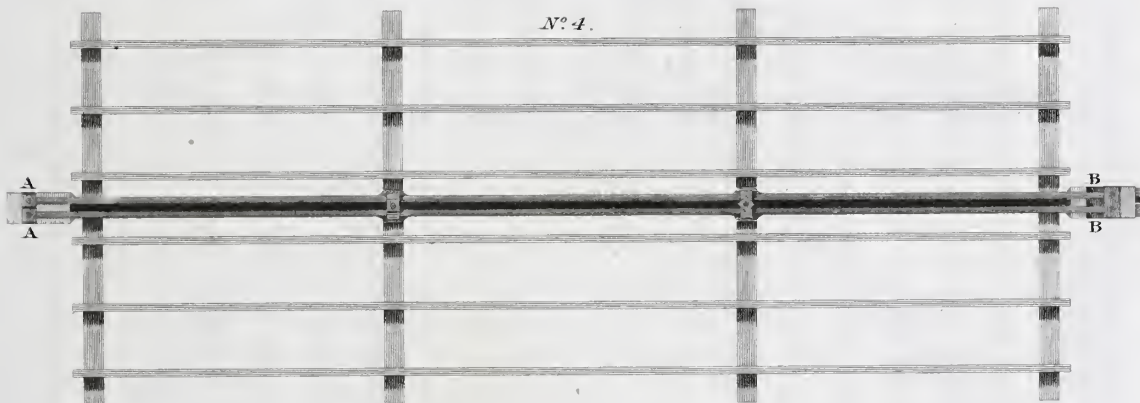
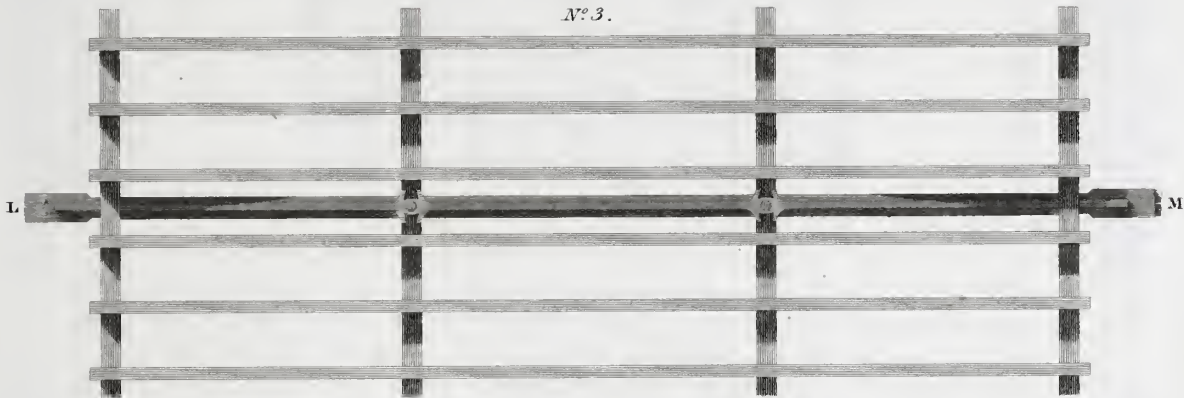
Fig. 4



CARPENTRY,

TRUSS GIRDERS.

PLATE II.



CARPENTRY,

TRUSS PARTITION.

PLATE 1.

Fig. 1.

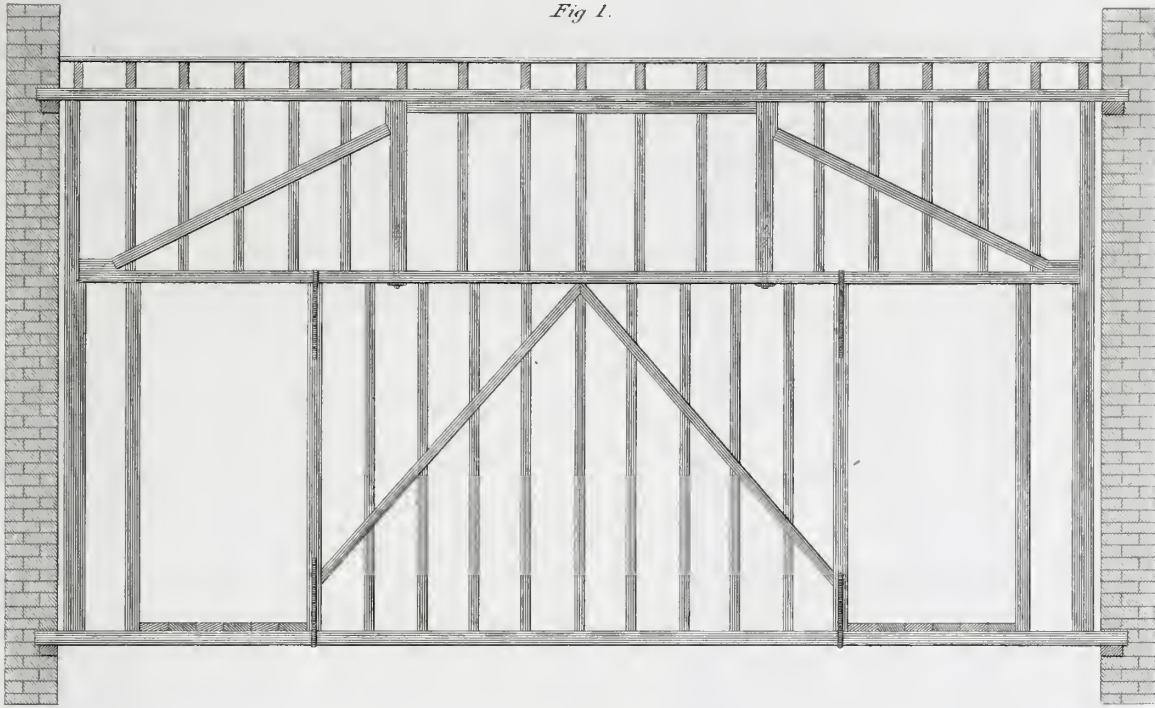
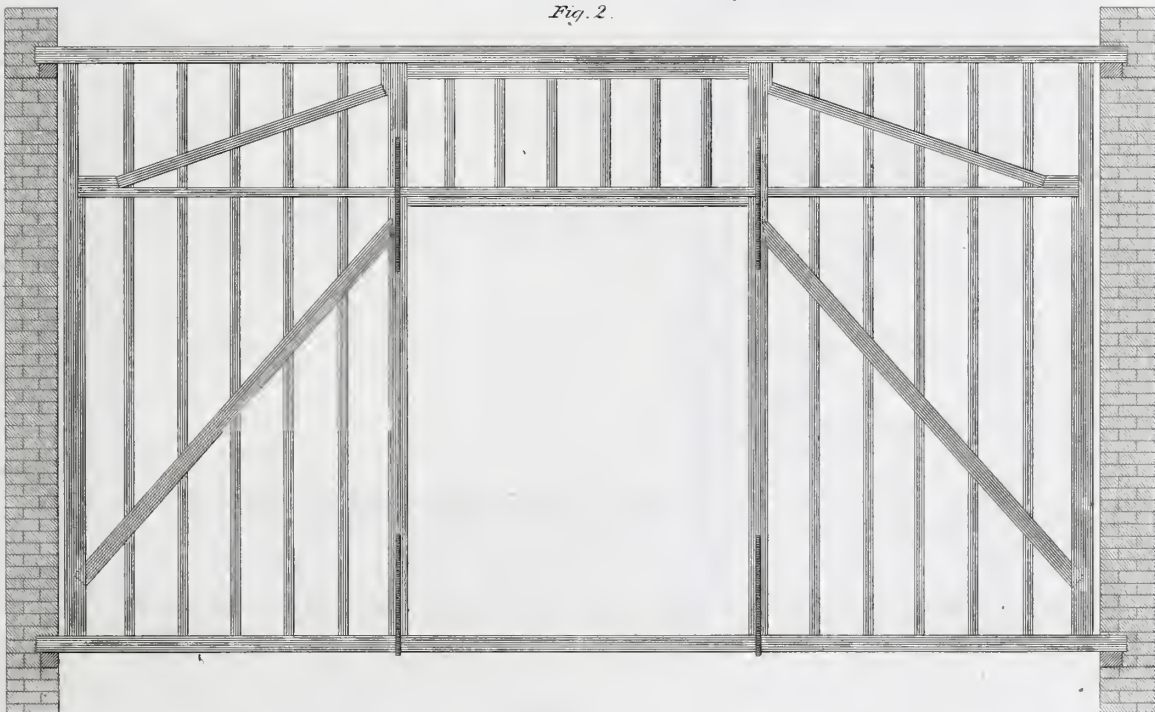


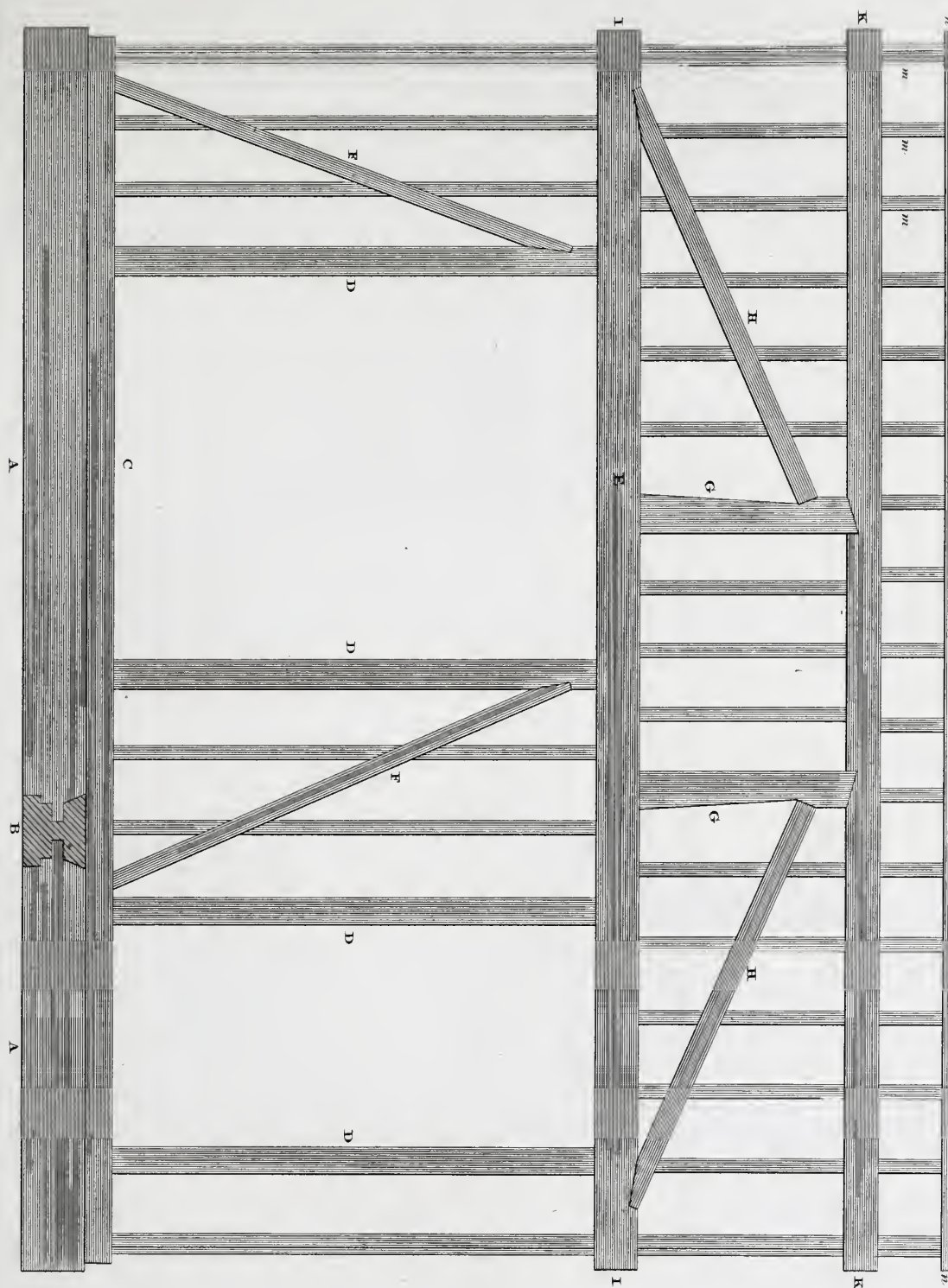
Fig. 2.



CARPENTRY,

TRUSS PARTITION.

PLATE II.



CARPENTRY.

TRUSSES EXECUTED.

PLATE I.

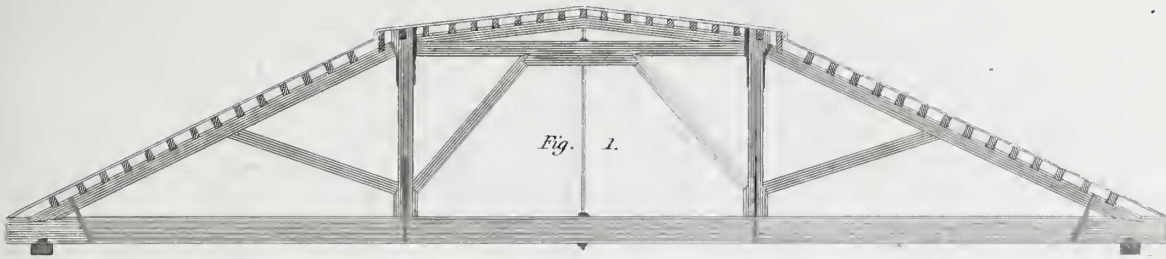


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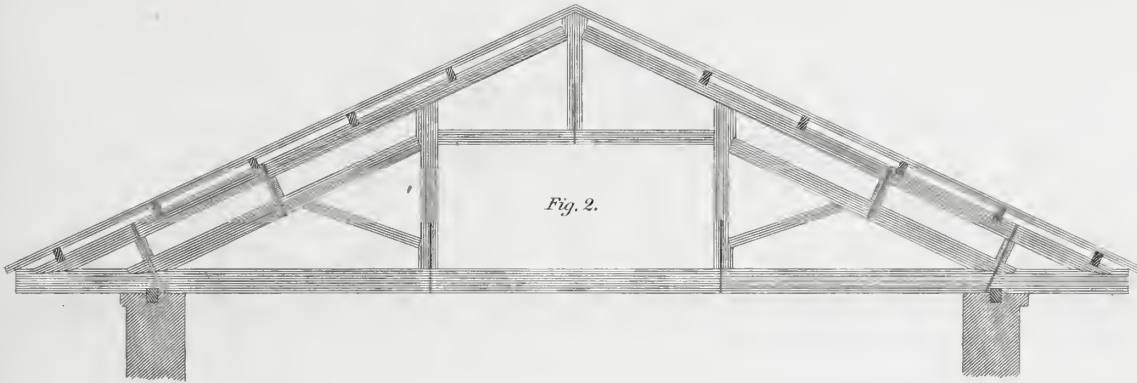


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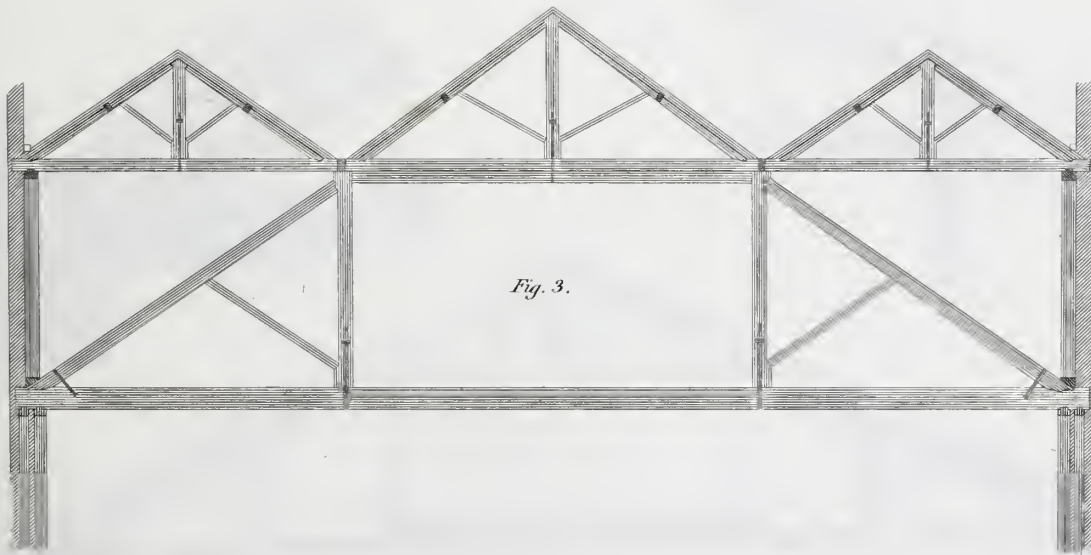


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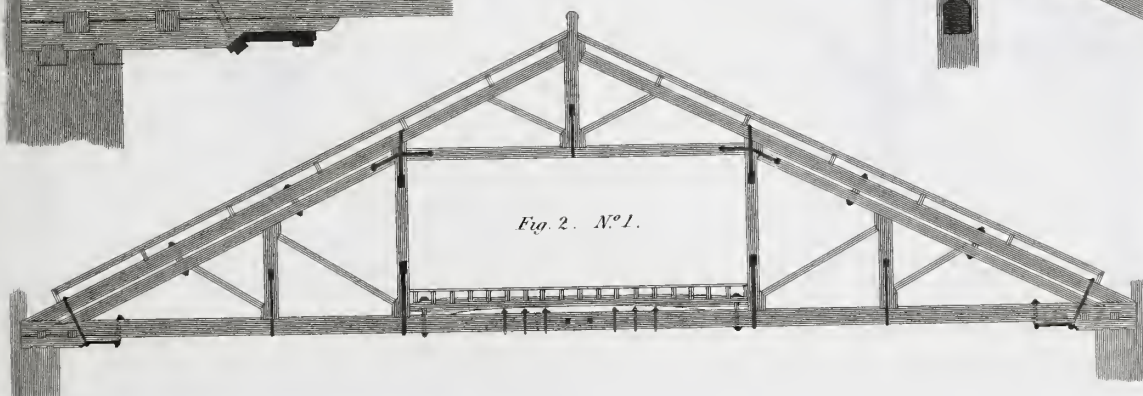
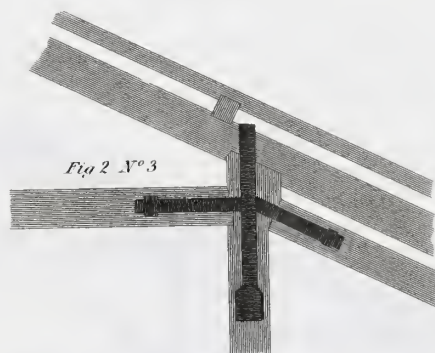
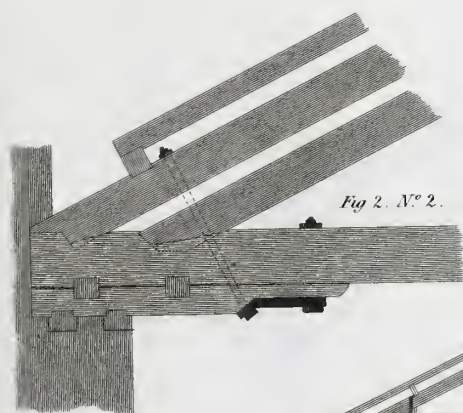
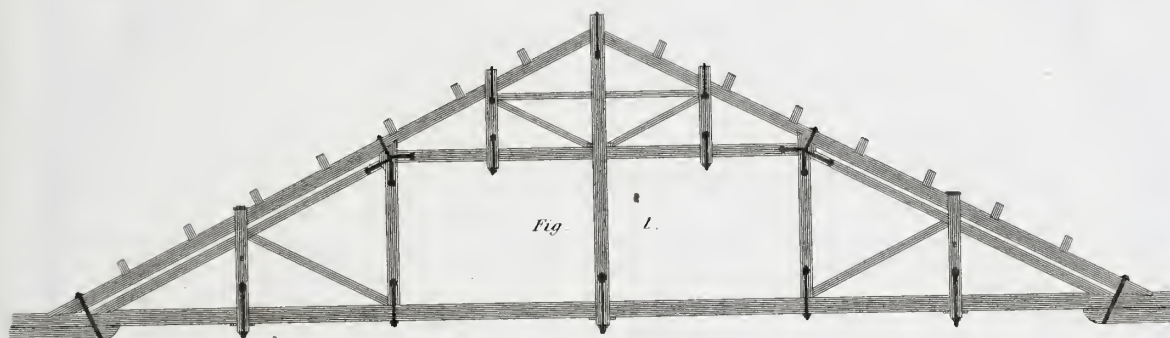
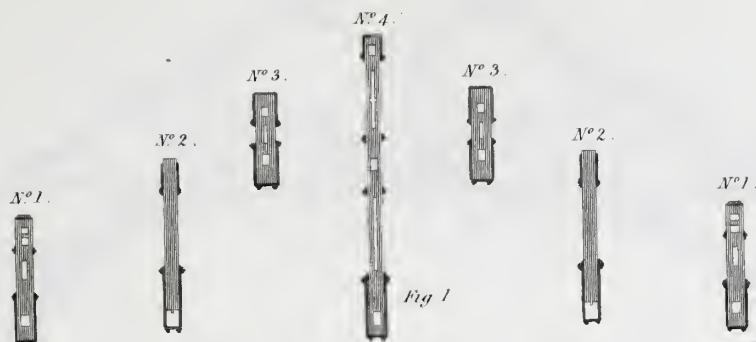
Measured & Drawn by P. Nicholson.



CARPENTRY.

TRUSSES EXECUTED.

PLATE II.



Drawn by P. Nicholson

CARPENTRY.

TRUSSES EXECUTED.

PLATE. III.

Fig. 1.

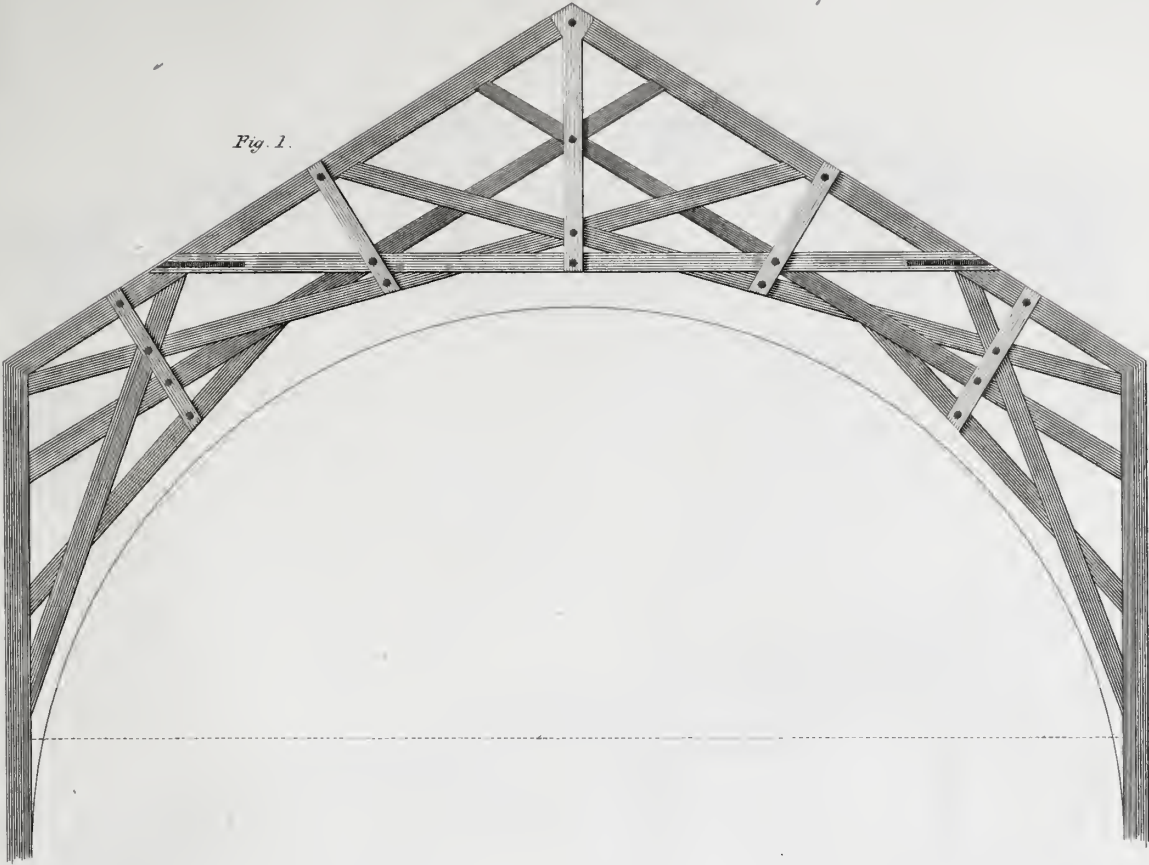
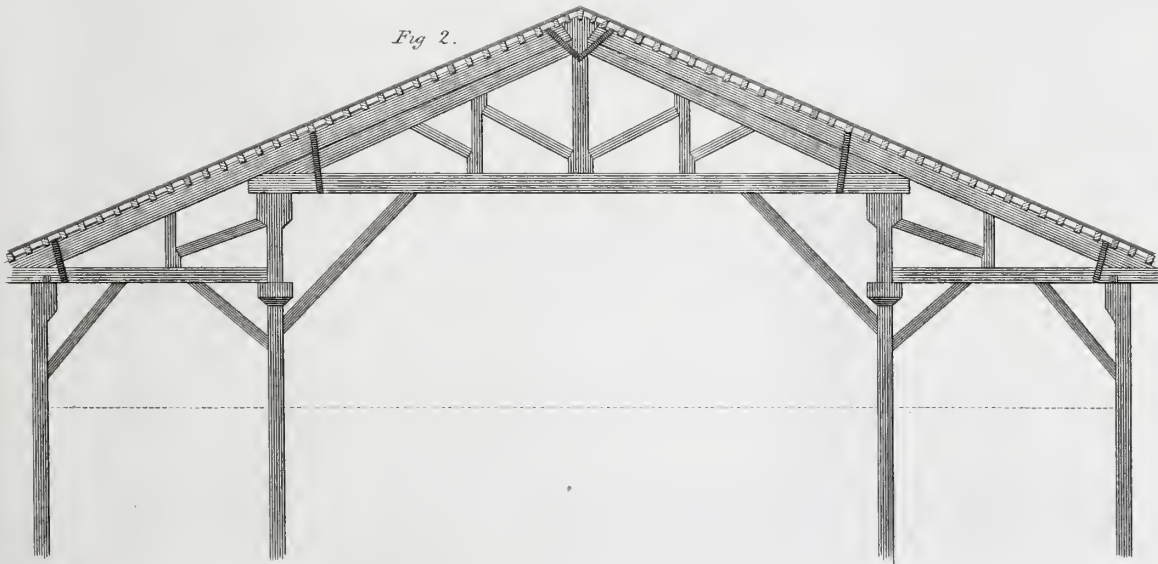


Fig. 2.



CARPENTRY.

TRUSSES EXECUTED.

PLATE IV.

Fig. 1.



Fig. 2.

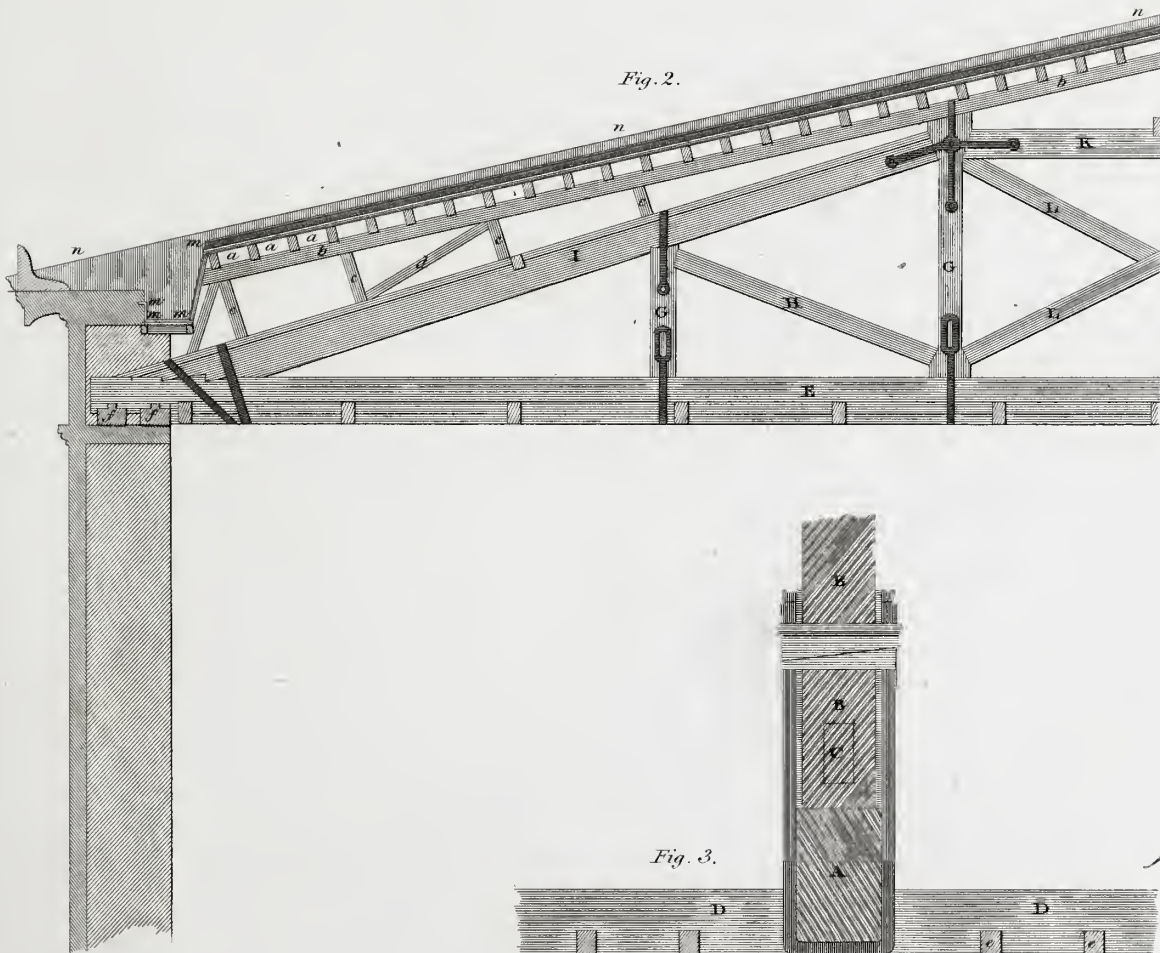
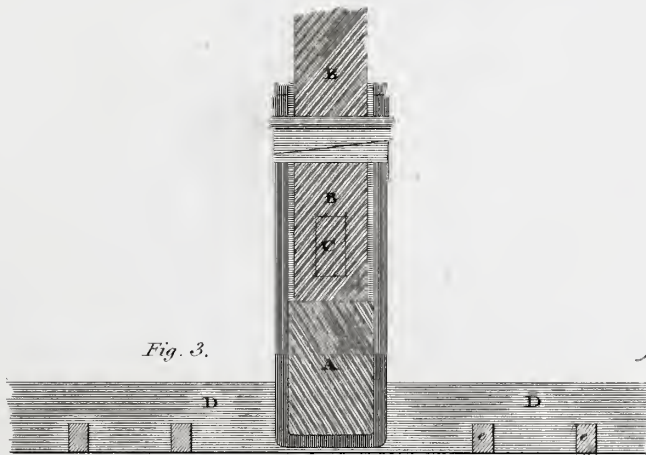
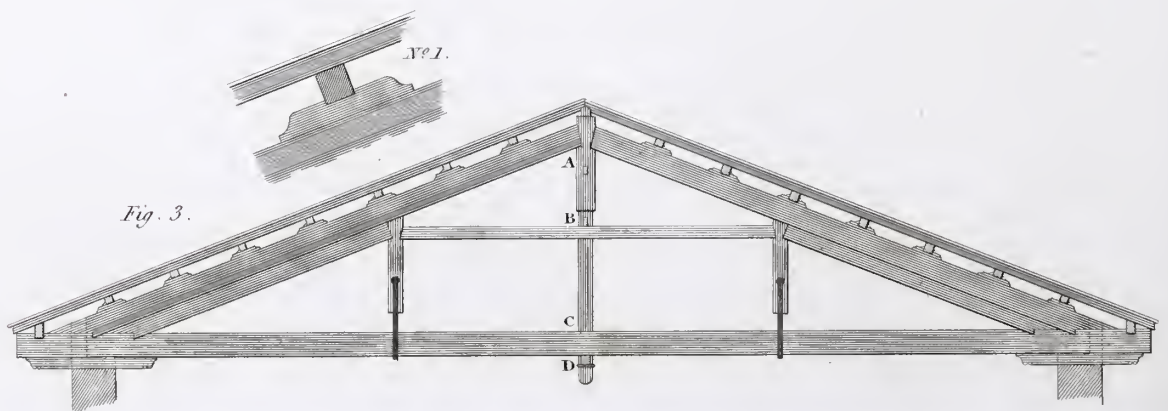
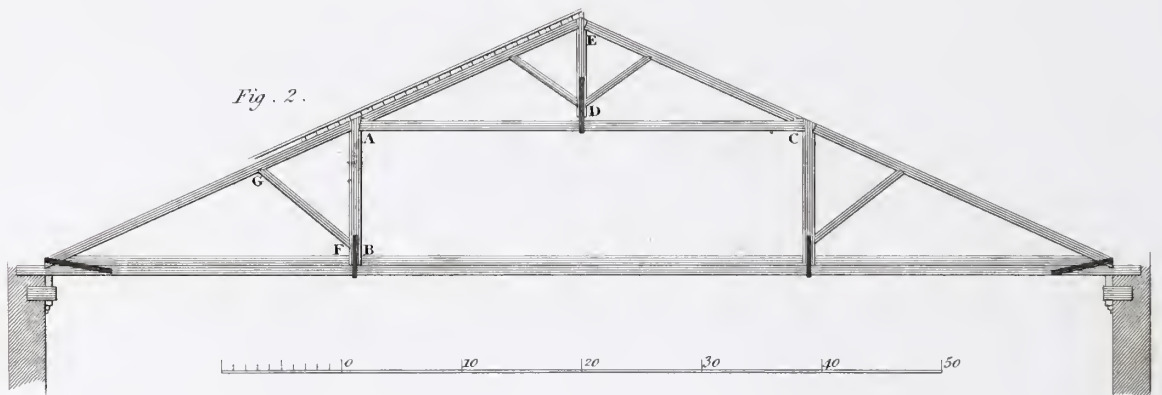
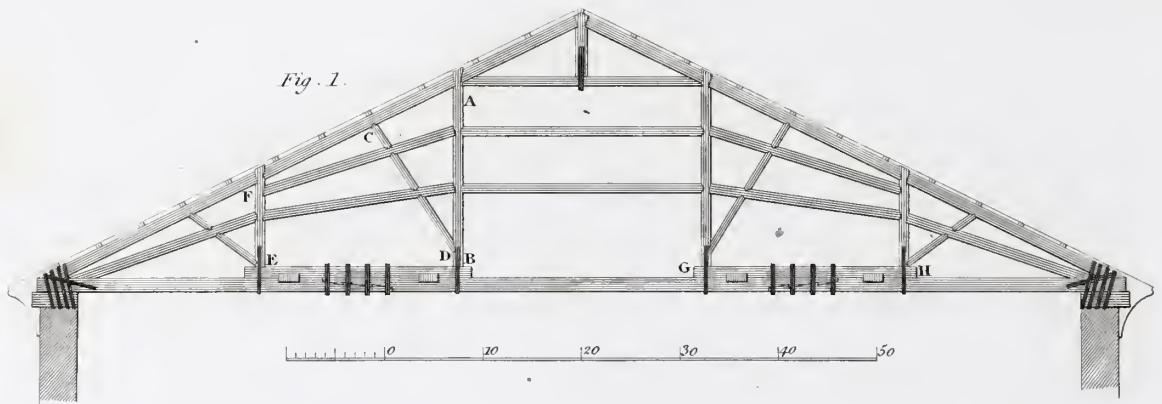


Fig. 3.





CARPENTRY.

PLATE VI.

ROOF.

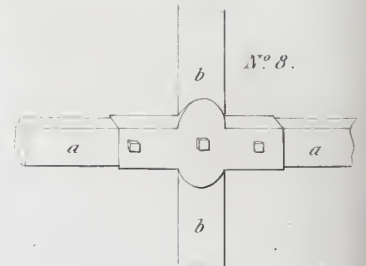
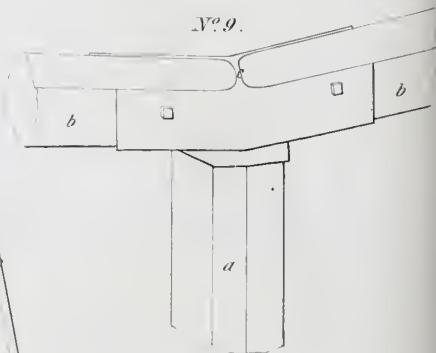
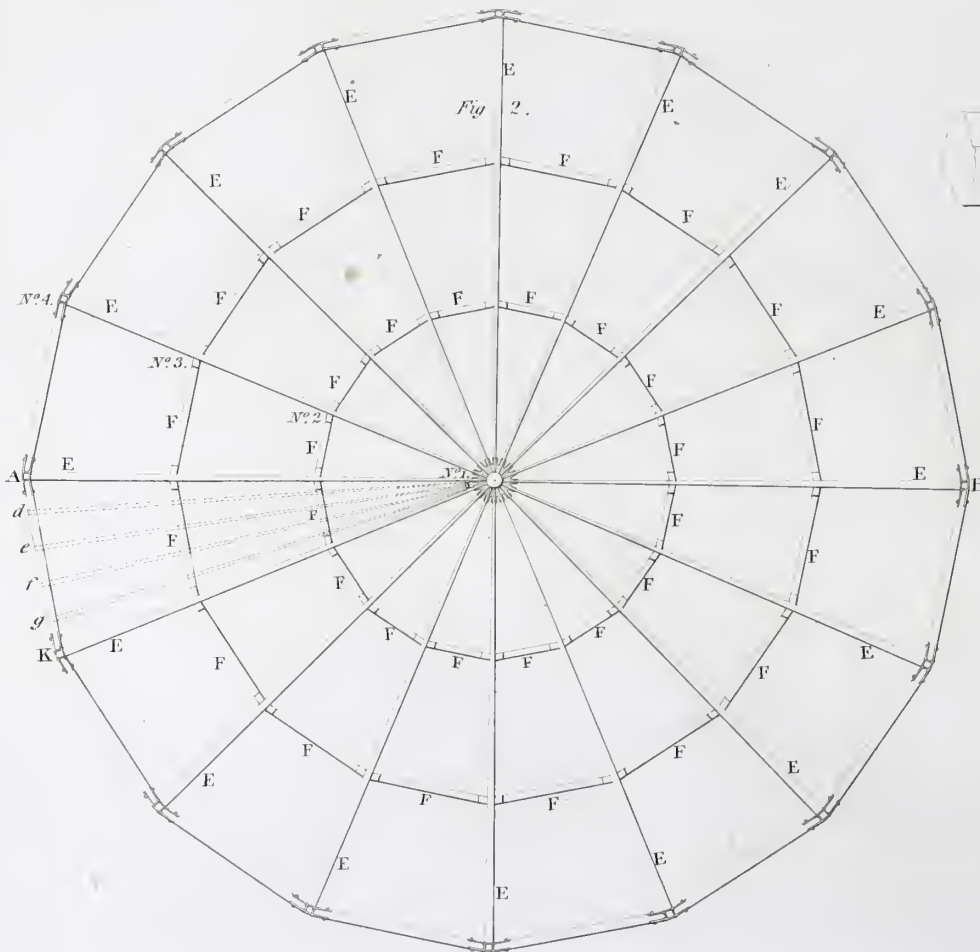
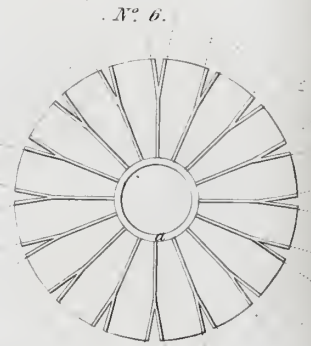
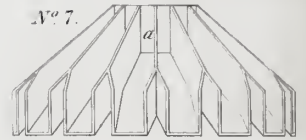
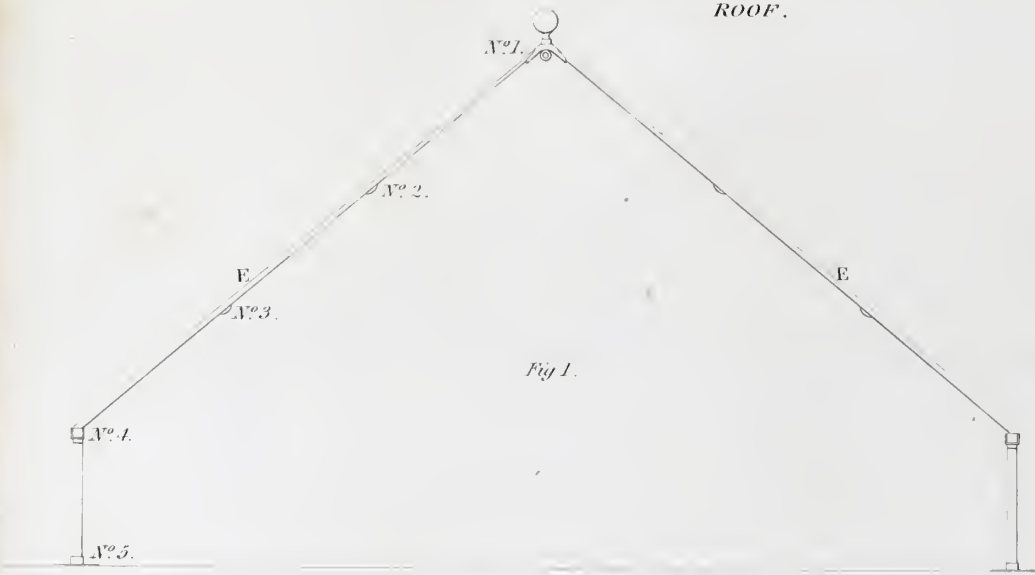


PLATE IV.

Truss of the roof of the new Church of St. Pancras.

Fig. 1, entire truss.

Fig. 2, half truss, showing the parts more distinctly.

PLATE V.

Fig. 1, truss of the roof of the Bourbonic Theatre, at Parma.

Fig. 2, truss of the roof of the Fenice Theatre, at Venice.

Fig. 3, truss of the roof of the Basilica of St. Paul's, without the walls of Rome. *No. 1* section of the purlin.

Most of the principles are either in two or three pieces: these have two or three scarfs; when one, it is in the centre, one pair of principles having their tie in one piece that is 78 feet long, without the least support from the trusses of the roof.

CIRCULAR ROOFS.

PLATE VI.

Plan and elevation of a conic roof, by Mr. Collinge.

No. 1 cast iron top cross, a plan and view drawn to a large scale, is shown at C and D. The dotted lines are the principal rafters E, which terminate at the points *a, a*, and are there bolted.

No. 2, cast iron sockets that receive the ends of the purlins *aa* at every angle, the principal rafter lying between them, and are bolted together as shown at F.

No. 3, cast iron sockets that receive the lower purlins, as *No. 2*.

No. 4, cast iron angles that receive the ends of the plates, on the under side of which is a socket for the upper part of the post *a* to slip into, as at G: *bb*, plates intersecting at *c*, where the principal rafter E pitches; the whole of which is tightly secured with bolts.

No. 5, cast iron step which rests on the surface with a socket, as H, to receive the bottom of the post.

In the Edinburgh Encyclopædia, under the article Carpentry, p. 528, published about ten years ago, Mr. Nicholson describes the properties of a circular roof with regard to strength in the following words:—

“A circular roof may be executed with timbers disposed in vertical planes, whether the ribs or rafters are convex, concave, or straight, without any tie between the rafters or

ribs, even though the wall plate were ever so thin, provided that it be only sufficient to sustain the weight of the roof, by joining the wall plate, so as to form a chain, a ring, or endless plate, and by strutting the rafters in one or more horizontal courses, without any danger of lateral pressure, or of the timbers themselves being bent by the weight of the covering.

“ But the same cannot be done with the roof of a rectangular building, for single parallel rafters would not only obtain a concave curvature, but would thrust the walls outwards.

“ Hence the means of executing circular roofs with safety are simple; but those for straight-sided buildings are complex, and require much more skill in contriving, according to the utility of the space between the rafters, which may be found necessary in forming more lofty or more elegant apartments, as in concave or coved ceilings.”

This description of circular roofs has been completely verified in the subsequent execution of the conic roof by Mr. Collinge. By this criterion, the immense trusses in the roof of St. Paul's Cathedral are unnecessary.

PLATE X.

Section of the roof of the dome of St. Paul's.

PLATE I.

Fig. 1, the most simple construction of a circular ribbed roof.

Fig. 2, circular ribbed roof, with a purlin for a building of greater extent than *fig. 1*.

Fig. 3 is a different construction without purlins, where, in every three consecutive rafters which pass through the axis, the bays on each side of the middle one are filled with rafters which stand in equidistant planes parallel to that in the middle. The edges of the parallel rafters on each side are circles of a less radius than that middle one. The principles of constructing spherical roofs depend entirely on the sections of a sphere.—See Sphere, p. 109, Primary Problems.

Fig. 4 is another method still, where the bays are filled in with strutting pieces which have the same curvature as the ribs themselves, and of which the sides are in planes passing through the centre of the sphere. This method of construction is easily executed, and well calculated to resist the effort of falling or the violence of stormy weather.

TIMBER BRIDGE erected over the River Clyde, Glasgow.—PLATE I.

Fig. 1, general elevation of the bridge.

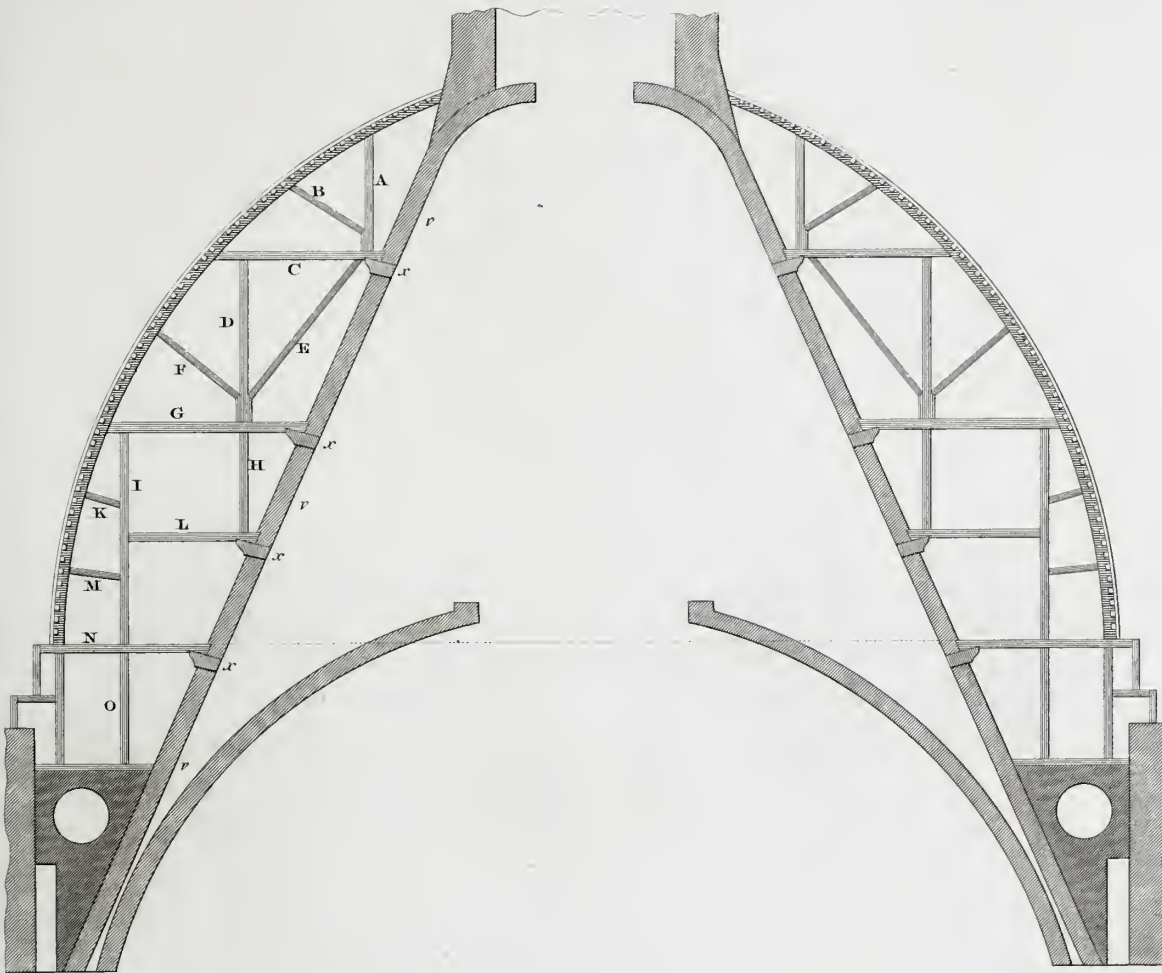
Fig. 2, longitudinal section of the same.

Fig. 3, joisting, with the plan of the fender piles.

CARPENTRY.

TRUSSES EXECUTED.

PLATE. X.





CARPENTRY.

FORMS OF CONSTRUCTION.

PLATE I.

Fig. 1.

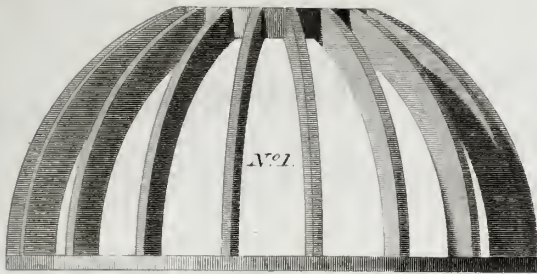


Fig. 2.

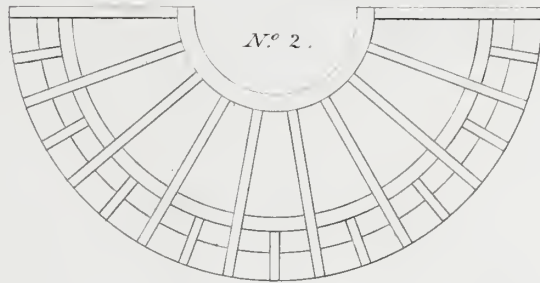
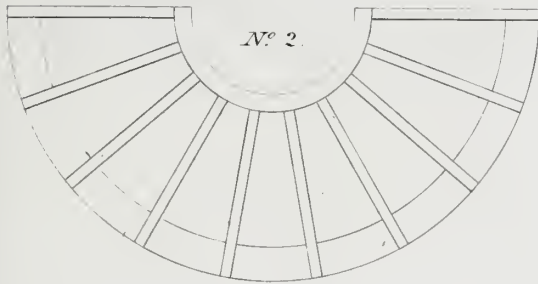
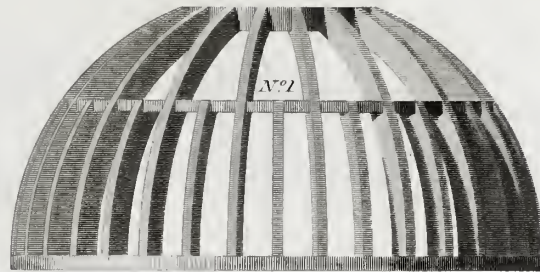


Fig. 3.

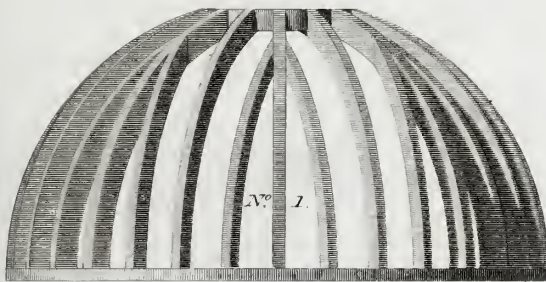
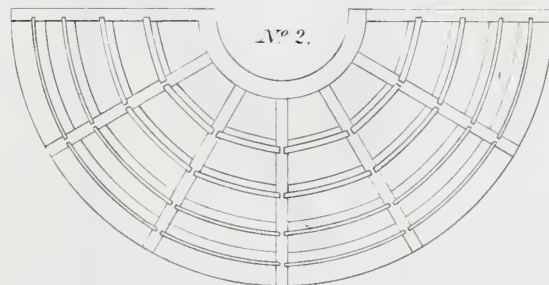
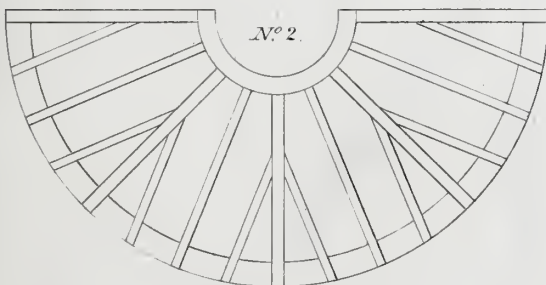
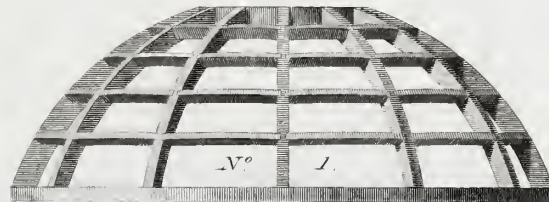


Fig. 4.



CARPENTRY,

TIMBER BRIDGE, ERECTED OVER THE RIVER CLYDE GLASGOW.

PLATE I.

Fig. 1.



Fig. 2.



Scale of feet.



Fig. 3.



Fig. 6.

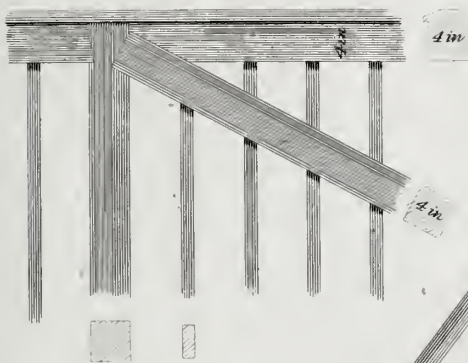


Fig. 4.

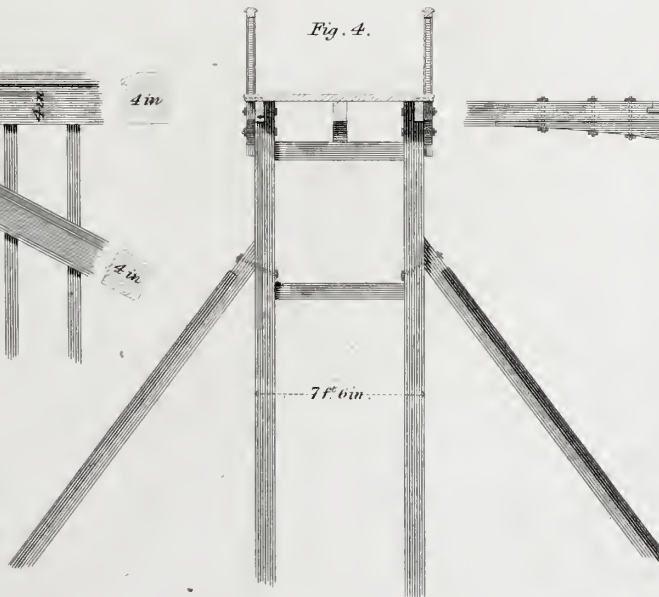
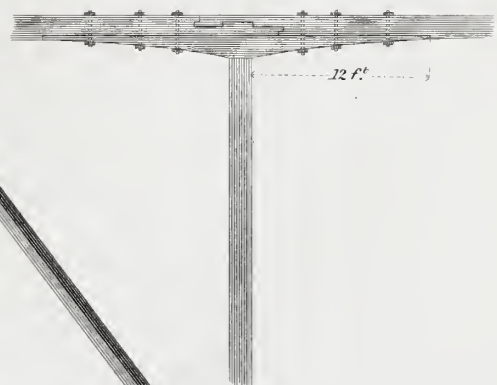


Fig. 5.





CARPENTRY,

SCARFING BEAMS.

PLATE 1

Fig. 1.

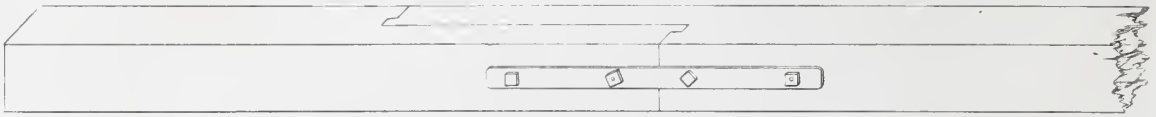


Fig. 2.

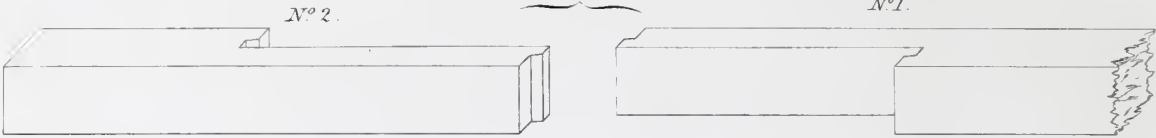


Fig. 3.

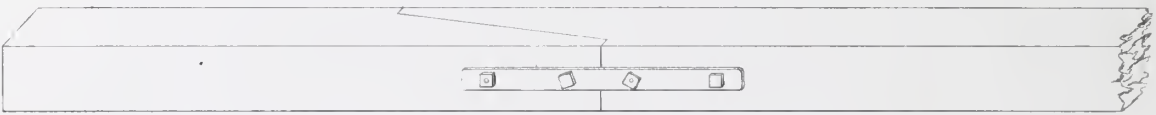


Fig. 4.

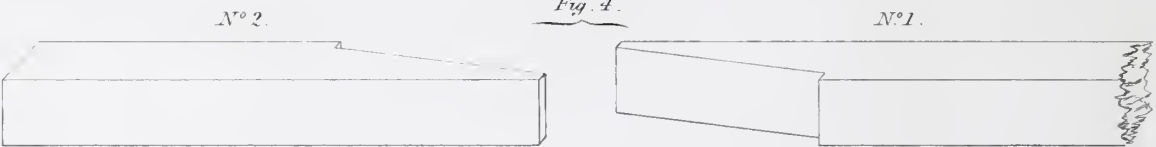


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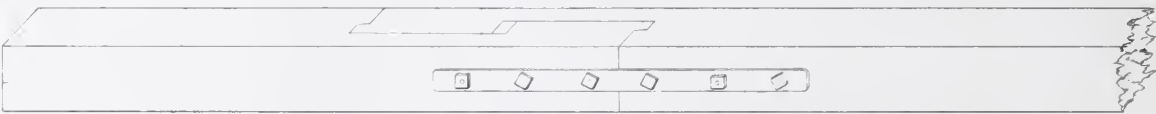


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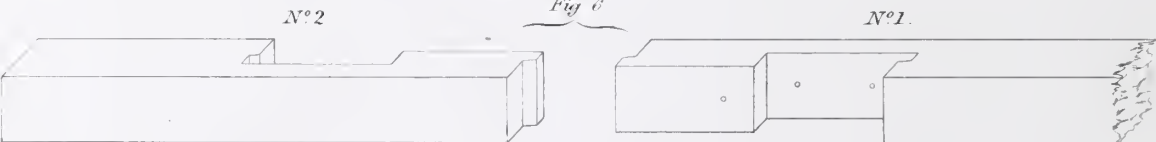


Fig. 7.

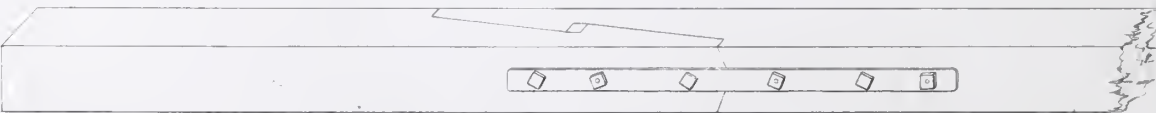


Fig. 8.

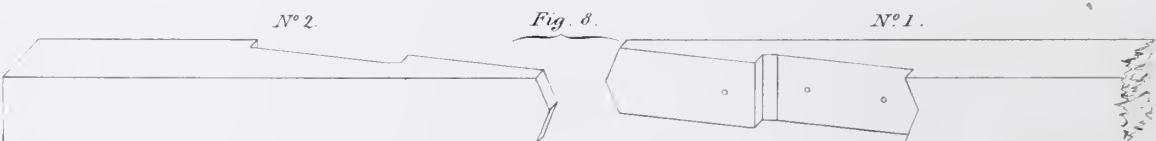


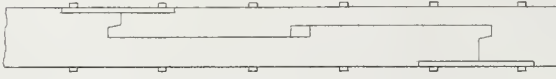
Fig. 1.



Fig. 2.



Fig. 3.



Introduced by P. Nicholson

Fig. 4.

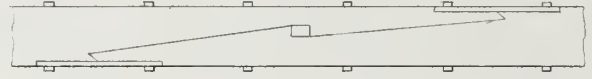


Fig. 5.

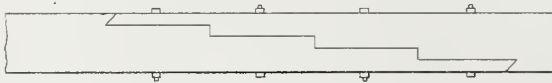


Fig. 6.

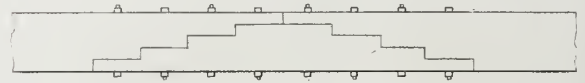


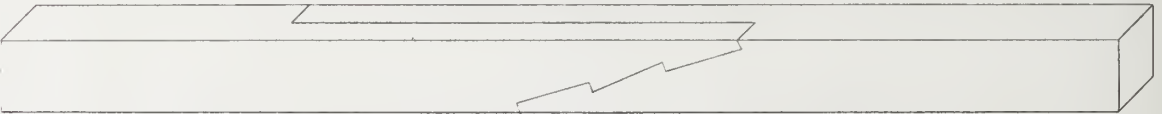
Fig. 7.



Fig. 8.



Fig. 9.



Introduced by A. Morton.

Fig. 10 N° 1.

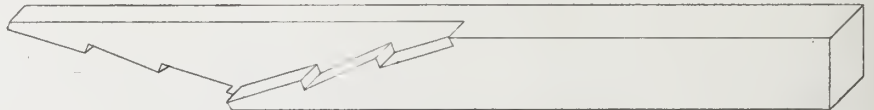


Fig. 10. N° 2.

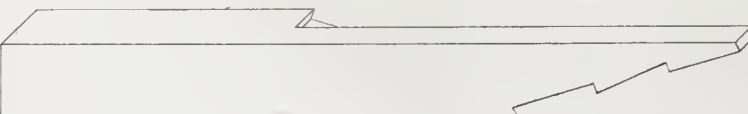


Fig. 4, transverse section.

Fig. 5, method of joining the beams over the posts.

Fig. 6, method of joining the trusses of the railing, by which the bridge is supported.

SCARFING BEAMS.

PLATE I.

Fig. 1, a plain parallel scarf bolted together.

Fig. 2, the same before the two pieces are joined.

Fig. 3, a scarf with an oblique joint bolted together.

Fig. 4, the same before the two pieces are joined.

Fig. 5, a tabulated parallel scarf bolted together.

Fig. 6, the same before bolted together.

Fig. 7, a tabulated oblique joint bolted together.

Fig. 8, the same before the two pieces are joined.

PLATE II.

The figures themselves are so plain, as not to require any description. *Fig. 8* is a method of lengthening beams by building them in thicknesses.

Joinery.

JOINERY is the art of employing wood in the finishing of rooms.

Plate 1 (subtitled *Mitering*) exhibits the sections of the various forms of wood joined at an angle.

Figs. 1 and 2, methods of joining boards, framing, or dado at an internal angle.

Figs. 3, 4, 5, 6, 7, 8, 9 and 10, exhibit the joining of boards, &c. at an external angle.

In *figs. 1, 2, 3*, the position of the grain of the wood is most frequently parallel to the direction of the edges of the section; but when employed in the construction of troughs, it is parallel to the mitre line.

In *figs. 4 and 5*, the external angle being that which is exposed to sight, is rounded or beaded.

Figs. 6, 7, 8, and 10, are what are properly denominated mitres.

Fig. 6 is the most common form of a mitre.

Fig. 7 a lapped mitre, which is much stronger than *fig. 6*.

Fig. 8 a lapped and tongued mitre, used in the construction of the pews of St. Pancras Church.

Fig. 9, dove-tailing.

Fig. 10, secret dove-tailing.

FORMS of HINGED JOINTS and their HINGES.

Plate 1 exhibits the two parts hinged together.

Nos. 2 and 3, the two parts before hinged, applicable to doors and their hanging stiles.

Plate 2, method of placing hinges on shutters, back flaps, &c. In *fig. 1*, the axis of the knuckle of the hinge is exactly opposite to the joint. In *fig. 3*, when it is required to throw the back flap at a given distance AB, we must place the centre of the hinge at half that distance. *No. 2* shows the same joint opened to a right angle. *Fig. 3* a rule joint; *No. 1* the two parts hinged together in a line; *No. 2*, the one opened to a right angle upon the other. *Fig. 4*, another method in order to throw the back flap the contrary way to

JOINERY.

MITERING.

PLATE 1.

Fig. 1.

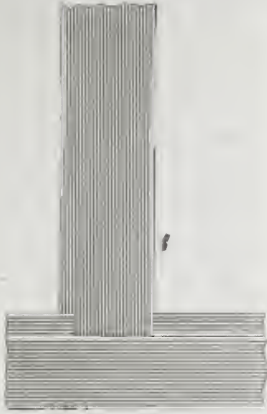


Fig. 2.



Fig. 3.



Fig. 4.

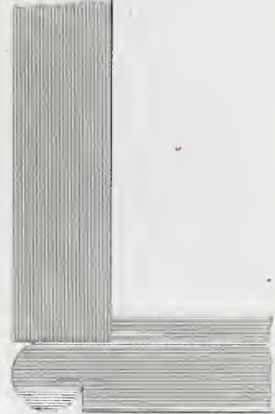
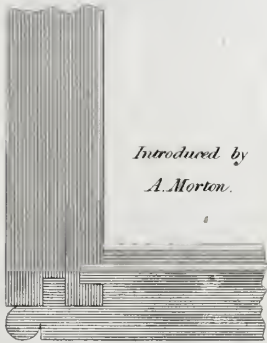


Fig. 5.



*Introduced by
A. Morton.*

Fig. 6.



Fig. 7.



Fig. 8.



*Introduced by
A. Morton.*

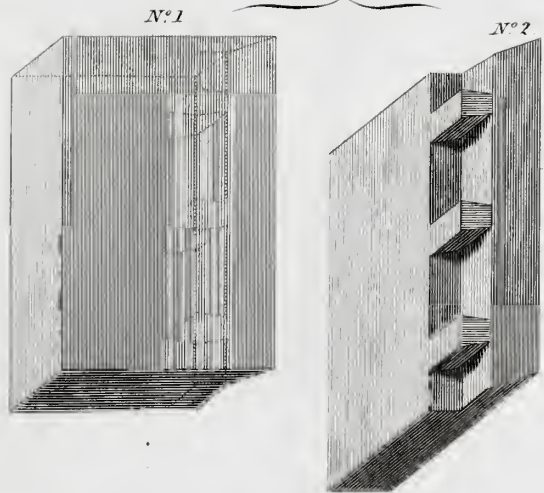
Fig. 9.



N^o 1.

N^o 2.

Fig. 10.



N^o 1.

N^o 2.

Introduced by P. Nicholson.



JOINERY,

FORMS OF HINGED JOINTS & OF THEIR HINGES.

PLATE I.

Fig. 1.

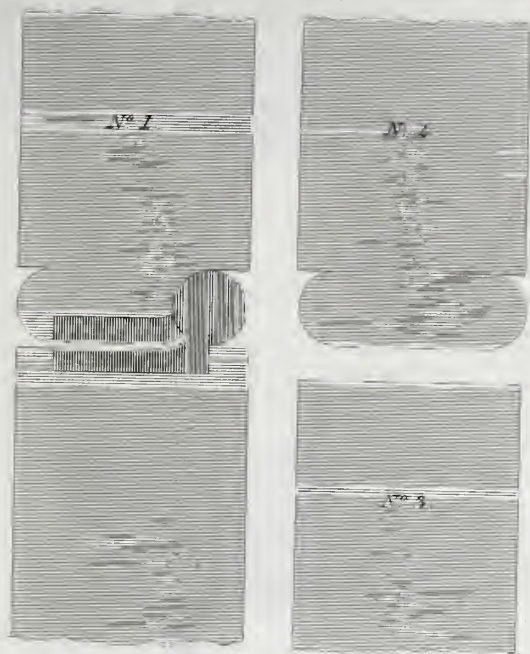


Fig. 2.

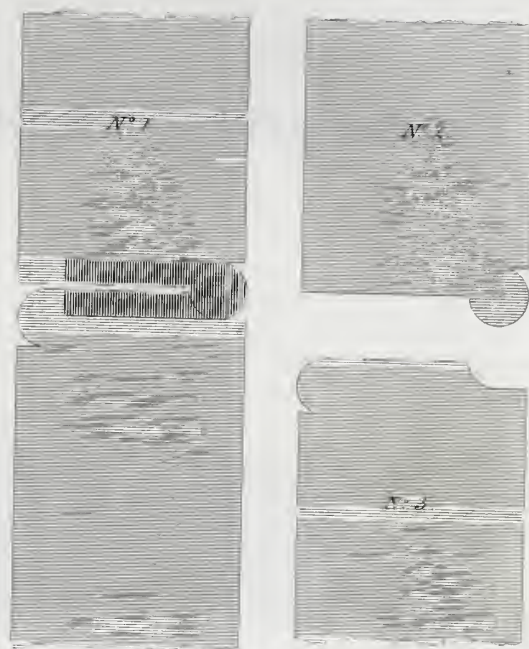


Fig. 3.

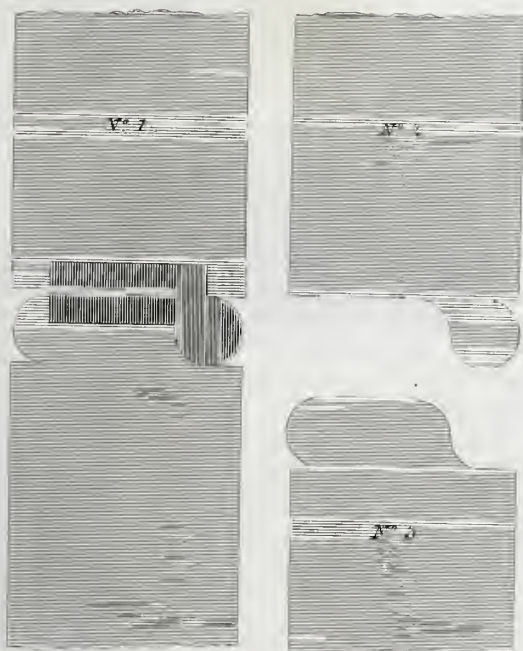
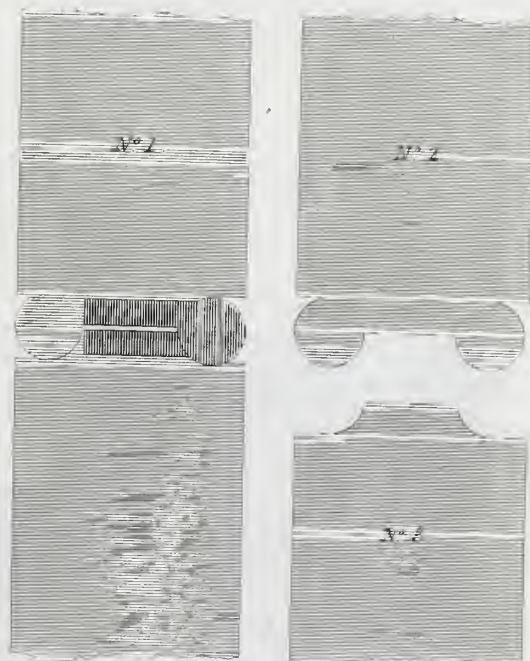


Fig. 4.



JOINERY,

FORMS OF HINGED JOINTS & OF THEIR HINGES.

PLATE II.

Fig 1.

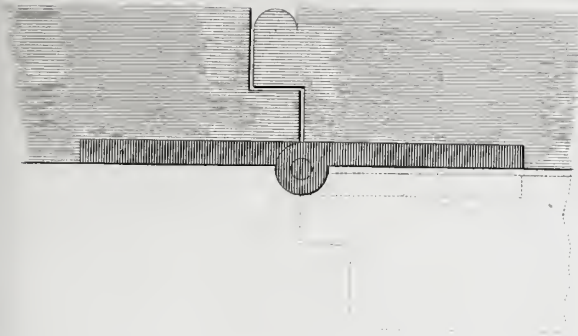


Fig 2 N°1

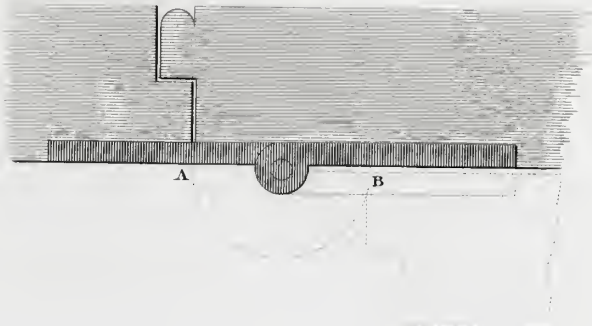


Fig 2. N° 2

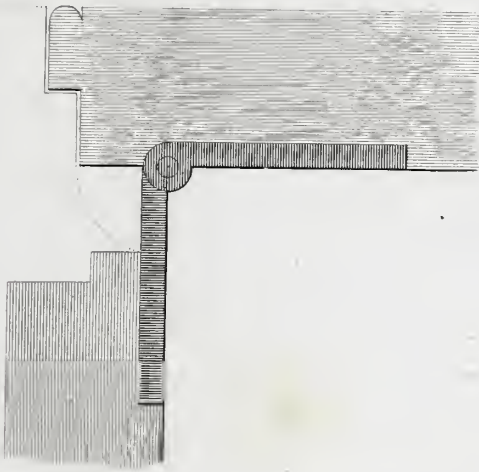


Fig 3. N° 1.

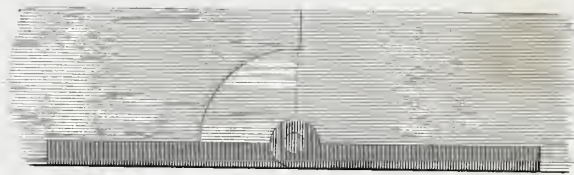


Fig 4. N° 1

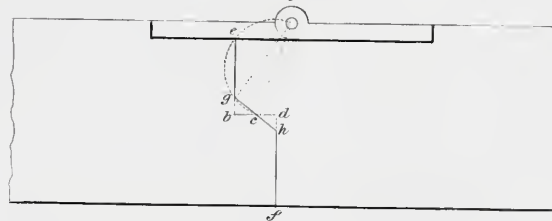


Fig 3. N° 2.

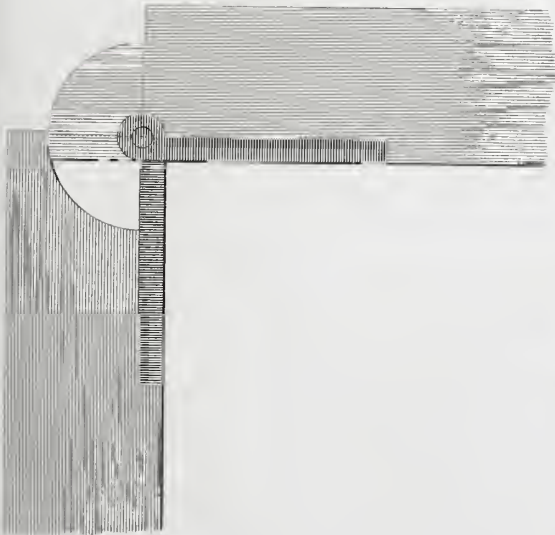
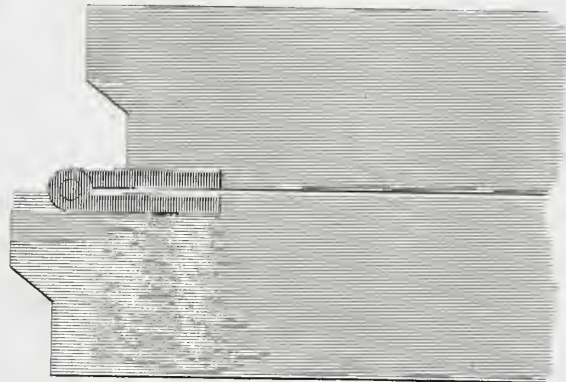


Fig 4. N° 2.



JOINERY,

FORMS OF HINGED JOINTS & OF THEIR HINGES.

PLATE III.

Fig. 1.

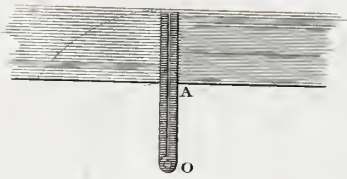


Fig. 2.

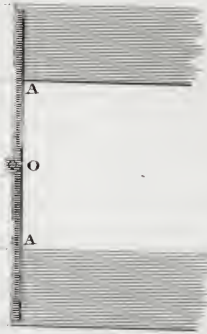


Fig. 3.

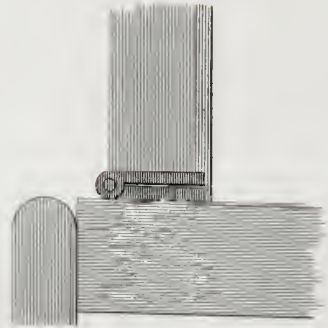


Fig. 4.

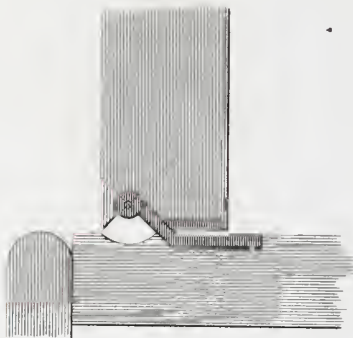


Fig. 5.

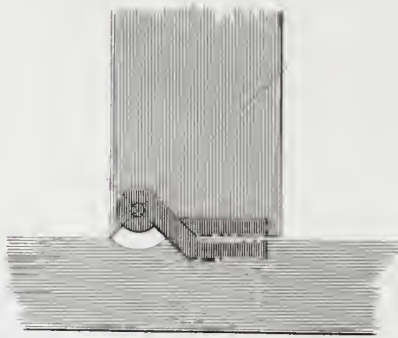


Fig. 6.

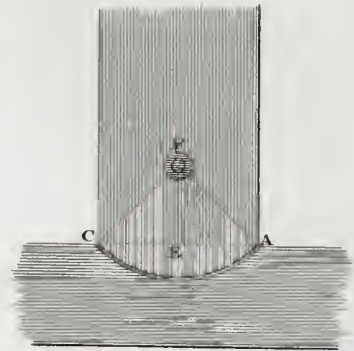


Fig. 7.

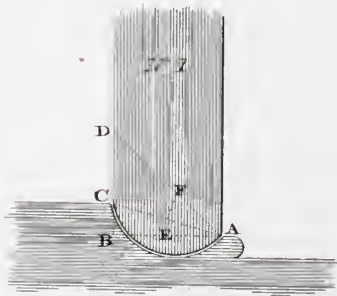


Fig. 8. N°1.

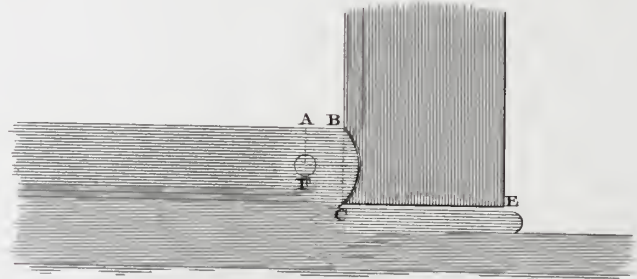


Fig. 7.

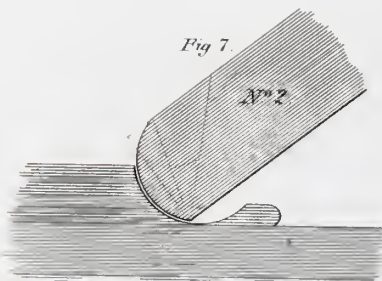
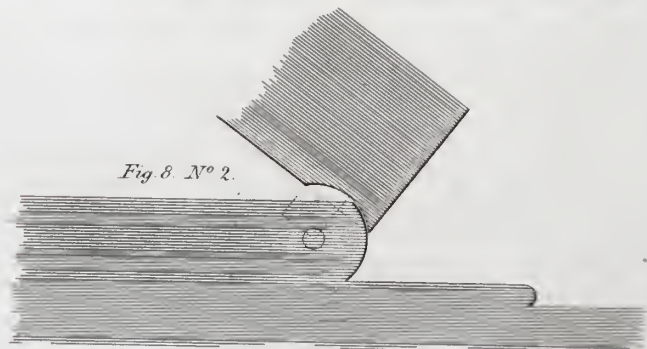
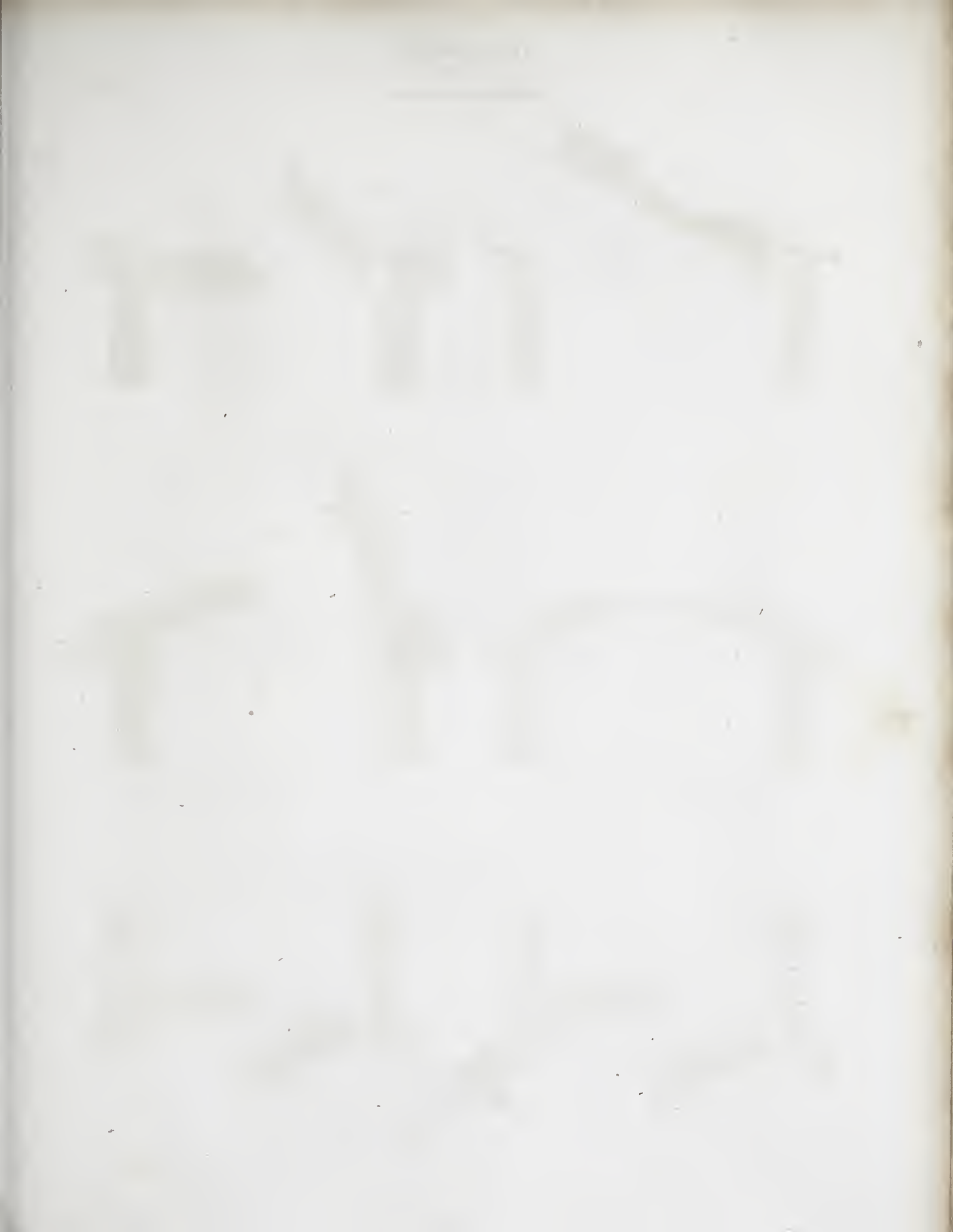


Fig. 8. N°2.

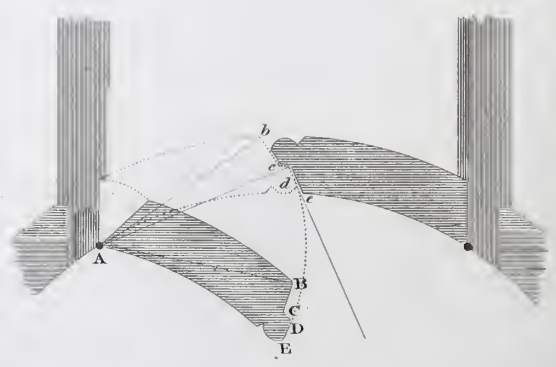
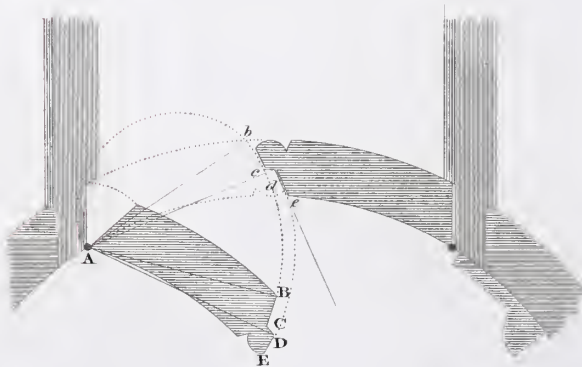
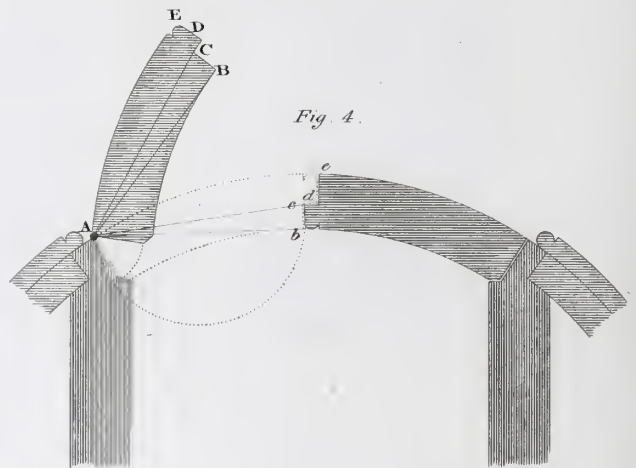
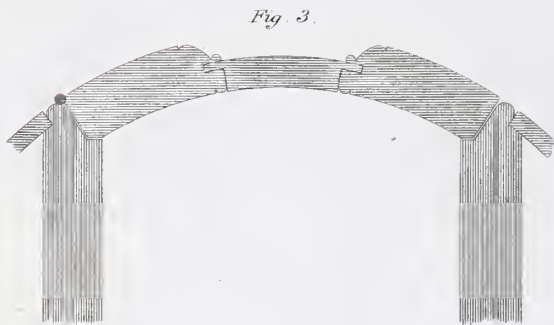
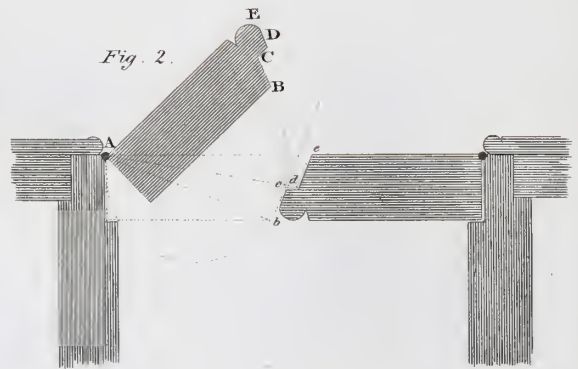
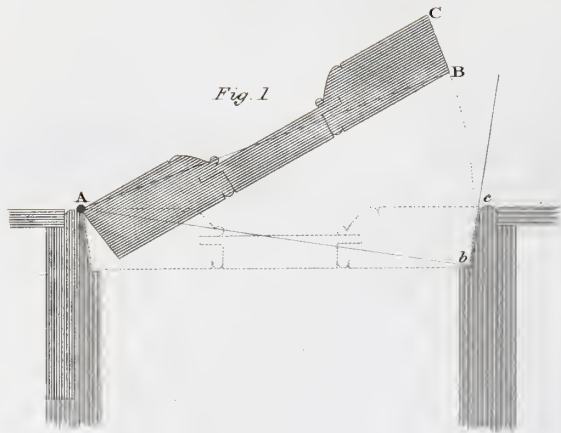




JOINERY.

HANGING OF DOORS.

PLATE 1.



JOINERY.

FOLDING DOORS.

PLATE. IV.

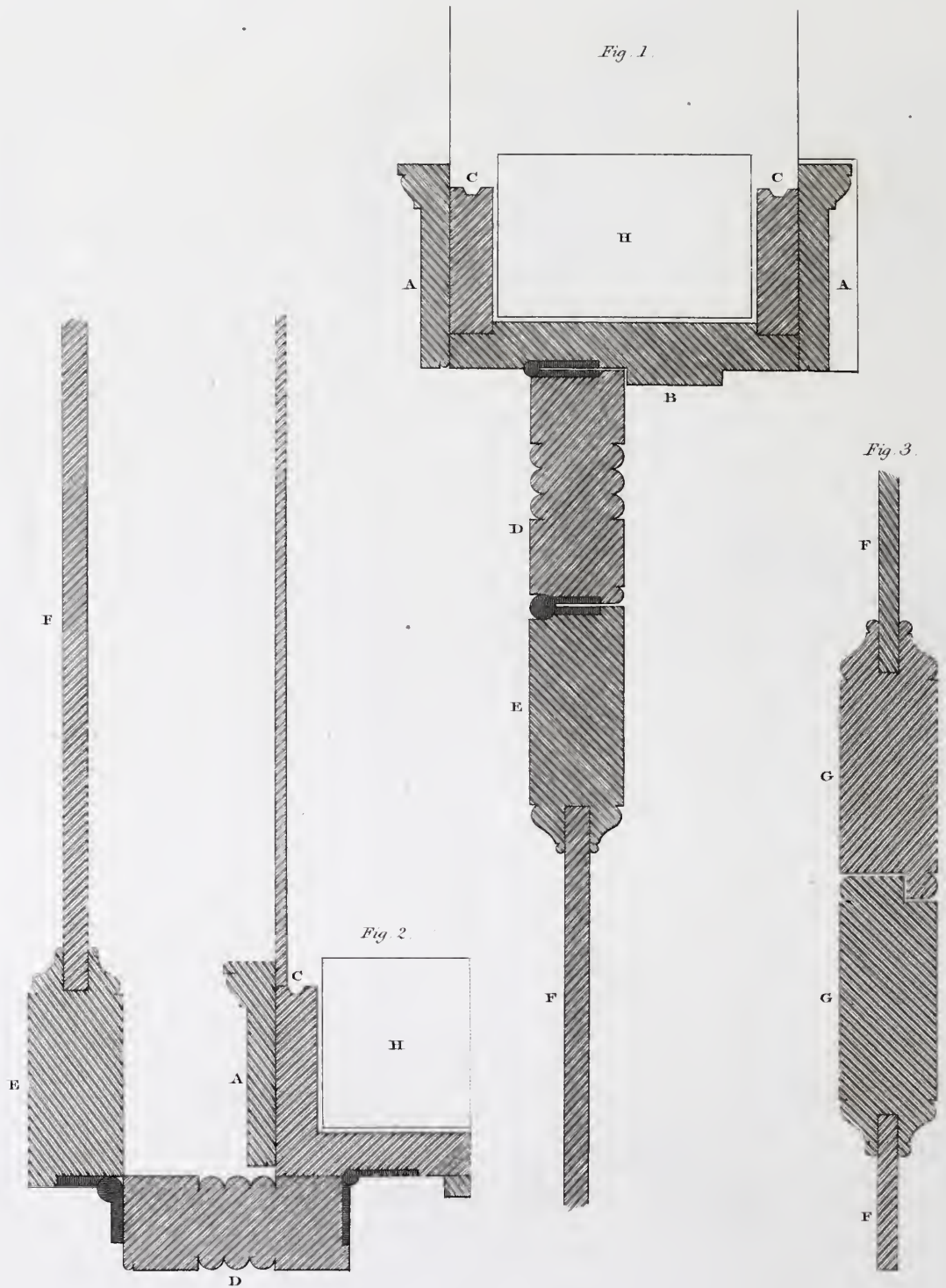


fig. 2. Let *eb* and *fd* be the parallel lines of the join; and let *bd* meet *fd* and *eb* perpendicularly, so as to make *eb* and *fd* equal. Bisect *bd* in *c*, and draw *Oc*, and describe the semicircle *cgo*, meeting *be* in *g*. Through *c* draw *gh*, meeting *fd* in *h*; then *ogh* is a right angle: and as *Og* is perpendicular to *gh*, *og* is the shortest of all the lines that can be drawn from *o* to the straight line *gh*.—No. 2, the same folded backwards.

Plate 3, forms of hinged shutters and their joints.

Fig. 1, joint with the proper hinge for the doors of pews, so as to clear a cornice or other moulding. *Fig. 2*, the same opened.

Fig. 3, joint and hinge connecting the sash frame and shutters.

Fig. 4 and 5, concealed joints.

Fig. 6, centre for a door.

Fig. 7, another centre for a door.

In No. 1, draw *AB* parallel to the jamb, meeting the other side in *B*. Make *BD* equal to *BA*, and join *AD* and *AC*. Bisect *AC* by a perpendicular *EF*, meeting *AD* in *F*; then *F* is the centre of the hinge. No. 2 is the same in the act of opening.

Fig. 8 is another form in order to accomplish the same object.

HANGING DOORS.—PLATE I.

Method of hanging doors so that all the points of the moving edge of the door may pass the surface of the edge of the jamb, or the surface of the edge of the fixed door, in the act of opening or shutting.

GENERAL PRINCIPLE.

Let *A*, *fig. 1*, be the centre of the hinge, and *B* the remote point on the other side, and moving edge of the door. Join *AB*, and draw *BC* perpendicular to *AB*; then *ACB* will be the bevel required.

If the surface of the edge of the door is formed into a rabbet, so that the ledge may stop the door from passing to the contrary side to that on which it revolves, each of the planes which is connected by the middle one must be formed in the same manner as before. Thus, in *figs. 2, 4, 5, 6*, the parts *BC, DE*, are each formed separately, as No. 1.

FOLDING DOORS.—PLATE IV.

Fig. 1, jamb lining, grounds, and architraves, with hanging style, and part of the doors hinged to the hanging style.

Fig. 2, one half of the jamb, with the part of the door opened and turned parallel to the wall.

Fig. 3, the two meeting styles.

SASHES.—PLATE I.

Fig. 1, elevations and sections of the various parts of a sash.

No. 1, section of part of the sash frame.

No. 2, section of one of the styles of the sash, the moulding being denominated astragal and hollow.

No. 3, section of the sash bar.

No. 4 and 5, section of meeting rails of sash.

No. 6, elevation of crossing bars according to the section *No. 3*.

No. 7, longitudinal section of the horizontal bar and of the dowel, and the transverse section of the vertical bar, showing the best method of joining the two together. This method is called *franking*.

Fig. 2, section of a sash with an astragal only, exhibiting the method of preventing the sashes from shaking.

CONSTRUCTION OF SHUTTERS, &c.

PLATE I.

Fig. 1, horizontal section of the sash frame and shutters through one side.

A, inside lining of the sash frame.

B, pulley piece or pulley style.

C, outside lining.

D, back lining.

E, inside bead.

F, parting bead of the sashes.

G, G, weights to balance the sash frame.

H, parting strip.

I, back lining of boxing.

J, ground.

K, front shutter hung to the inside lining A of the sash frame by the hinge *u*.

L, M*, back flaps hinged together at *w*, and to the front shutter at *v*.

N, architrave-pilaster.

No. 2 is a vertical section through the bottom rail of the sash, through the sill of the sash frame, and through a part of the back of the window.

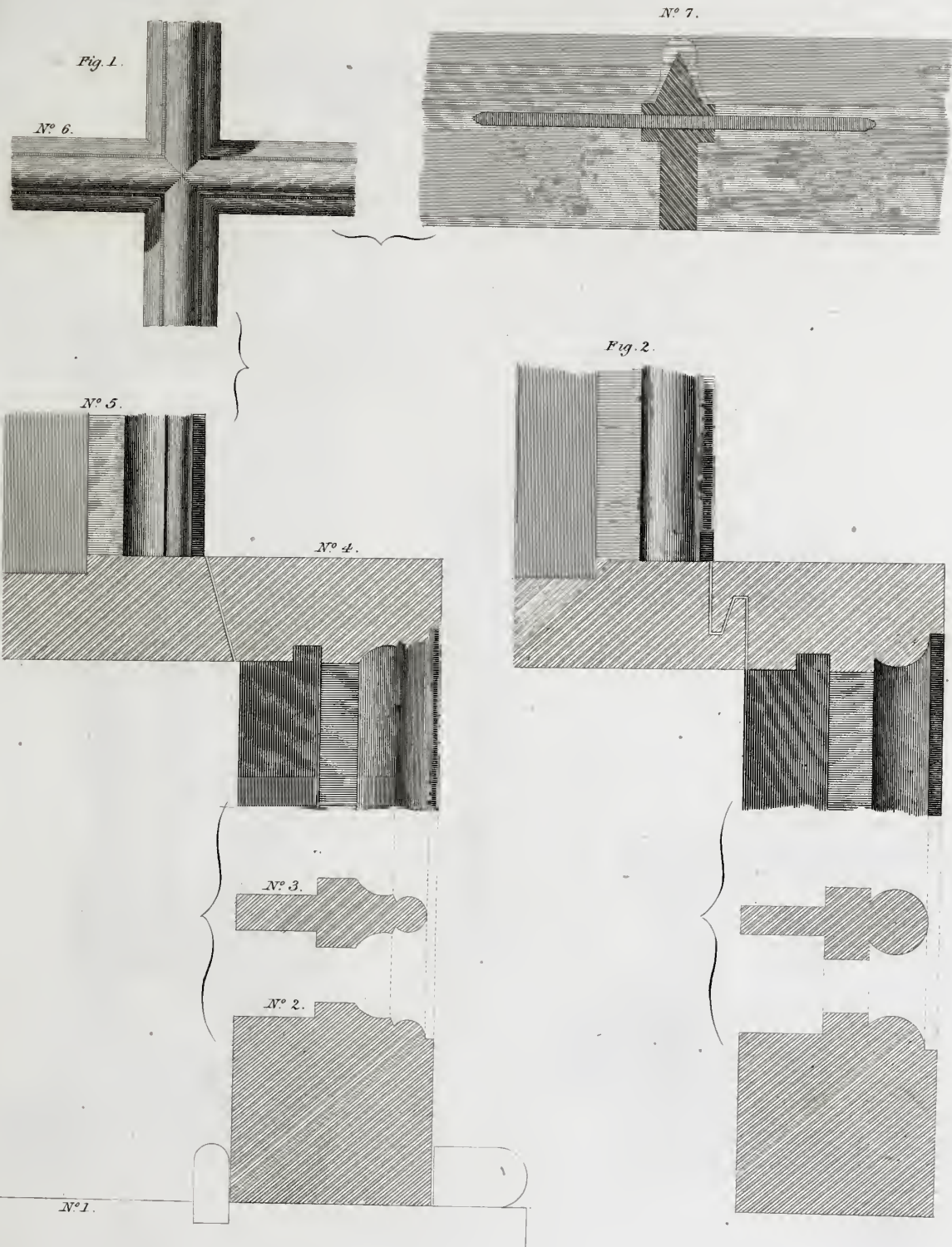
O, bottom rail of sash.

* By a mistake in the drawing, the sides of the flap M are reversed.

JOINERY,

SASHES.

PLATE . I.



JOINERY.

CONSTRUCTION OF SHUTTERS.

PLATE I.

Fig. 1. N^o 1.

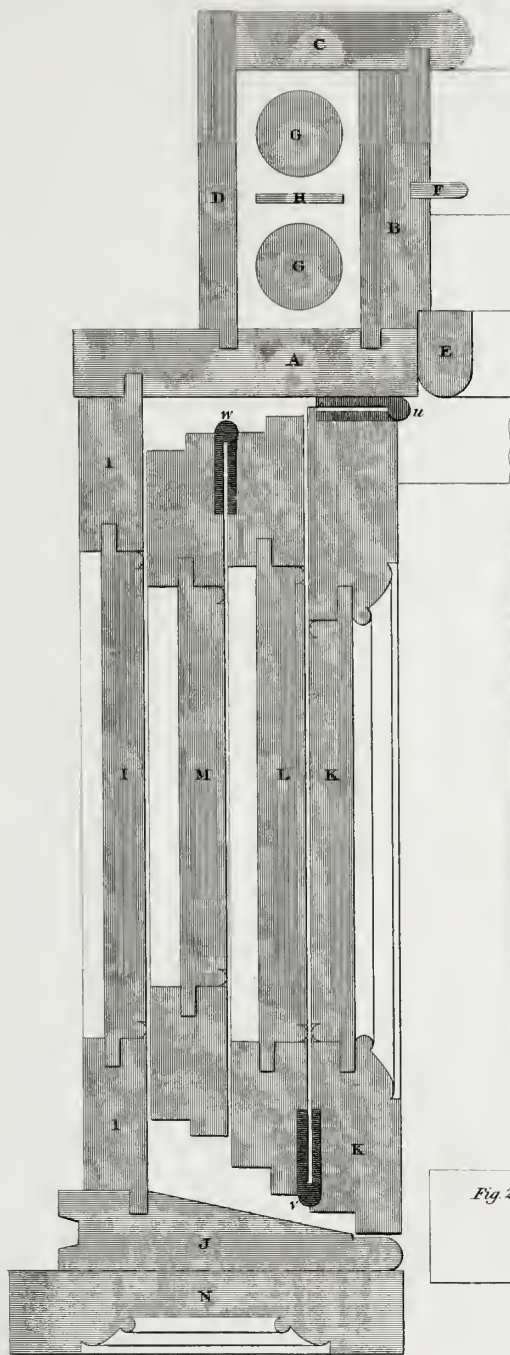


Fig. 1. N^o 2.

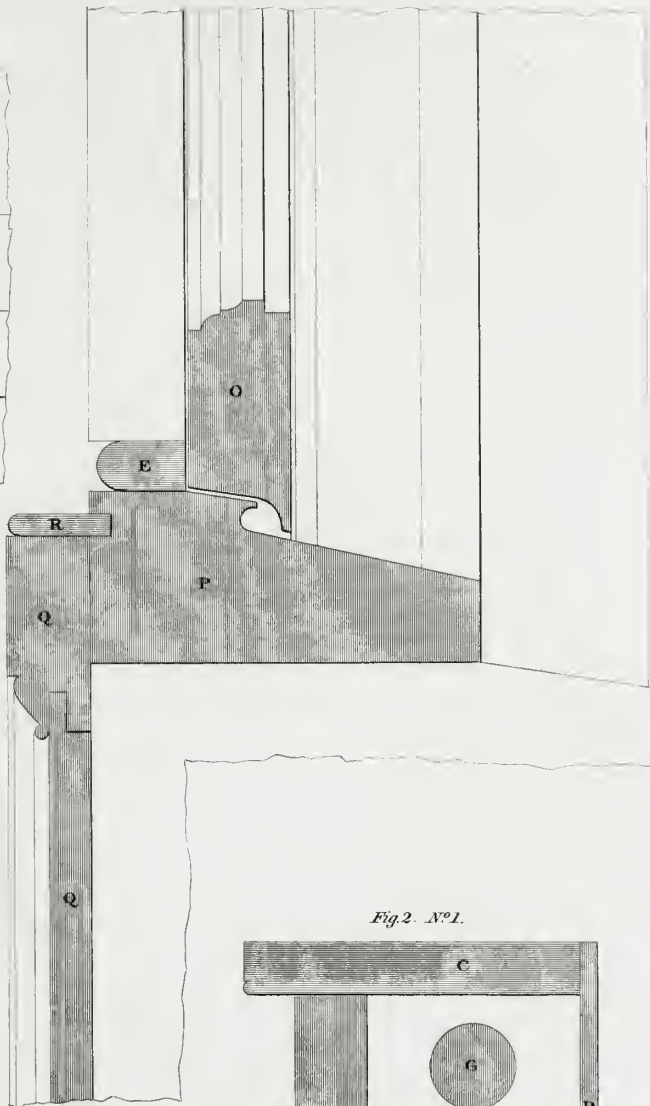


Fig. 2. N^o 1.

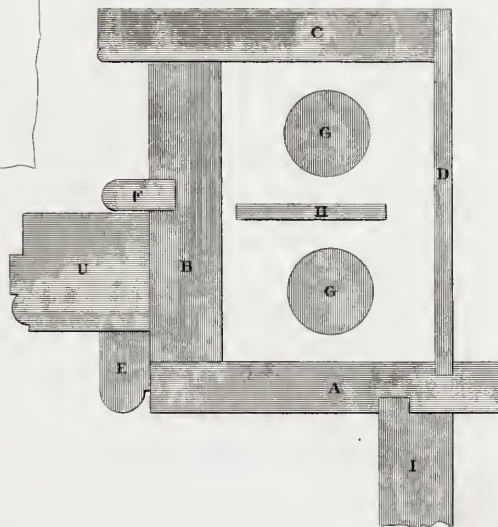
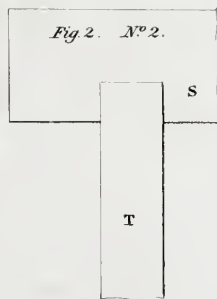


Fig. 2. N^o 2.



JOINERY.

CONSTRUCTION OF SHUTTERS.

PLATE. A.

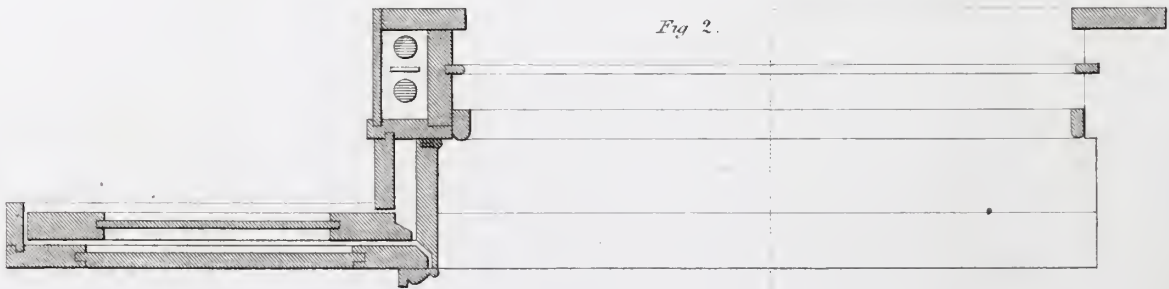
Fig 3



Fig 1



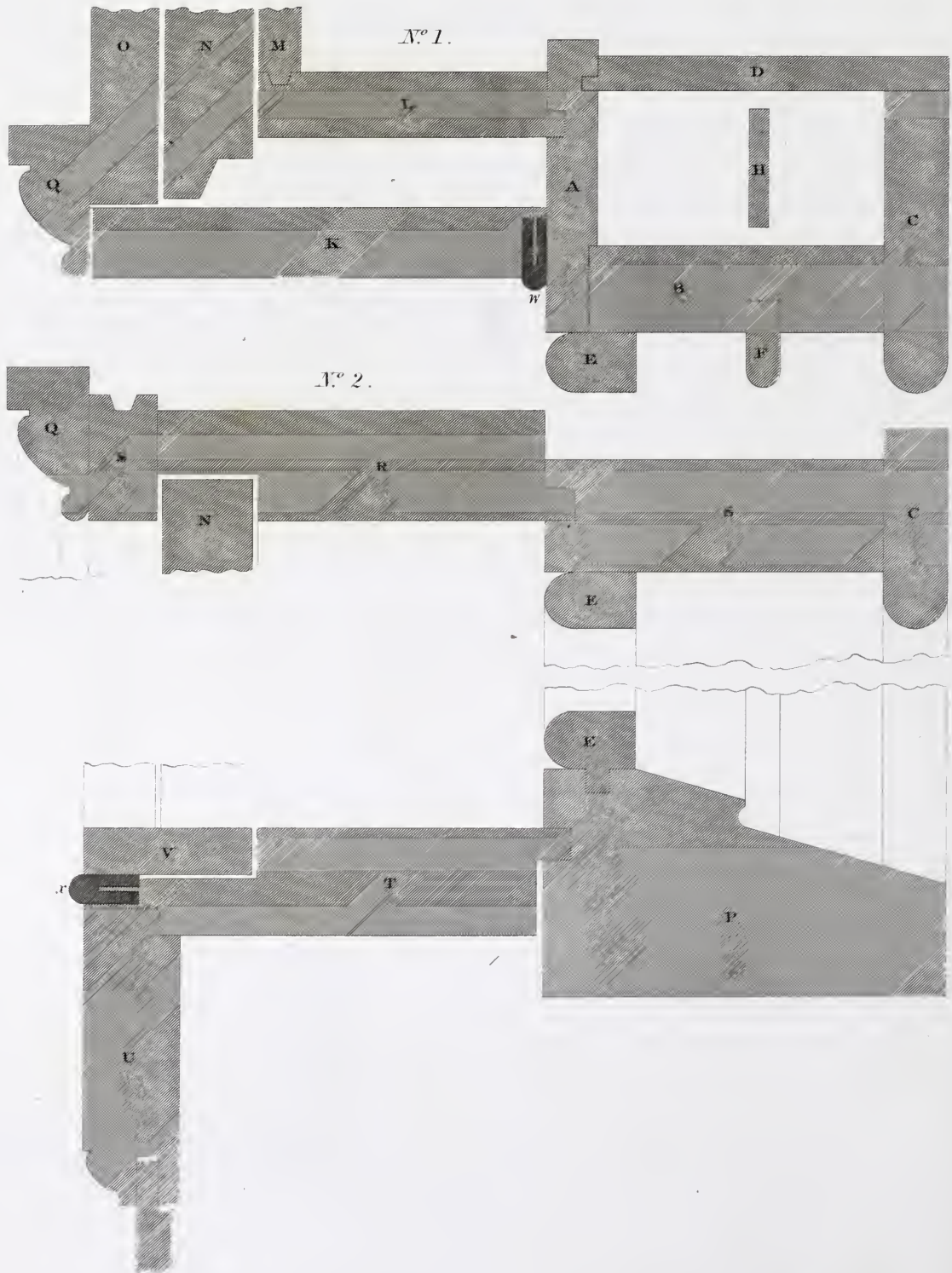
Fig 2.



JOINERY.

CONSTRUCTION OF SHUTTERS.

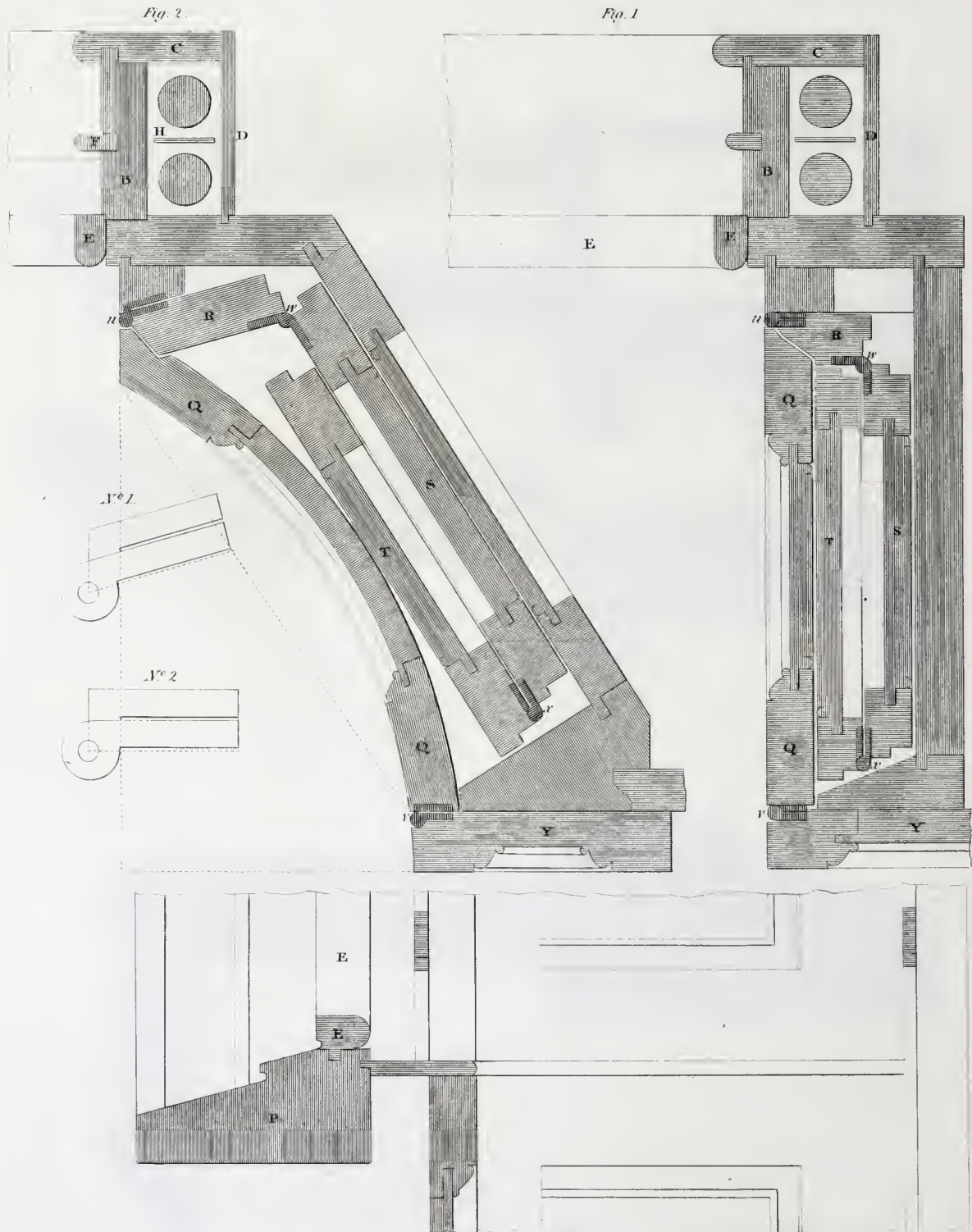
PLATE II.



JOINERY.

CONSTRUCTION OF SHUTTERS.

PLATE III.



Drawn by P. Vichelsen.

London. Published by John Day & Co. June 20, 1822.

P, sill of the sash frame.

Q, back of recess of window.

R, coping bead.

Fig. 2, sections of a more common sash frame.

No. 1, horizontal section through one side of the sash frame.

No. 2, part of the elevation of the head and pulley style of the sash frame.

S, part of the head.

T, part of the pulley style.

PLATE A.

Fig. 1, elevation of the inside of a window inserted in a wall, which is not of sufficient thickness to admit of the shutters on each side being contained in a boxing which does not project from the inside of the wall into the room. This is done by means of a sliding shutter within, and parallel to the surface of the wall; and the sliding shutter on each side is covered by a piece of framing, of which the outer face is flush with the surface of the plaster.

Fig. 2, horizontal section through the sash frame and shutters.

Fig. 3, vertical section through the sash frame.

PLATE II.

The corresponding parts have the same letters of reference as in Plate I.

In *No. 1*, horizontal section through sash frame; L, M, casing of wall for the shutter.

K, a door hung to the sash frame.

N, part of the sliding shutter.

O, part of the framed board covering the shutter.

Q, architrave moulding.

No. 2, a vertical section through the head of the sash frame and soffit of the window.

R, soffit.

S, P, part of ground.

PLATE III.

The design of this plate of the sections of shutters is to exhibit the same finish, whether the shutters are folded together and inclosed in the boxing, or unfolded and extended over the aperture, so as to exclude the light. This is obtained by means of a door hung to the architrave, which is opened when the aperture of the window is required to be closed; which being done, the door is again shut.

Fig. 1, design for a straight door.

Fig. 2, design for a door circular on the plan.

PLATE IV.

An entirely new method, by leaving a sufficient recess in the brick work, so that the shutters may fall in the space between the back of the sash frame and the jamb.

FITTING AND CUTTING WINDOW SHUTTERS.

PLATE I.

Fig. 1 exhibits the inside elevation of a window, where the sash and sash frame are out of the square.

Fig. 2, a vertical section through the window.

Young men, who are not aware of the difficulty which the obliquity of the sash frame occasions, are liable to spoil their work by cutting the shutters square to the joints, which run upwards; by this means, the transverse joints will not be parallel to the horizontal bars of the window, as they ought to be.

PLATE II.

This plate exhibits the method of cutting the shutters, which is as follows:—

Having first fitted the shutters which are hung to the sash frame in the boxings to the full length, revolve them on their hinges on the face of the window; fit in the intermediate parts or back flaps for each half of the aperture, with the proper rabbets on the edge of each flap. Draw a straight line parallel to the top and bottom ends of the shutters in the middle of the breadth of the meeting bars of the two sashes; then the hinges must be placed in a straight line perpendicular to this line or to the ends, and as near to each joint as may be found convenient.

SKYLIGHTS.—PLATE I.

Fig. 1, square skylight.

Fig. 2, octagonal ditto.

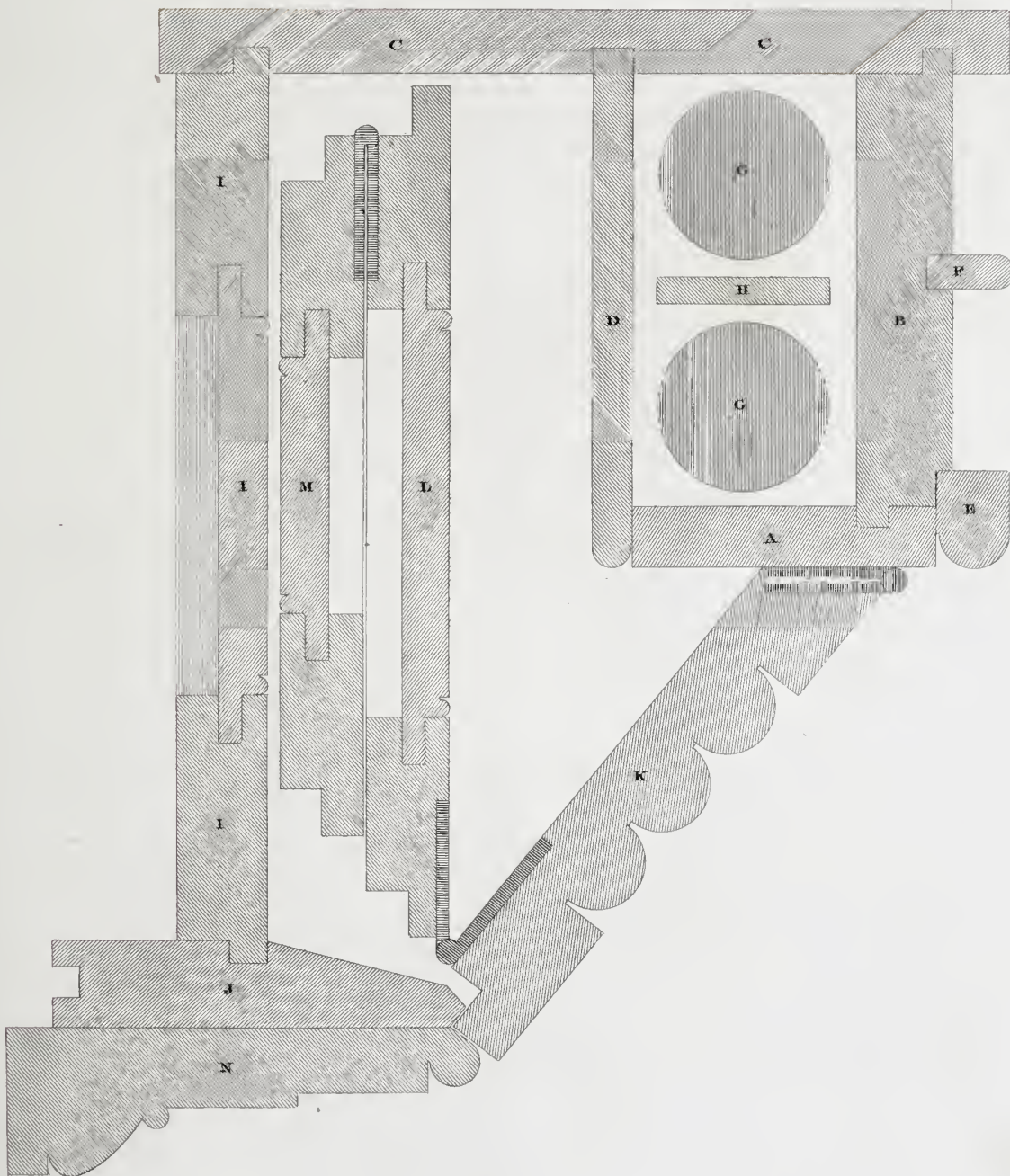
Each of these figures show the backing of the hips in the same manner as the hip of a roof.

Figs. 3 and 4, skylights upon elliptic plans. In order to space out the ribs for each quarter.

JOINERY.

CONSTRUCTION OF SHUTTERS

PLATE IV.



JOINERY,

FITTING & CUTTING WINDOW SHUTTERS.

PLATE I.

Fig 2

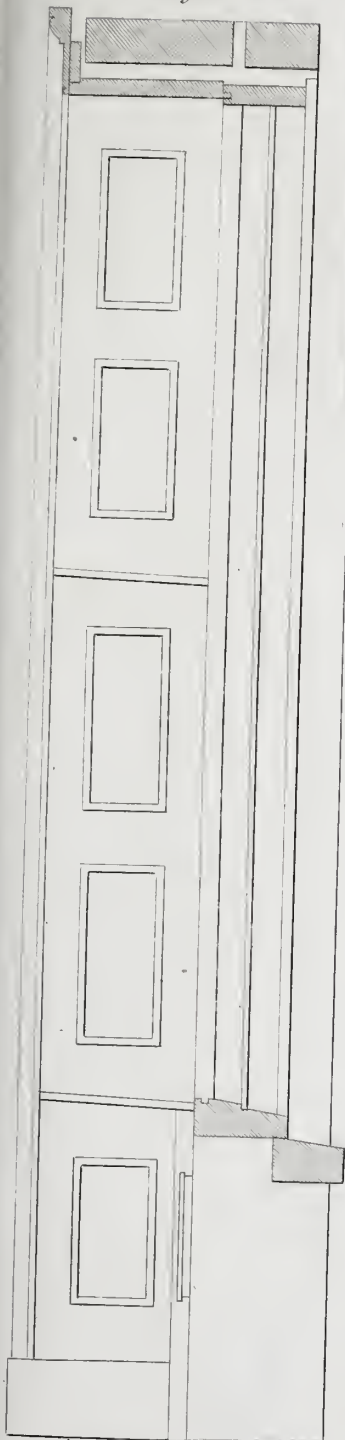


Fig 1



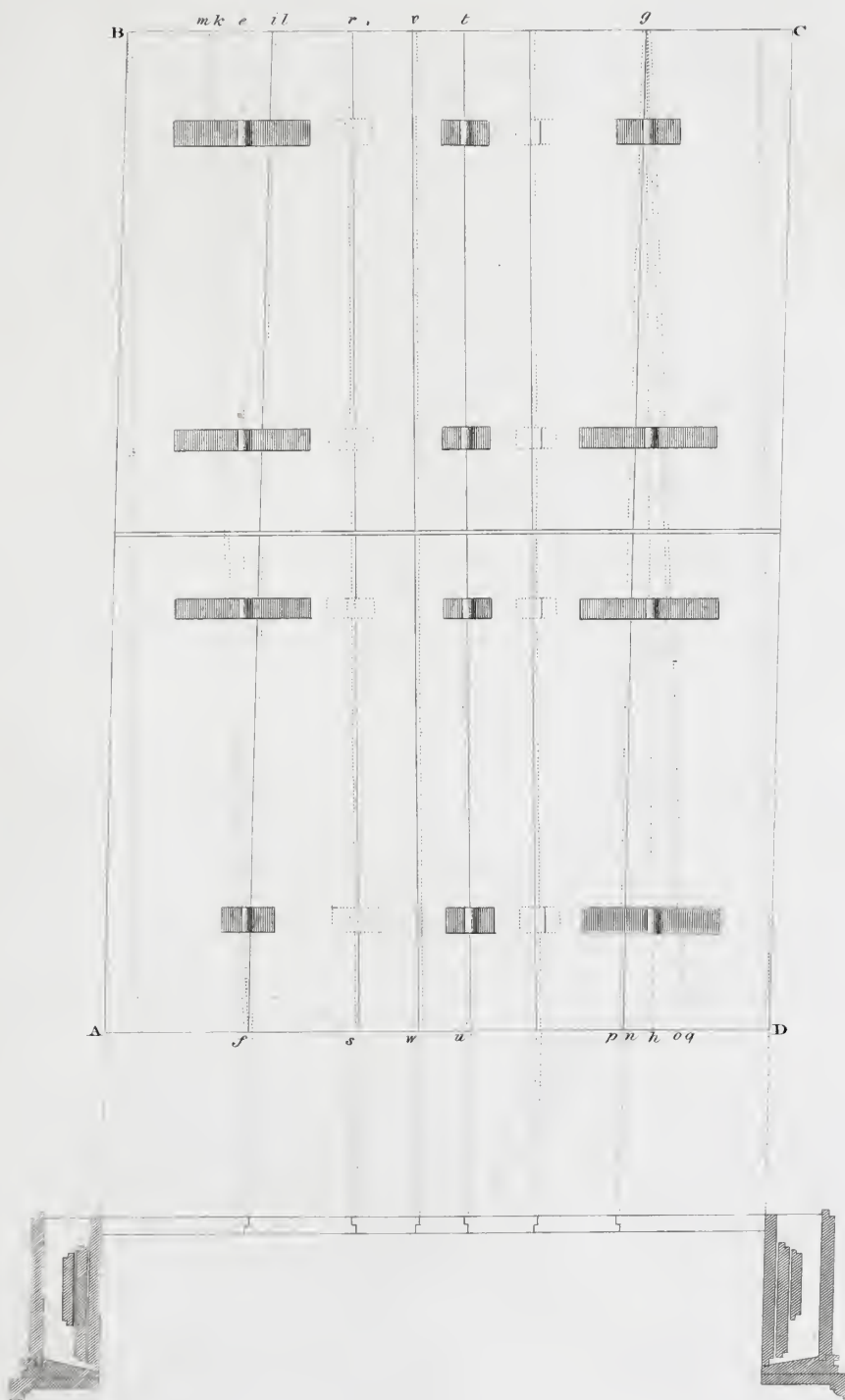
Introduced by P. Nicholson.

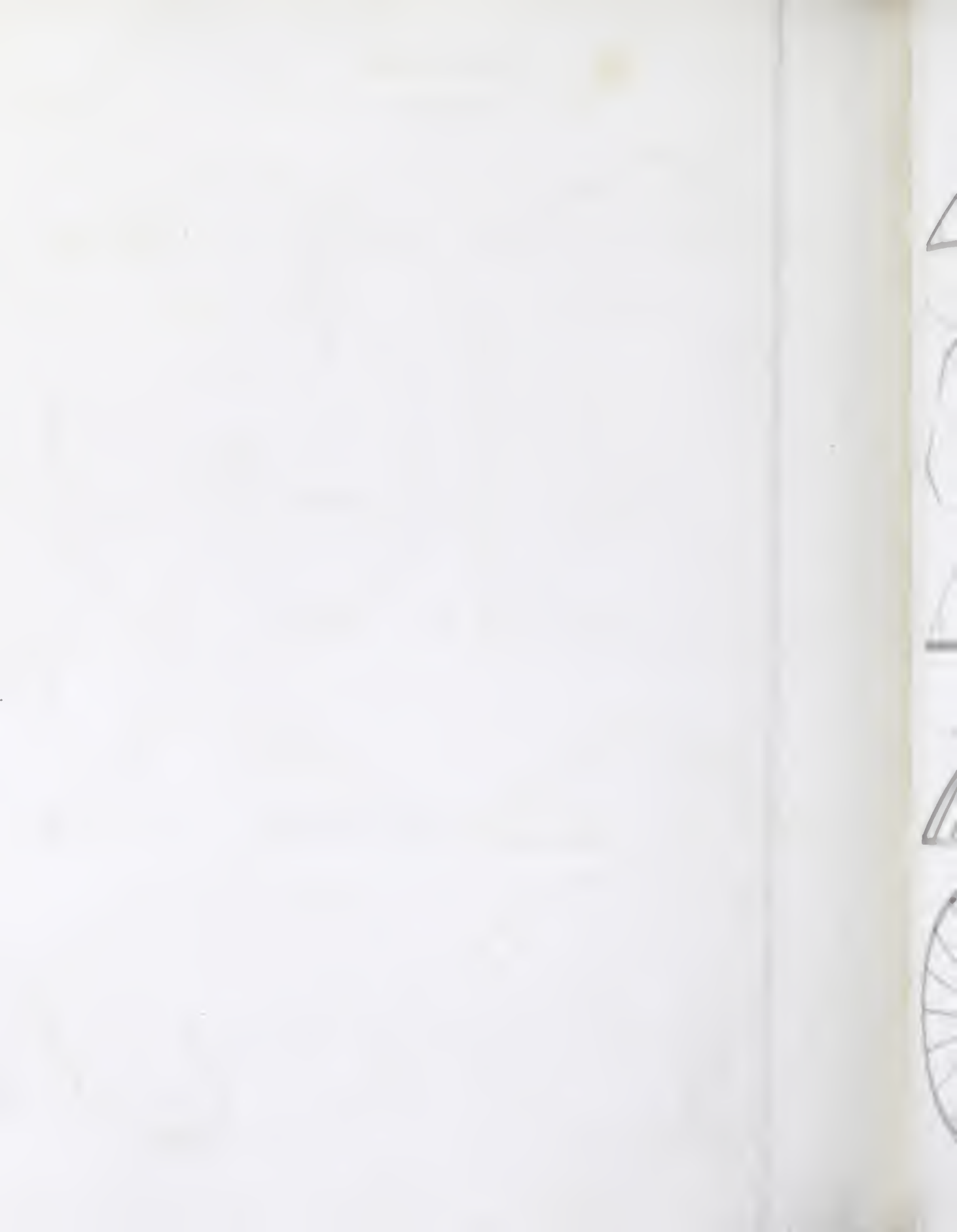
London, Published by John Day & Co. Oct. 20. 1822.

JOINERY,

FITTING & CUTTING WINDOW SHUTTERS.

PLATE, II.





JOINERY.

SKYLIGHTS

PLATE I.

Fig 1



Fig 2.

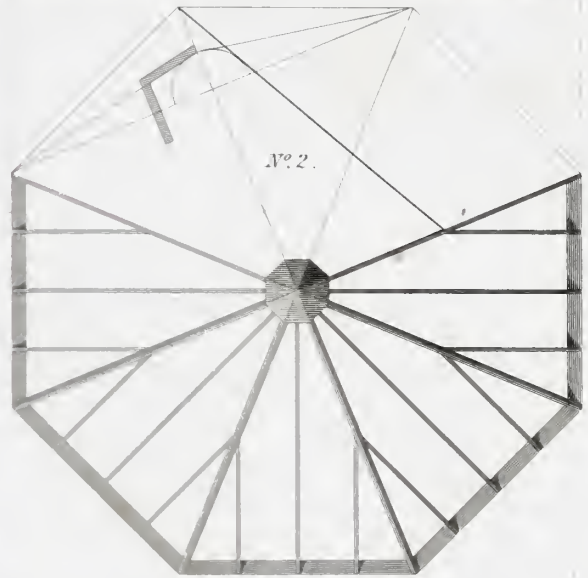
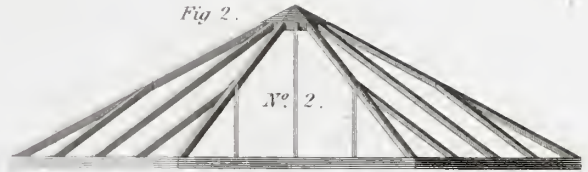


Fig 3

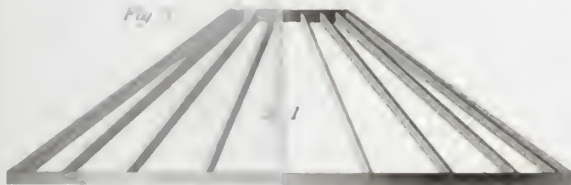
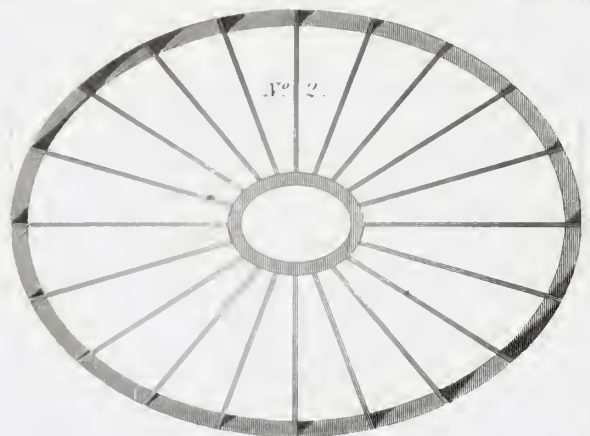
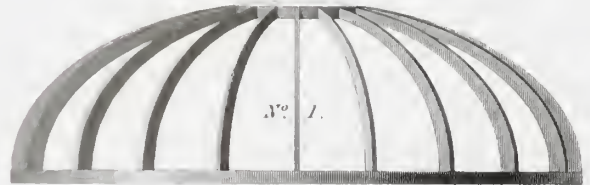


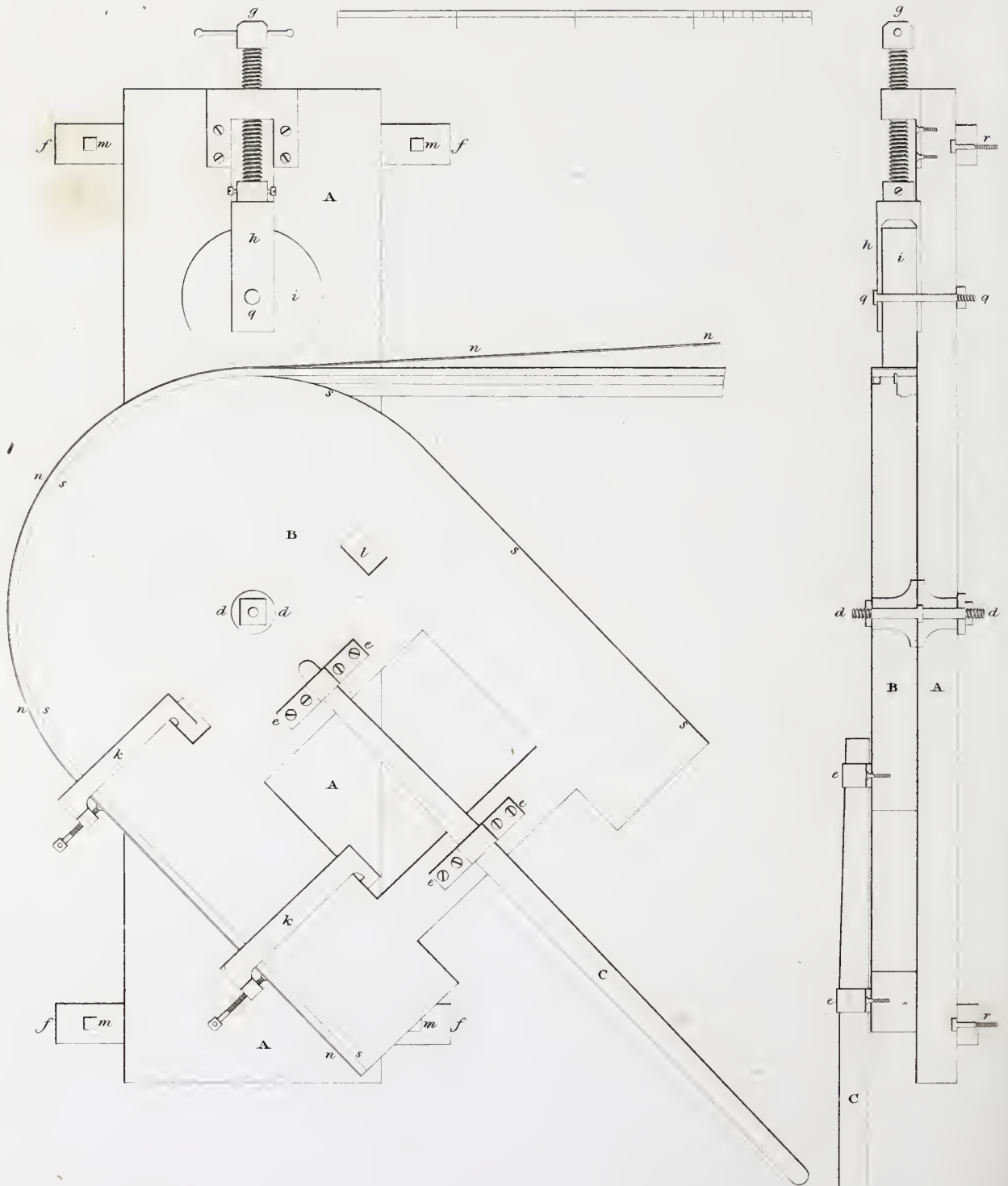
Fig 4.



JOINERY.

BENDING MACHINE.

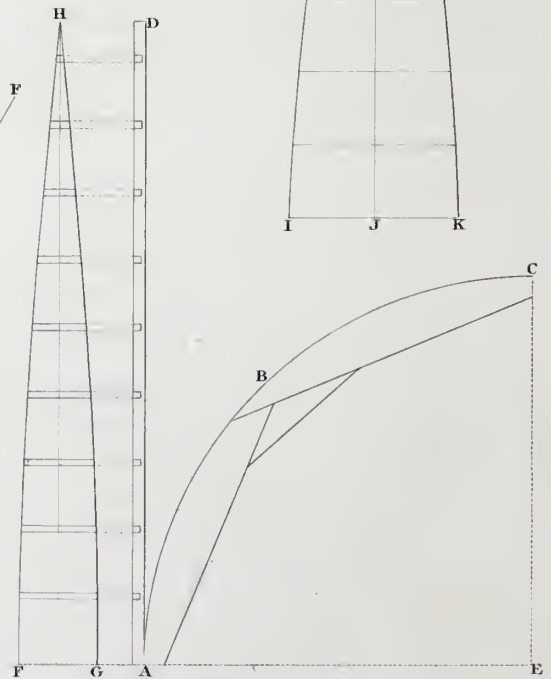
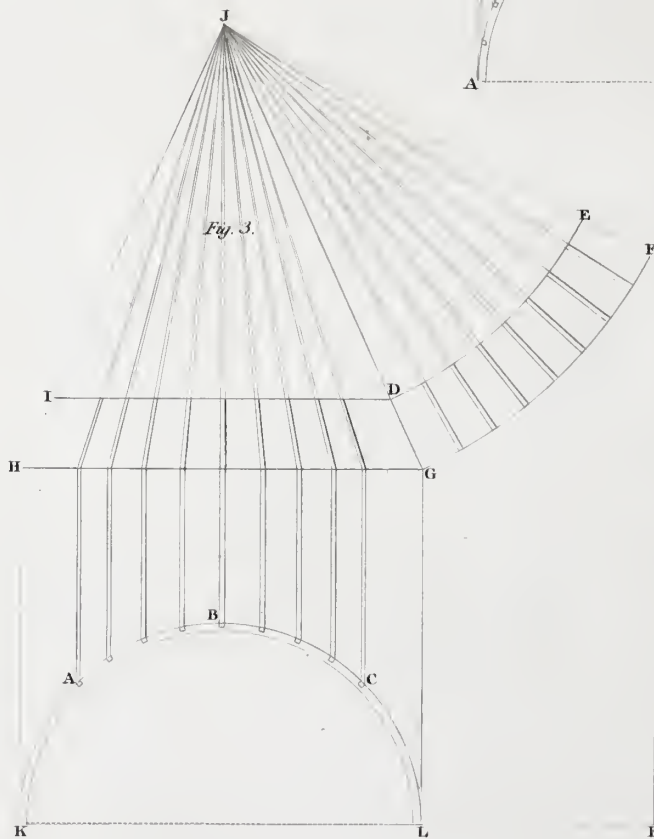
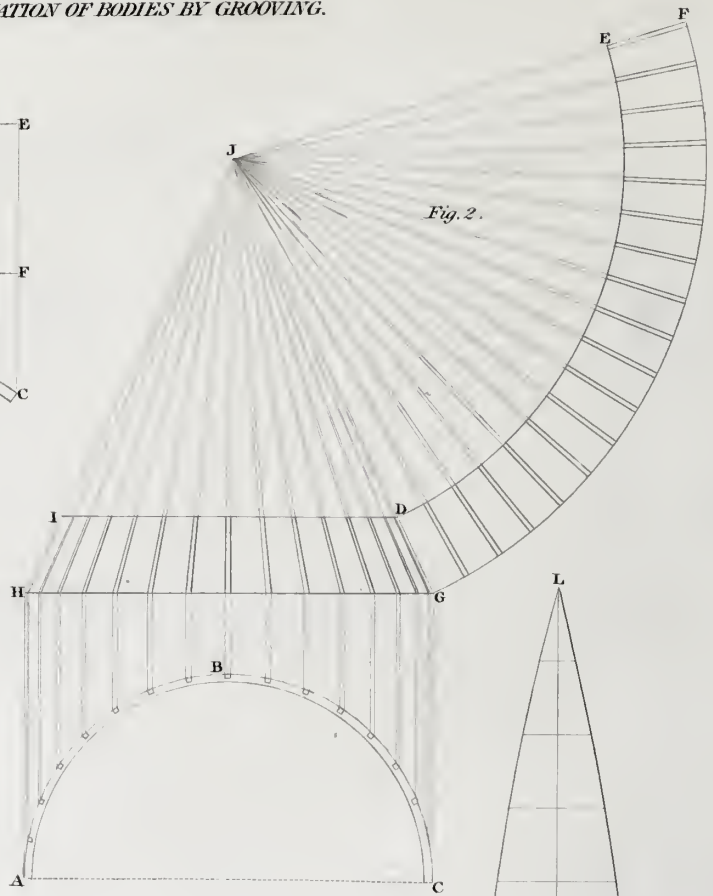
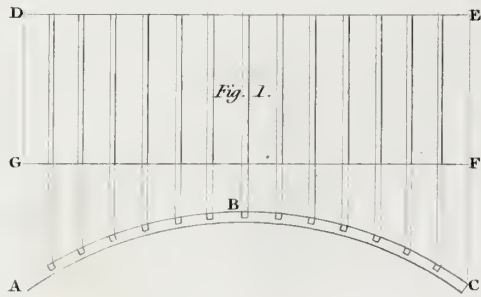
PLATE. I.



JOINERY.

PLATE I.

INCURVATION OF BODIES BY GROOVING.



Join AB, and bisect AB by a perpendicular, which make equal to the half of AB; then C being the extremity, join AC and BC. From C, with a radius equal in length to the perpendicular, describe an arc, meeting AC in 0, and BC in 5. Divide 0.5 into five equal parts, the number intended to be in each quarter, and through the points of division 1, 2, 3, 4, draw the lines meeting AB in the points *d, e, f, g*. Through the points *d, e, f, g*, draw lines from the centre of the figure to meet the elliptic curve, and the points of intersection are the places of the feet of the bars as required.

Fig. 4 is seldom executed, on account of the difficulty of incurvating the glass.

The plan in No. 2 is equally divided.

BENDING MACHINE.—PLATE I.

This plate represents a machine for bending sash bars, styles, beads, &c.

The plan is represented on the left side of the plate. AA represents the bed of the machine, which may be a plank suitable to the articles to be bent; *f, f, f, f* represents bearers screwed to the bed, and likewise screwed down to a work bench, as shown at the section on the right hand: MM represents the heads of the screws; B shows a templet (commonly called a cylinder by workmen), the centre of which is at *d, d*, and is supposed to be employed bending a sash style and bead at the same time, as shown in the section.

Suppose the style intended to be bent to be worked to its proper rabbet and mouldings, and the templet rabbeted to receive it and the bead also; then suppose the style to be fastened to the straight part of the templet by means of small cramps, as represented at KK. *nnnn* represents a piece of iron hoop which is pressed close to the templet by means of the wheel *ii* and the screw *gg*; the cylinder is supposed to be in the act of being moved round by means of the lever *cc*, and when brought far enough round, may be confined by cramps as on the other side.

INCURVATION OF BODIES BY GROOVING.—PLATE I.

Fig. 1 exhibits the method of forming a concave cylindric surface.

ABC is a transverse section, and DEFG the elevation of the back.

Fig. 2, the method of forming a concave conic surface for a half cone, ABC being the transverse section, and DGHI the elevation of the back.

Fig. 3 exhibits the method of forming a concave conic surface for a segment less than a semi-cone, ABC being the portion required, and DGHI the elevation.

The remaining figure shows how a concave spherical surface is formed in gores, AD being the section, FGH the gore, and ABC the templet to bend it upon.

ELLIPTIC OR CIRCULAR ARCHIVOLT ON A CIRCULAR PLAN.

PLATE I.

Fig. 1, plan and elevation.

Divide the inner curve of the elevation into any number of equal parts at the points 1, 2, 3, &c., and draw the line *sa*, 1*b*, 3*c*, 4*d*, &c. perpendicular to the chord of the opening, meeting the wall line in the points *a*, *b*, *c*, *d*, &c. In the line MN, *Fig. 2*, set off the distances *ab*, *bc*, *cd*, &c. equal to the developements of *ab*, *bc*, *cd*, *fig. 1*. To the line MN draw the perpendiculars *b1*, *c2*, *d3*, &c., and make the heights of these perpendiculars equal to those in the elevation, *fig. 1*; through all the points *a*, 1, 2, 3, &c. draw a curve. Make *aM* equal to the breadth of the archivolt, and draw the outer curve MHPIN parallel to the inner curve. Here the veneers are made in three lengths.

RAKING MOULDINGS.—PLATE III.

Given the inclination of a moulding to the horizon, and a section of that moulding, to find the section of the return moulding.

Let BE, *fig. 1*, be the line of inclination, and let CDM be a section of the moulding. Draw a line through A parallel to EB. Through M draw MD perpendicular to the line passing through A, meeting it in P. Draw M*m* parallel to EB, and draw any line af parallel to the horizon above the moulding. In af make ap equal to AP, and draw *pm* perpendicular to af, and *m* will be a point in the curve. This may be applicable to the skirting of a stair, where the passages return both above and below.

In order to avoid the trouble of making mouldings to various sections, torus skirting is most frequently executed, as is shown in *figs. 2* and 3.

Fig. 4 shows the method of finding the angle bars of shop fronts. No. 1 is the given bar, Nos. 2 and 3 are angle bars found from No. 1. Thus *rm*, No. 3, is equal to RM, No. 1. In No. 2, make *ap* equal to AP, No. 1, and draw *pm* parallel to *aE*.

STAIRS.—PLATE I.

Fig. 1, plan and elevation of a dog-leg stair without winders.

Fig. 2, plan and elevation of a dog-leg stair, with winders in one half of the turning.

HAND RAILING.

PLATE I.

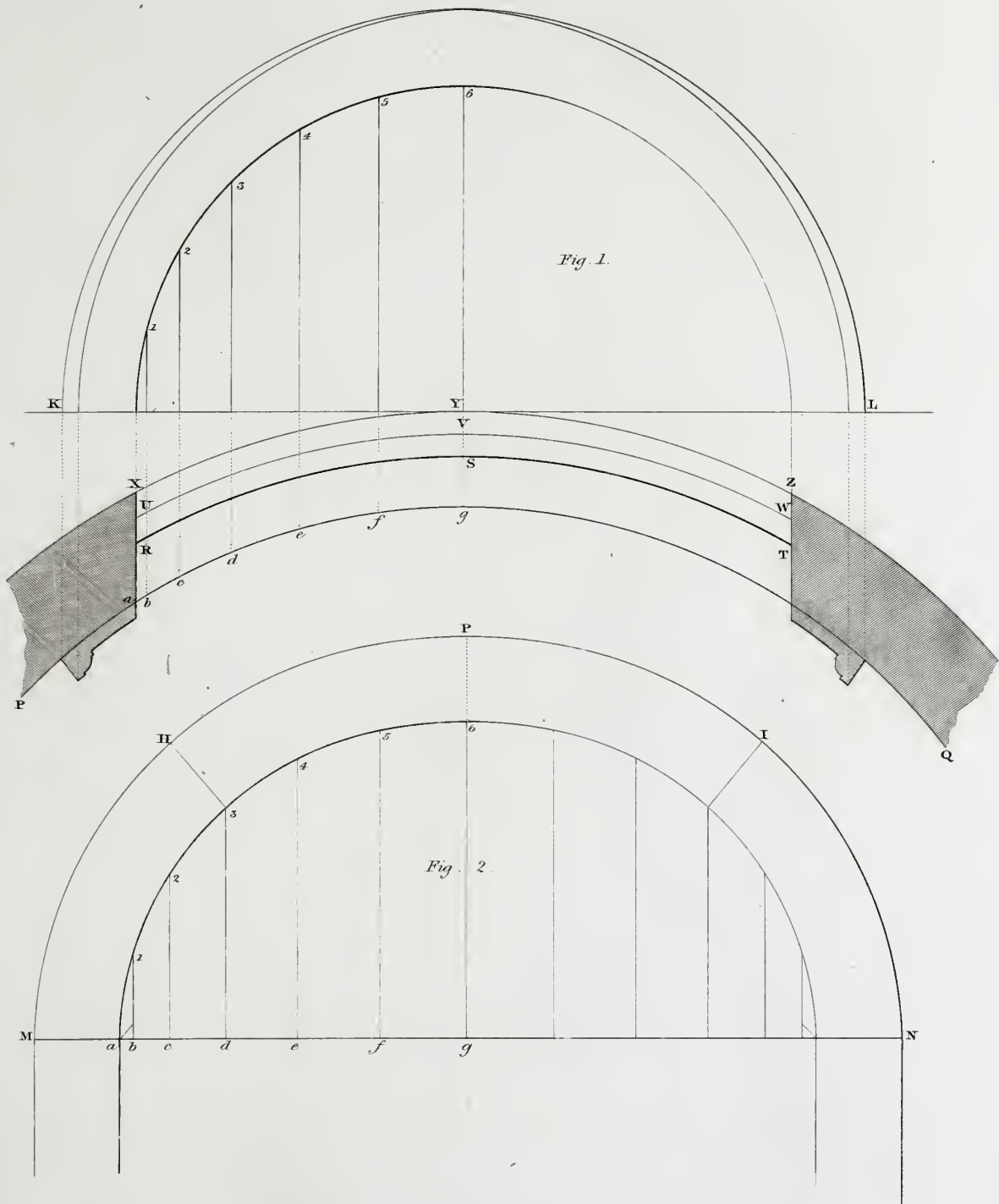
Fig. 1, exhibiting the cylinder with the rail squared.

Fig. 2 exhibits the method of describing the rail on the elevation, *fig. 1*.

JOINERY,

ELLIPTIC OR CIRCULAR ARCHITRAVE ON A CIRCULAR OR ELLIPTIC PLAN.

PLATE. 1.





RAKING MOULDINGS.

Fig. 1.

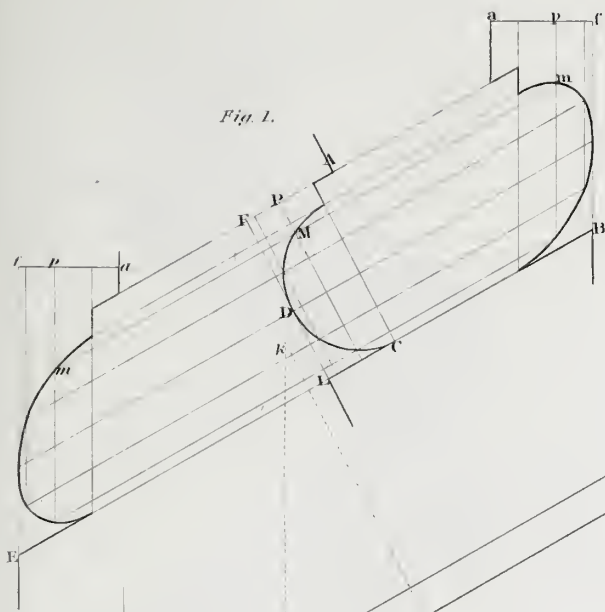


Fig. 2.

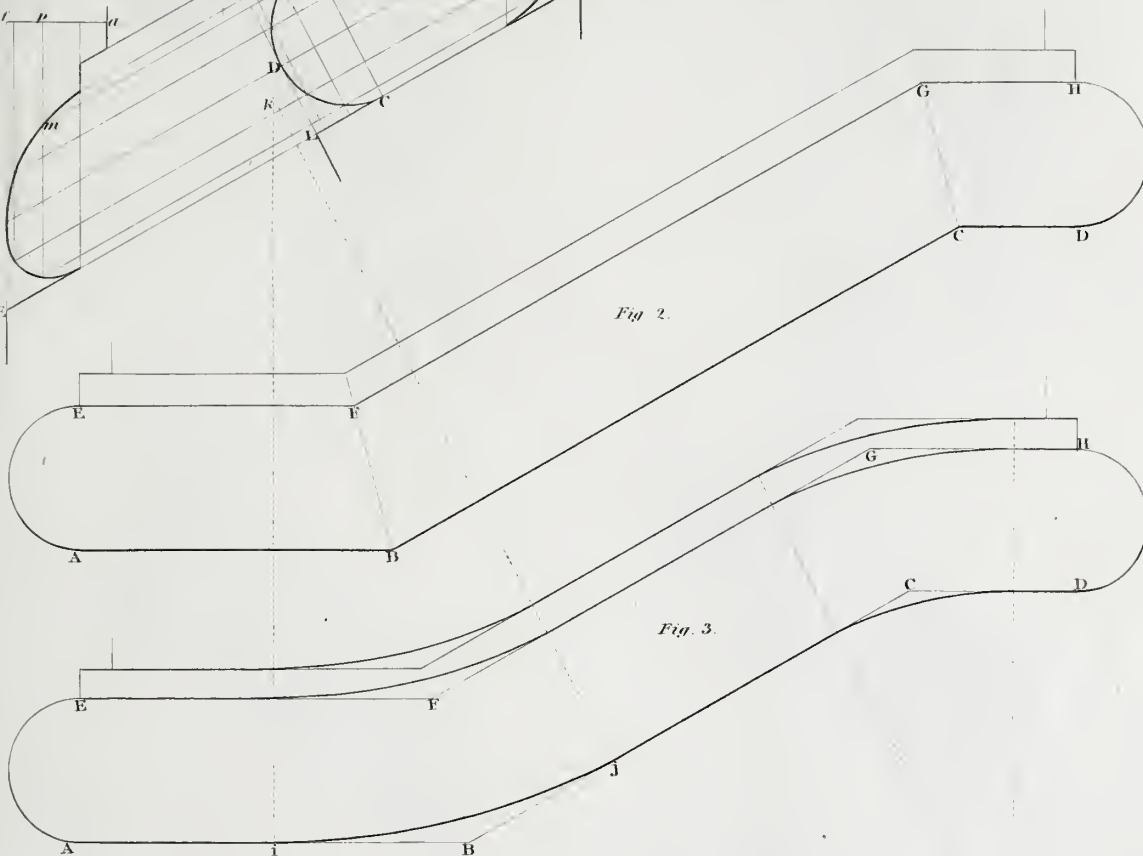


Fig. 3.

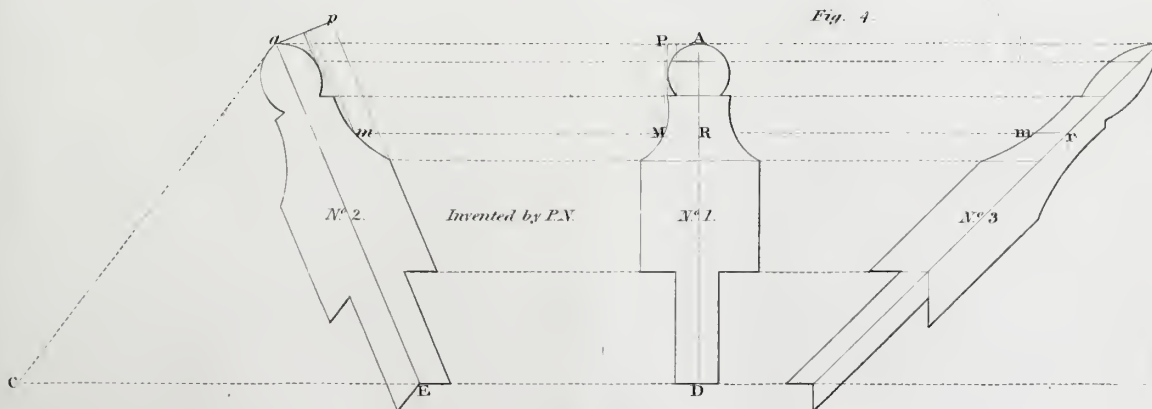


Fig. 4

~~N.º 2.~~

Invented by P.N.

N.º 1.

No. 3

Drawn by P. Nicholson.

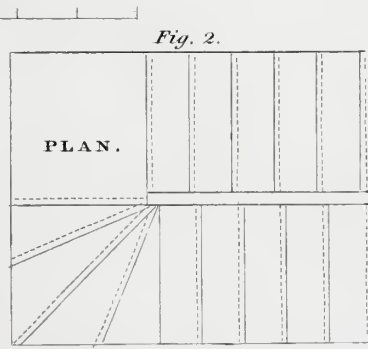
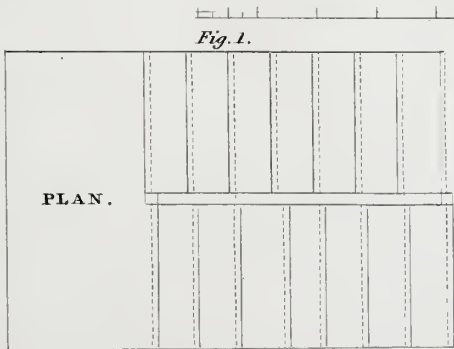
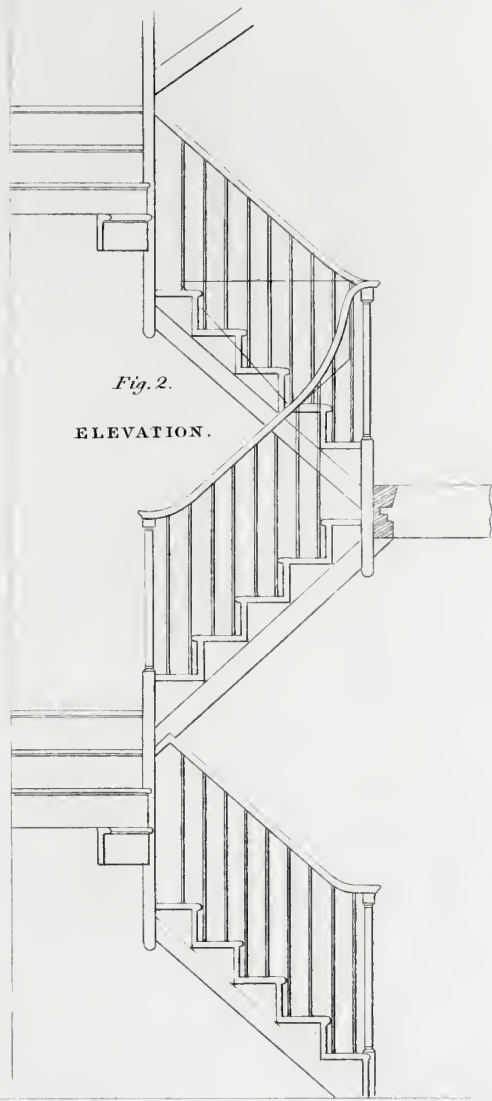
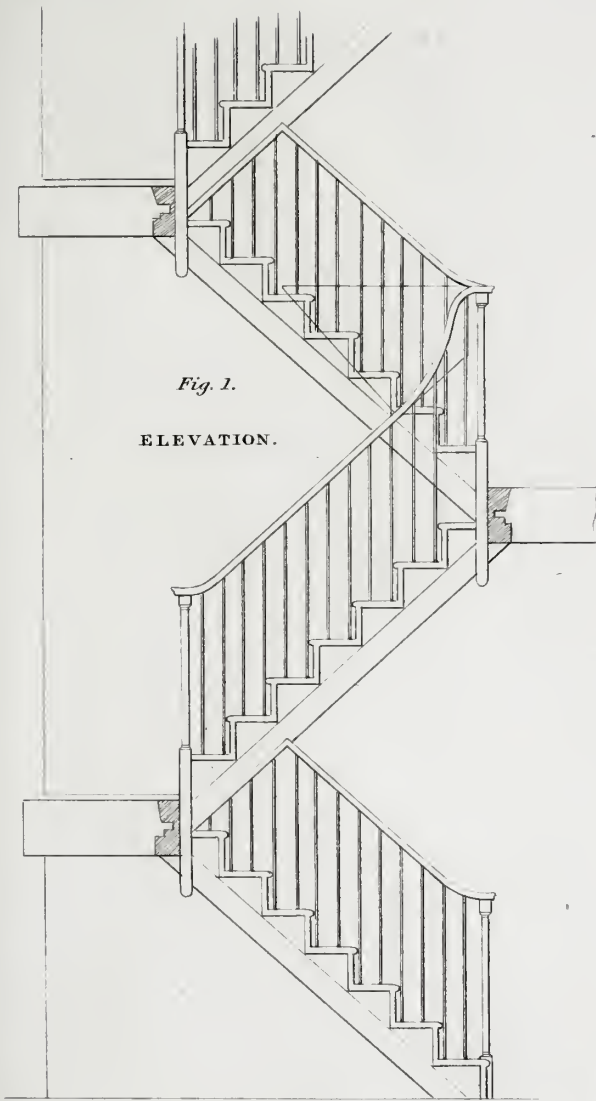
London. Published by John Day & Co. June 20 1892



JOINERY.

STAIRS.

PLATE I.





JOINERY.

HAND RAILING.

PLATE I.

Fig. 2.

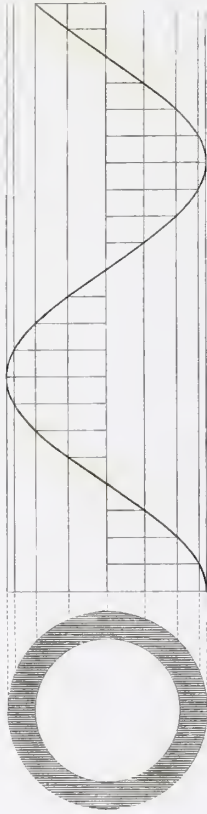


Fig. 1.

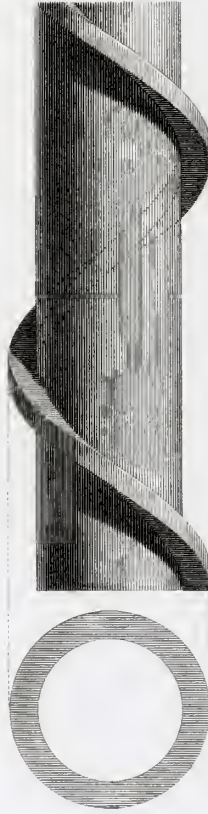


Fig. 3.



Fig. 4.



Fig. 5.

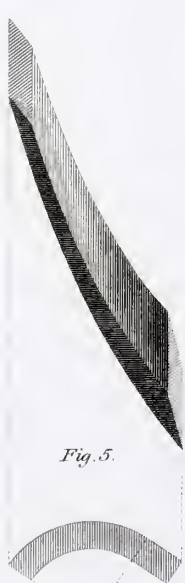


Fig. 6.

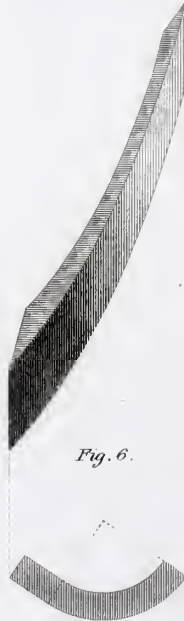


Fig. 7.



Fig. 8.



JOINERY,

HAND RAILING.

PLATE VII.

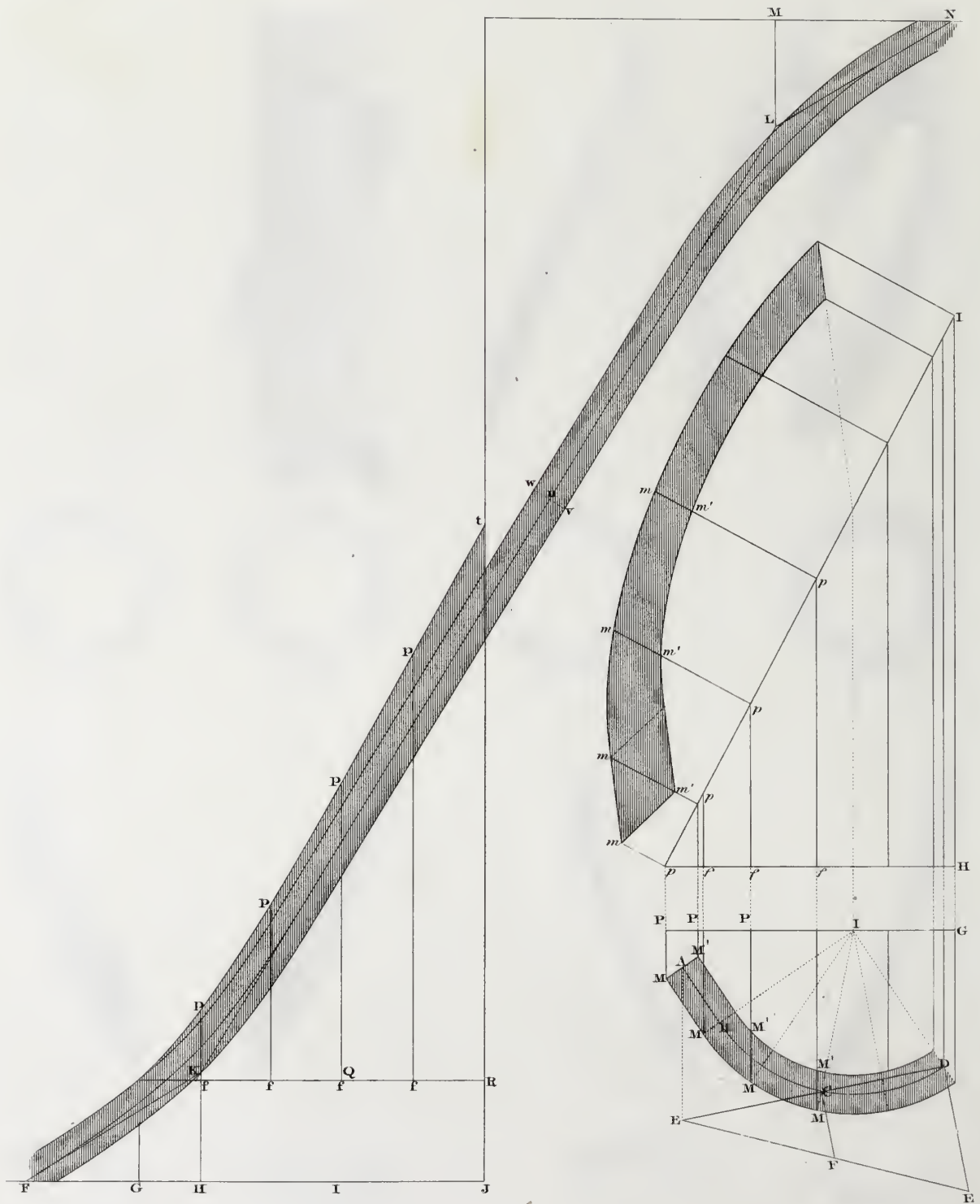


Fig. 3 shows the position of the cutting plane, and the portion of the cylinder contained between two parallel planes; and as the cylinder is supposed to be hollow, *fig. 4* shows the same for the entire cylinder and for the half.

Figs. 5 and *6* show the concave and convex side of the same for one quarter, taking in a small portion of the straight rail.

Figs. 7 and *8* exhibit *figs. 5* and *6* completely squared.

Figs. 5 and *6*, the first state of the pieces as cut from the rough plank. *Figs. 7* and *8*, the pieces entirely squared.

PLATE VII.

To find the moulds of a hand-rail with 8 winders in the turning or 4 in each half.

Let *MMM*, &c. be outside of the plan, and *M'M'M'*, &c. the inside of the same, *ABCD* a line passing through the middle of its breadth, the part *AB* being straight, and *BCD* one fourth of the circumference of the circle; the point *C* in the middle of the arc *BD*, *A* at one extremity of the line, *ABCD* and *D* at the other.

Divide the quadrant *BCD* into any number of equal parts, which in this example are 4. Draw the straight line *RK*, and make *RK* equal to the developement of the quadrant *MMM*, &c. on the convex side. Draw *KH* perpendicular to *RK*, and make *KH* equal to the height of a step. Draw *HF* parallel to *RK*, and make *HF* equal in length to the breadth of a flyer, and join *FK*.

Draw *Rt* perpendicular to *RK*. In *Rt* make *Ru** equal to the height of 4 winders, and join *uK*, curve of the angle at *K*. Through *u*, draw *vw* perpendicular to *uK*. Make *uv* and *uw* each equal to half the breadth of the falling mould, and draw the upper and lower edges of the falling mould, as seen in the figure.

Join *DC*, and produce *DC* to *E*. Draw *DE* and *CF* perpendicular to *DE*. Make *DE* equal to one fourth (or any part) of the height from *R* to the upper edge of the falling mould in the perpendicular *Rt*, and *CF* equal to one fourth (or the same part) of the height from *Q* to the upper edge of the falling mould in the perpendicular *QP*, that *DF* was of the height in the perpendicular *Rt*. Join *EF*, and produce it to meet *DC* in *E*. Join the dotted line *EA*. Draw *GP* through the centre *I*, or at any convenient distance from the plan perpendicular to *EA*, and draw *Mp* parallel to *EA*, meeting *GP* in *P*. At any convenient distance, draw *Hp* parallel to *GP*. Make the perpendicular of the face mould equal to its corresponding height on the falling mould, and draw the straight line *pI*; then drawing ordinates *PM*, *PM*, &c. and *pm*, *pm*, &c. from the points *P*, *P*, *P*, &c. and *p*, *p*, *p*, &c. where lines parallel to *HI* meet *PG* and *PI*, and making *pm* equal to

* *u* is, by mistake, placed too far up.

the corresponding PM on the plane both outside and inside, we shall have the face mould required. By making the middle of the falling mould at the heights, as the nomings of the steps, the same face mould will apply to the upper and lower parts of the rail over the circular part.

The top line PPP, &c. is left on the falling mould to regulate its position, when bent upon the convex surface, as the line PPP, &c. will fall into the plane surface of the top of the plank. The line PPP, &c. is obtained by making the perpendiculars fP, fPfP, &c. equal to the corresponding perpendiculars fp, fp, fp, &c.

PLATE VII*.

This plate exhibits the construction of the face mould and falling mould, supposing the rail to be raised higher upon the winders than upon the flyers. Here the curve line *stuvwx* will coincide with the top surface of the plank, and the lower edge of the falling mould above. The method of proceeding to find the falling and face moulds is so similar to the description of Plate VII, that any farther explanation would rather retard than facilitate the progress of the learner.

PLATE VIII.

This plate shows the application of the trammel to the description of the face mould.— See the article *Stereotomy*, Primary Problems, from p. 99 to 102 inclusive.

PLATE IX.

This plate exhibits the method of finding the elevation of the squared piece so as to get it out of the least quantity of stuff. The position of the plan is found in the same manner as in Plate VII, Hand Railing, and the heights are taken from those of the filling mould.

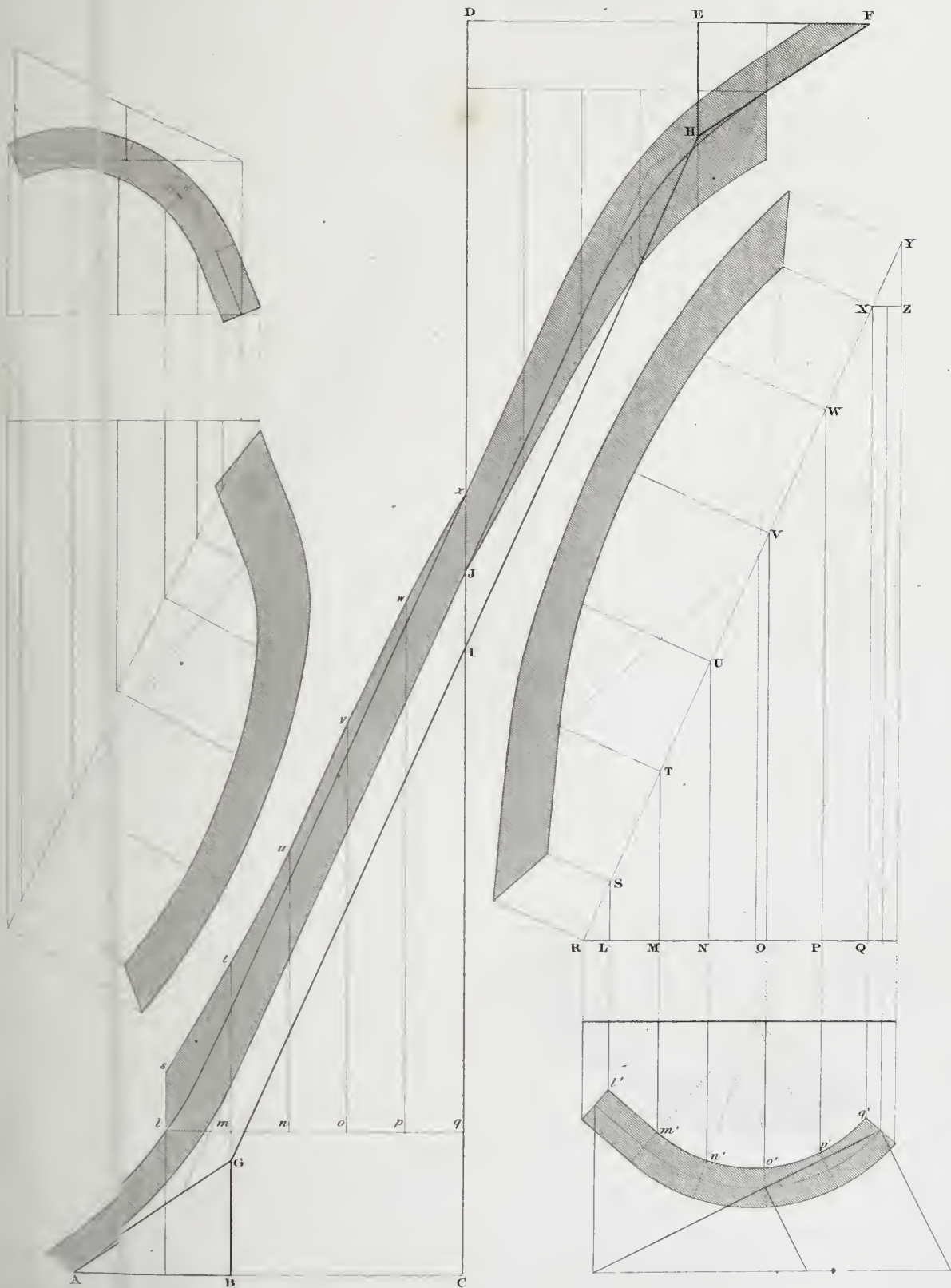
PLATE X.

This plate exhibits the elevation of the squared piece of the rail where the ordinates are taken perpendicular to the chord joining the extremities of the concave side of the piece. The heights are taken from the corresponding heights on the falling mould in the same manner as in the preceding plate. The intention of this plate is to show that the rail will require much thicker stuff than if the position of the plan had been found as in the preceding plate.

JOINERY,

HAND RAILING.

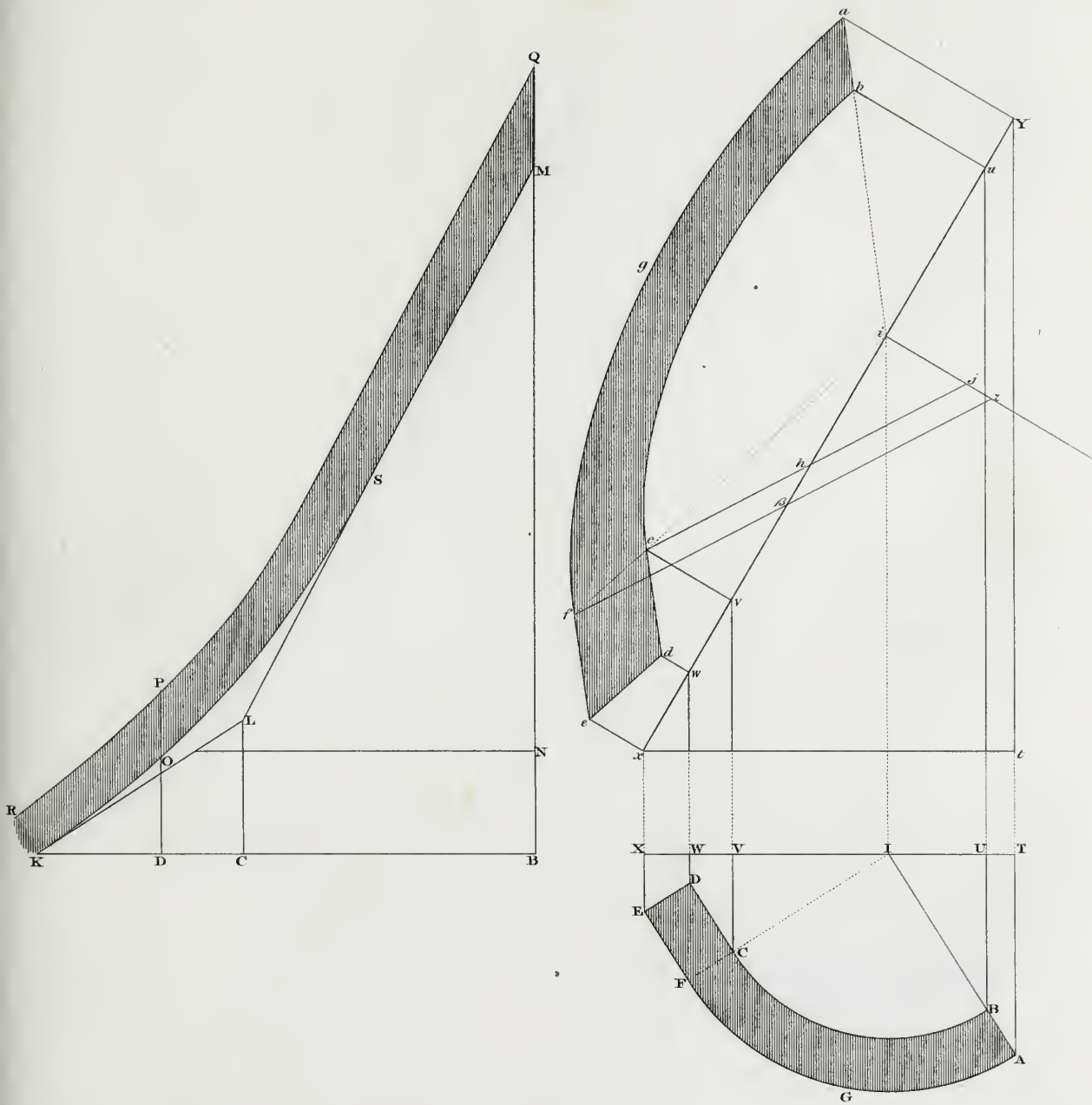
*PLATE. VII.**



JOINERY.

HAND RAILING.

PLATE VIII.

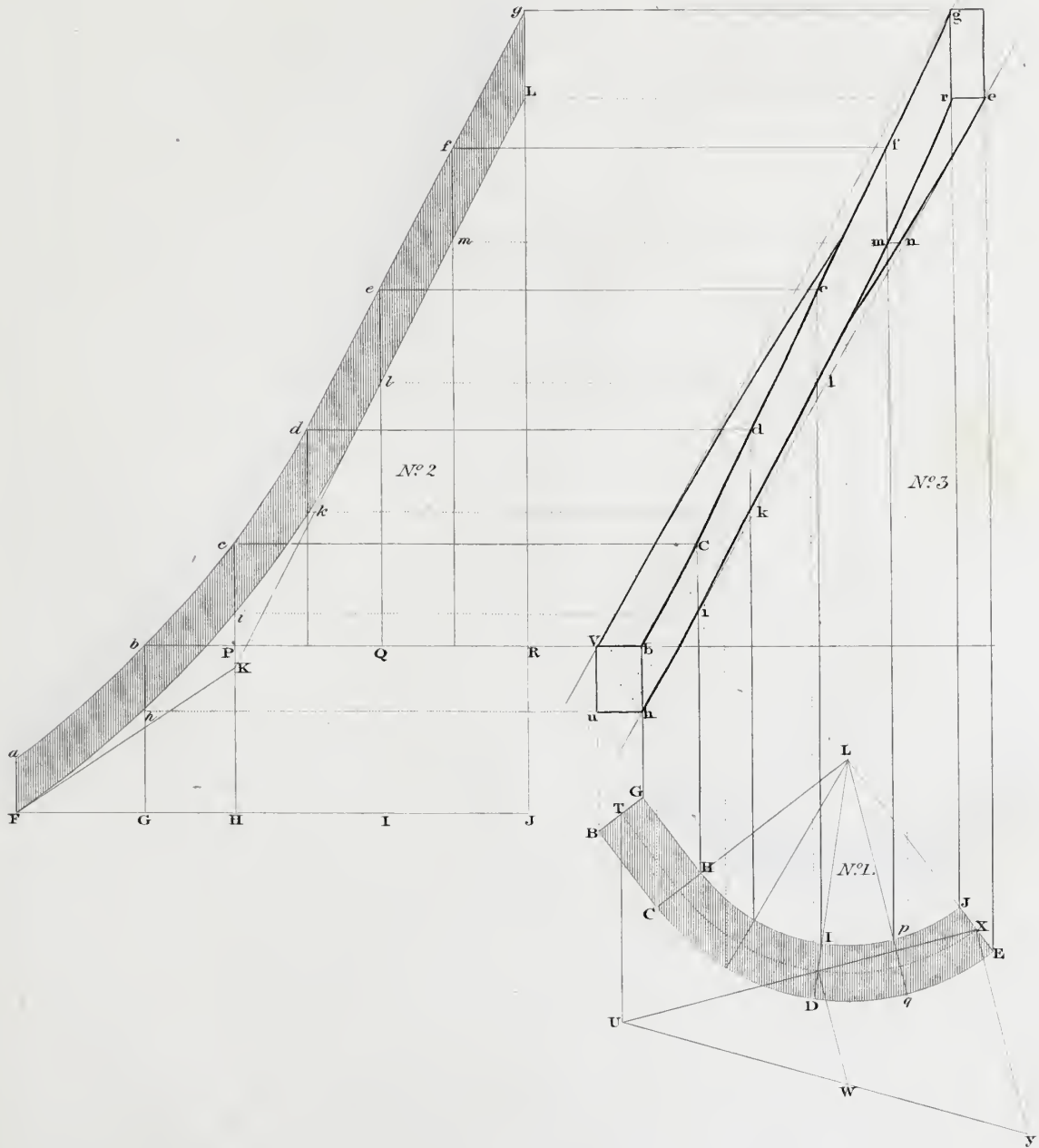




JOINERY.

HAND RAILING.

PLATE IX.



Invented by P. Nicholson.

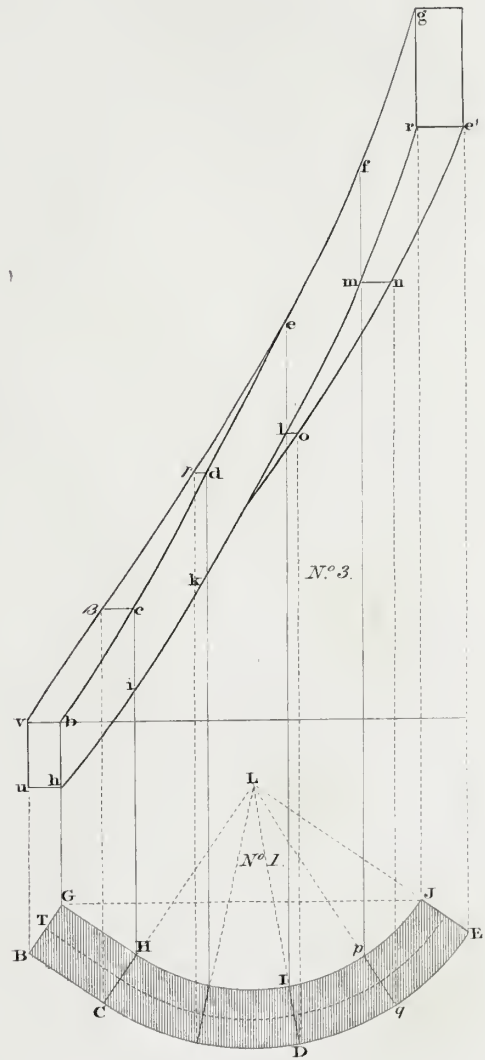
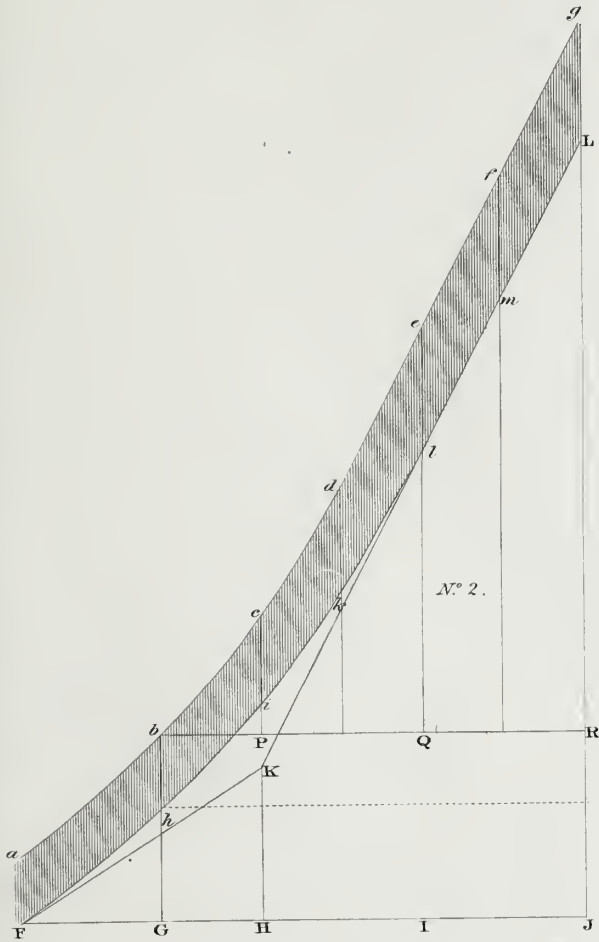
London, Published by John Day & Co Oct. 20. 1822.



JOINERY,

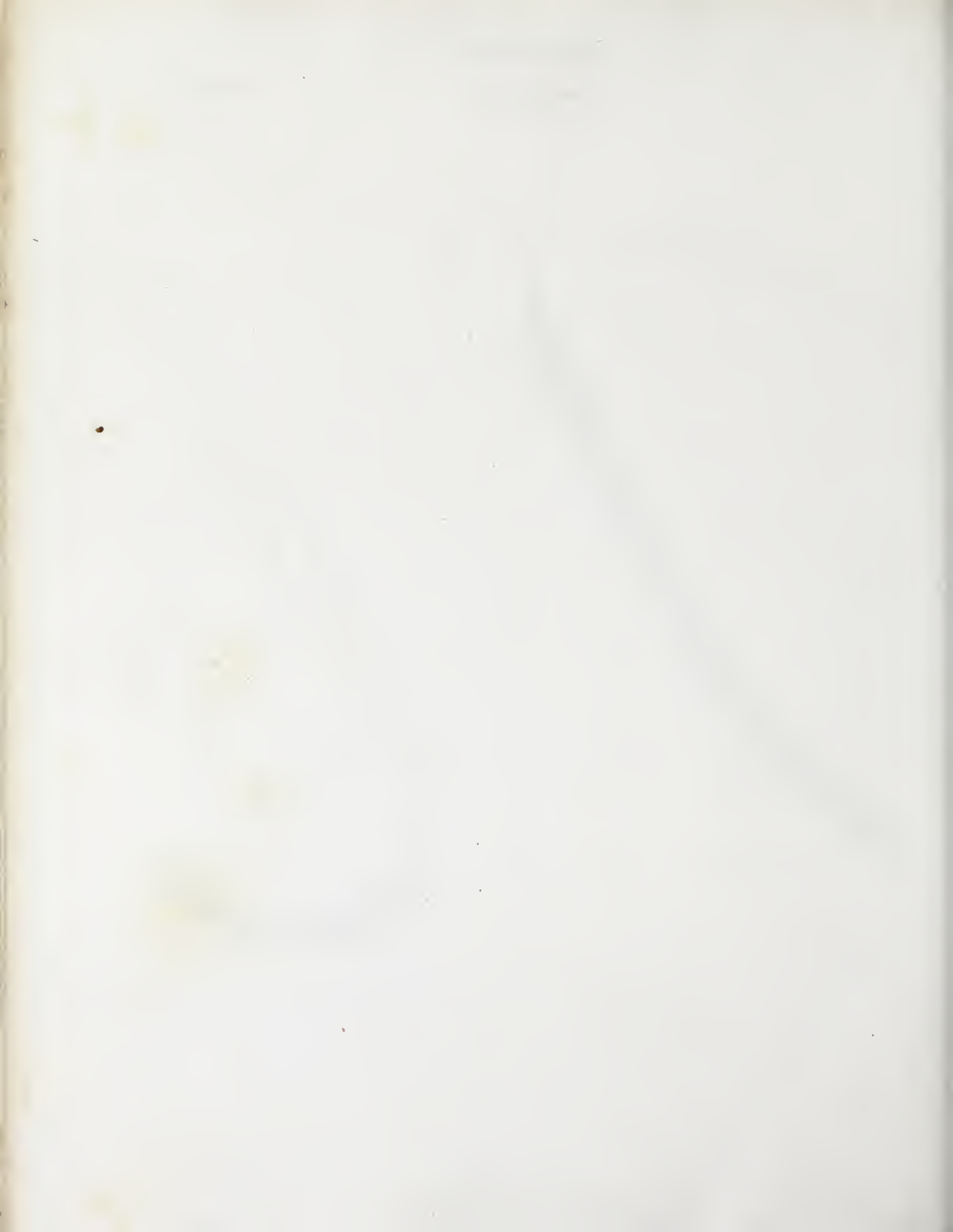
HAND RAILING.

PLATE X.



Invented by E. Nicholson.

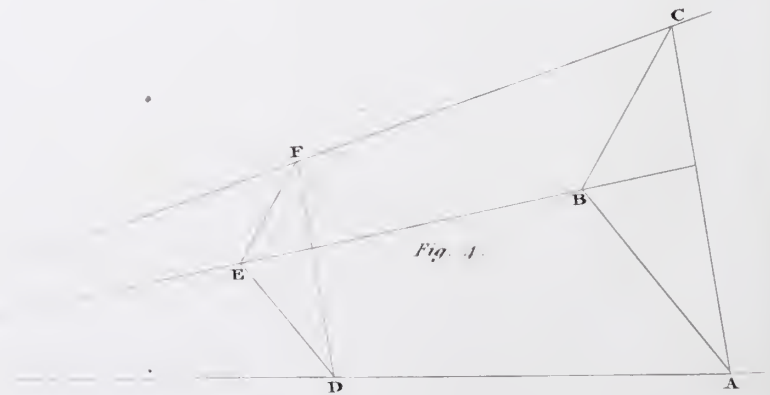
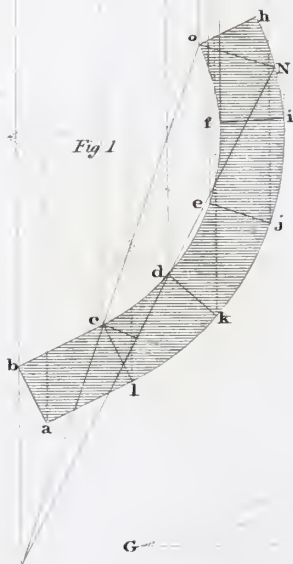
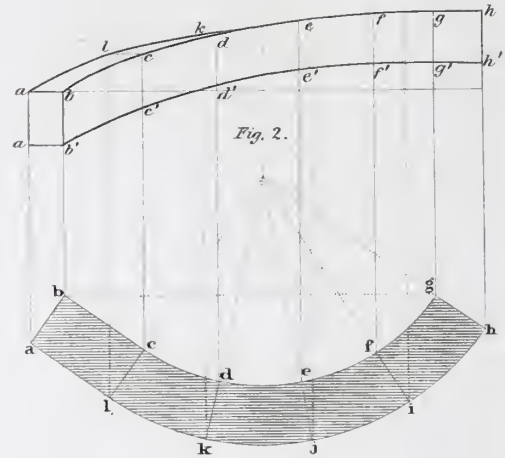
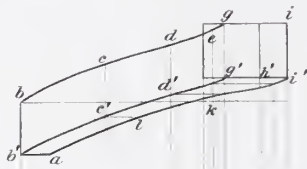
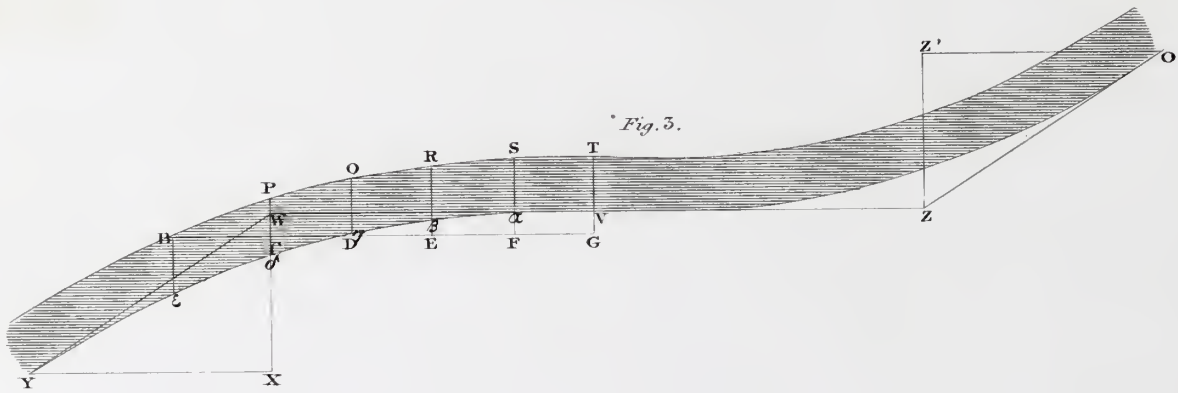
London, Published by John Day & Co. Oct' 20 1822



JOINERY,

HAND RAILING.

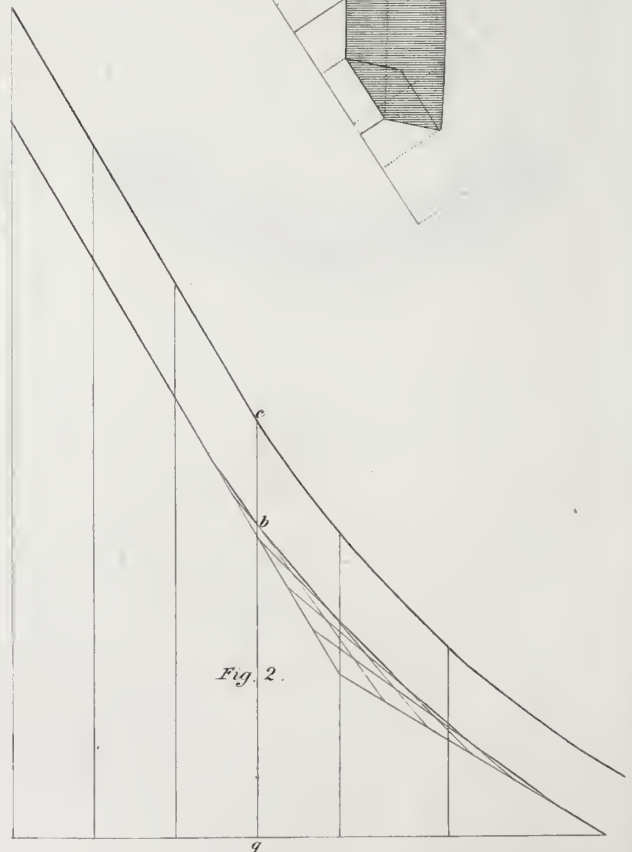
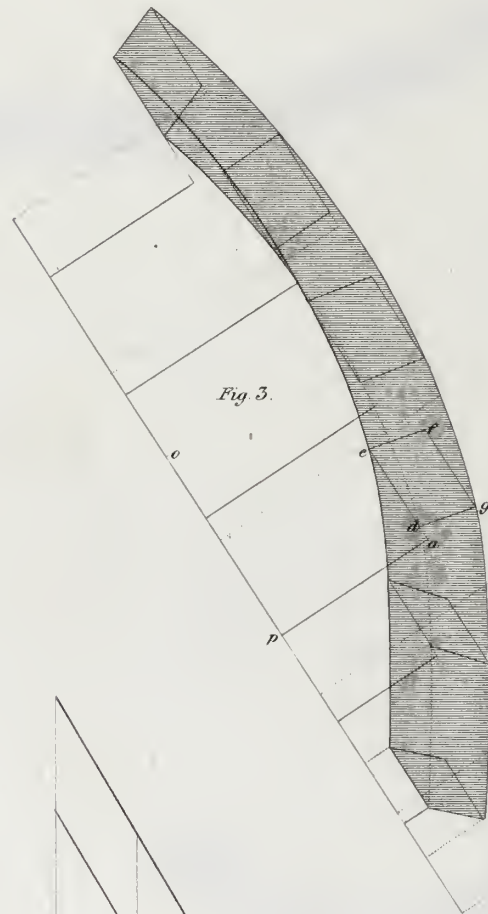
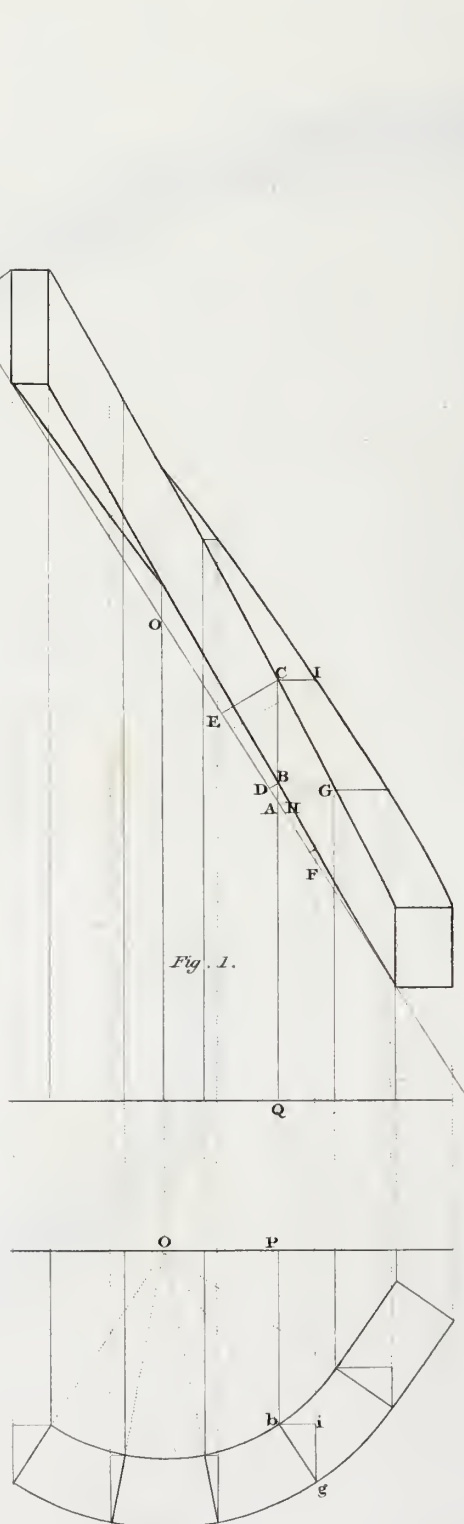
PLATE XI.



JOINERY.

HAND RAILING.

PLATE XV.



Invented by F. Nicholson.

London, Published by John Day & Co Octr 20. 1822.

PLATE XI.

Fig. 1, exhibits the plan and elevation, in order to get the rail out of the least quantity of stuff round a level half space.

Fig. 2 is the common method of finding the elevation of the rail piece by joining the points at the extremities of the concave side for a base line, and tracing it from the falling mould, *No. 3*. This position requires stuff considerably thicker than in *fig. 1*.

PLATE XV.

This plate is a method of tracing the face mould so as to apply only to one face of the plank, and cut the piece out square to the face, or in lines which are perpendicular to the face.

Find the elevation of the rail piece, and let *BCIG* be a section of the rail in the elevation. Produce *CB* to meet the base *OP* in *P*, and the concave side of the plan in *b*. From the centre *O*, and through *b*, draw the straight line *Og*, meeting the convex side of the rail in *g*. Draw *gi* parallel to *bP*, and *bi* perpendicular to *OP*. Let *O* be the point in the line *OF* which comes in contact with the extremities of the lower edge of the elevation corresponding to the centre *O* in the plan, and let the line *CQ* meet *OF* in *A*.

Draw *GH*, *BD*, *CE* perpendicular to *OF*, meeting *OF* in the points *H*, *D*, *E*.

In *fig. 3*, draw a straight line, in which take the distance *op* equal to *OA*, *fig. 1*. Draw *pg* perpendicular to *op*. In *pg*, take *pa* equal to *Pb*, *fig. 1*, and *ag* equal to *ig*, *fig. 1*.

Draw *ae* and *gf* parallel to *po*. In *ae*, make *ad* equal to *AD*, *fig. 1*, and *ae* equal to *AE*, *fig. 1*. Join *dg*, and complete the parallelogram *edgf*; then *e* is a point in the concave side of the face mould, and *g* a point in the convex side, so that, by finding all the sections in the same manner, we shall obtain as many points both in the concave and convex side of the falling mould as we please.

Perspective.

PERSPECTIVE is the art of representing any object on a plane, so that, if that plane be interposed between the eye and the original object, a ray or straight line passing from any point of the object to the eye will cut the plane in the corresponding point.

The plane on which the object is represented is called the *picture*; hence, to find the perspective of a straight line in any original object, we have only to find the representations in the picture of the two extremities of that line, and join the two points.

In this manner, we may find the representation in the picture of every straight line in the original object.

As the object of this treatise is to instruct mechanics who are supposed to understand the representations of objects geometrically; and consequently not ignorant of the nature of planes, although they may not be able to express themselves on the subject in the language of Euclid; yet, as their profession instructs them in the practice of forming plane surfaces at any given angles with each other, they must have the same idea of planes and their positions as the geometer, though they may give them other appellations. We cannot, therefore, imagine that the mechanic and geometer will understand each other in conversation; but the inspection of a drawing will present similar ideas: for this reason, we shall make no ceremony of introducing definitions in mathematical order, and proceed to explain the terms and principles of the art by means of a diagram.

FIRST PRINCIPLES.—PLATE I.

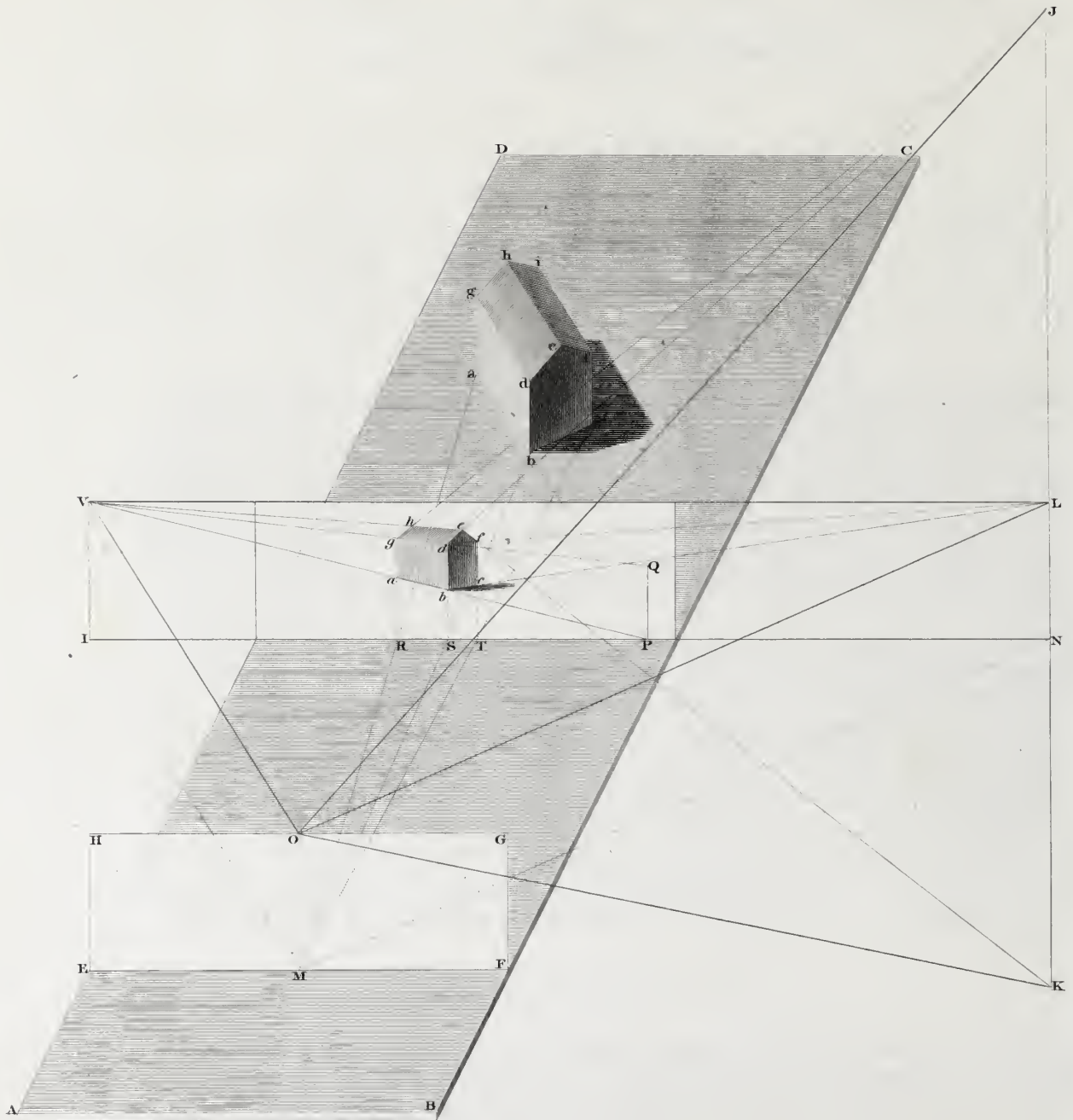
Let ABCD be a plane surface, representing the ground we walk upon; ab, bc* the wall lines of the two visible planes, which are supposed to be perpendicular to the ground plane; the front of the house being abdg, the gable bdefc*, the nearest side of the roof dghe, and the farther side efih.

* The letter c must be understood, having been omitted in the plate.

PERSPECTIVE.

FIRST PRINCIPLES.

PLATE I.



Introduced by P. Nicholson.

London Published by John Day & Co. Oct. 20 1822

Let M be the point in the plane $ABCD$, on which the spectator is supposed to take his position, called the *station point*; and let MO be a perpendicular to the ground plane, O being the position of the eye, called the *point of sight*.

Let $VLNI$ be the plane of the picture, perpendicular to and meeting the ground plane $ABCD$ in the straight line IN , which is called the *intersecting* or *ground line*; and if the straight line VL be parallel to IN , and at a distance from it equal to MO , the height of the eye, the line VL is called the *vanishing line* of the plane $ABCD$.

In order to find the representation of the house with respect to the picture,

Draw OL parallel to the wall line bc^* , and the point L in the vanishing line VI is called the *vanishing point* of the wall line bc^* , and of all straight lines parallel to bc^* .

In like manner draw OV parallel to the wall line ba of the front of the house, and the point V is the vanishing point of ba , and of lines parallel to ba , as dg , eh , fi .

Through the vanishing point L , draw KJ perpendicular to the vanishing line VL ; and through the point of sight O , draw OJ and OK parallel to the inclined lines de and ef , the meeting of the roof and gable ends.

From the visible extremities a , b , c^* of the ground plan of the house, draw the lines aM , bM , cM to the station point M , meeting the ground line IN in the points R , S , T .

In the plane of the picture, draw Rg , Sd , and Tf perpendicular to the ground line IN . Prolong the wall line ab of the front of the house to the intersecting line IN , and the point P , where these two lines meet, is called the *intersecting point* of the wall line ab .

Draw PQ in the plane of the picture perpendicular to the ground line IN , and make PQ equal to the height bd or ag of the walls of the house; Q is the intersecting point of the top, gd of the wall.

Join the intersecting point P and the vanishing point V by the line PV , meeting Rg in a , and Sd in b ; also join the intersecting point Q to the same vanishing point V by the line QV , meeting Rg in g , and Sd in d : then $abdg$ will represent the front $abdg$ of the house.

Again, join bL , meeting Tf in c , and join dL , meeting Tf in f . Draw gJ , dJ , and join Kf , and produce Kf to meet dJ in e . Draw eV , meeting gJ in h , and we have now the complete representation of the object.

HOUSE.—PLATE I.

Fig. 1 is the plan of a house to be represented in perspective from the station point O ; the straight line VL answers both the vanishing and intersecting lines.

If it were requisite to find the station point O , so as to view the object under a given

* The letter c must be understood, having been omitted in the plate.

angle, we have only to draw the lines which meet in O from the extreme points of the plan parallel to the sides of the isosceles triangle, of which the base is in line VL, the angle at the vertex being the given angle.

Fig. 2 is the elevation.

Fig. 3, the perspective representation, where every part is found in the same manner one after the other, as in the preceding diagram of the first principles.

In this example all the vanishing points are within the paper.

It is sufficient to say once for all, that all the perspective lines are found by lines joining their intersecting and vanishing point; and the terminations of these lines by lines drawn from the corresponding terminations of the originals to the station point; then drawing the perpendiculars from the points in the intersecting line to meet the lines drawn to the vanishing points.

O B E L I S K.

PLATE I.

The plate exhibits the plan of the obelisk. Here, as the parallel planes have various heights, we have produced the lines representing them on the plan to the intersecting line; the points P, O, N, M, &c. are the intersecting points.

In this example the station point is without the paper, and is supposed to be inaccessible; nevertheless the lines are drawn with the same accuracy, tending to the station point, as if that point were actually at command, by means of an instrument called a centrolinead, invented by Mr. Peter Nicholson, rewarded by the Society of Arts in 1815, and now manufactured by Mr. Dollond, St. Paul's Church-yard.

The two extreme lines tending to the station point are found in the same manner as in the preceding example, by a triangle shown at the bottom of the plate.

PLATE II.

This plate exhibits the method of putting the obelisk in perspective. Here the intersecting points of the parallel lines of the original object at the various heights are the upper extremities of the perpendiculars drawn from the intersecting points of the vertical planes passing along these lines, and are known by the same letters that are used on the plan, Plate I.

The elevation is here exhibited in order to ascertain the heights of the intersecting points; and to make this process easy, a section of all the planes which form one side of

PERSPECTIVE,

HOUSE.

PLATE. I.

Fig. 1.

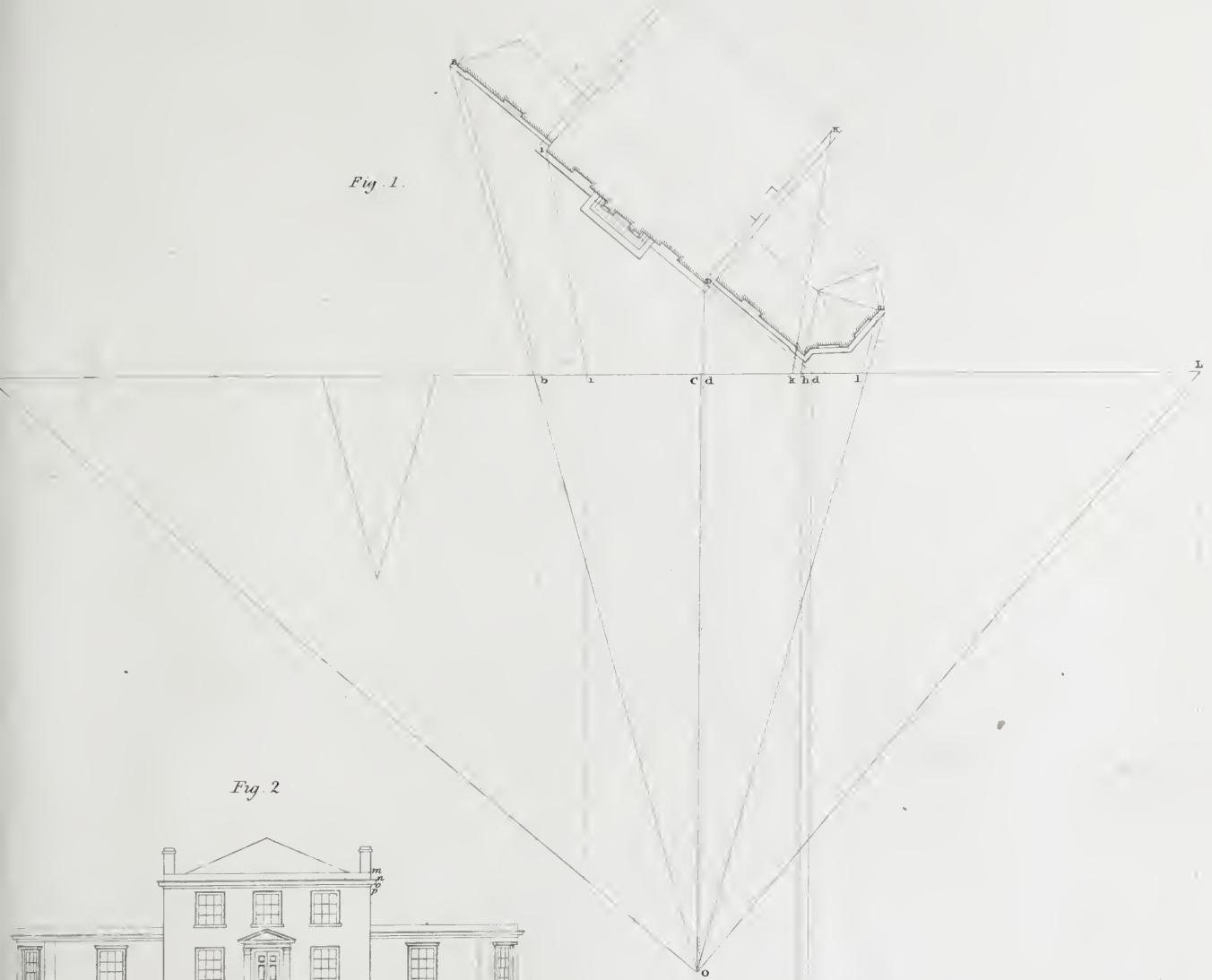


Fig. 2



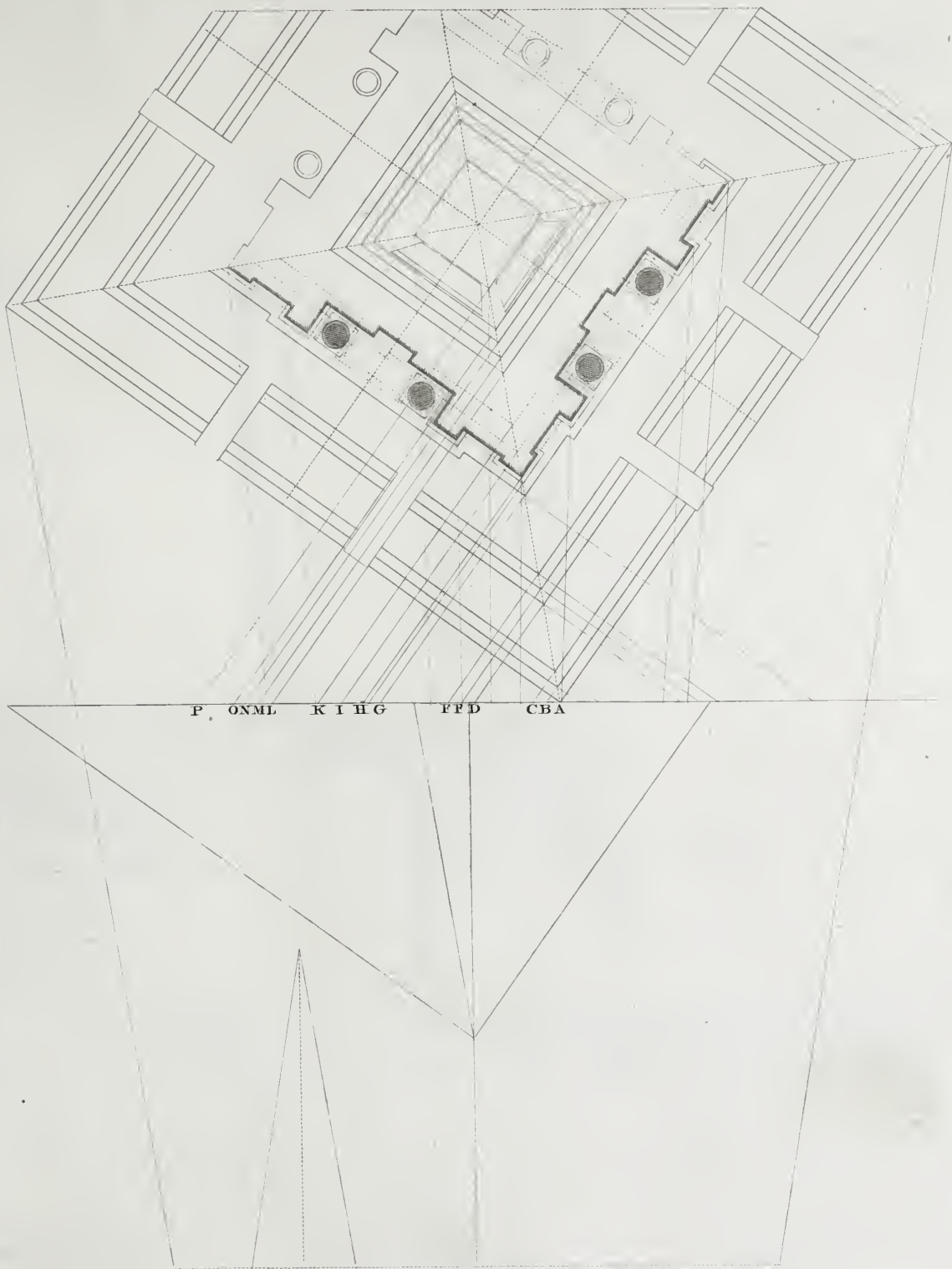
Fig. 3.



PERSPECTIVE,

OBELISK.

PLATE I.

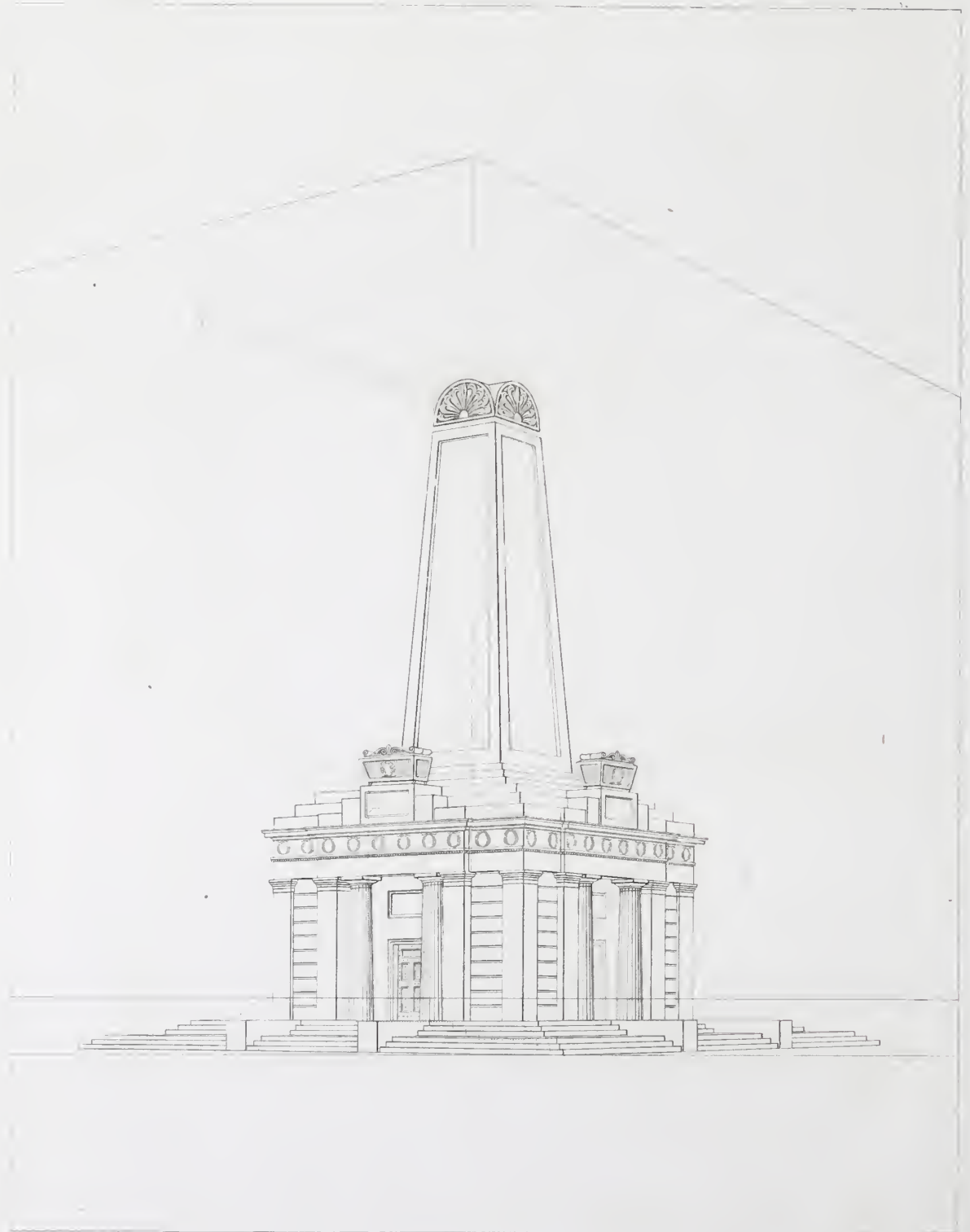




PERSPECTIVE.

PLATE

OBELISK.



Engraved by C. Armstrong

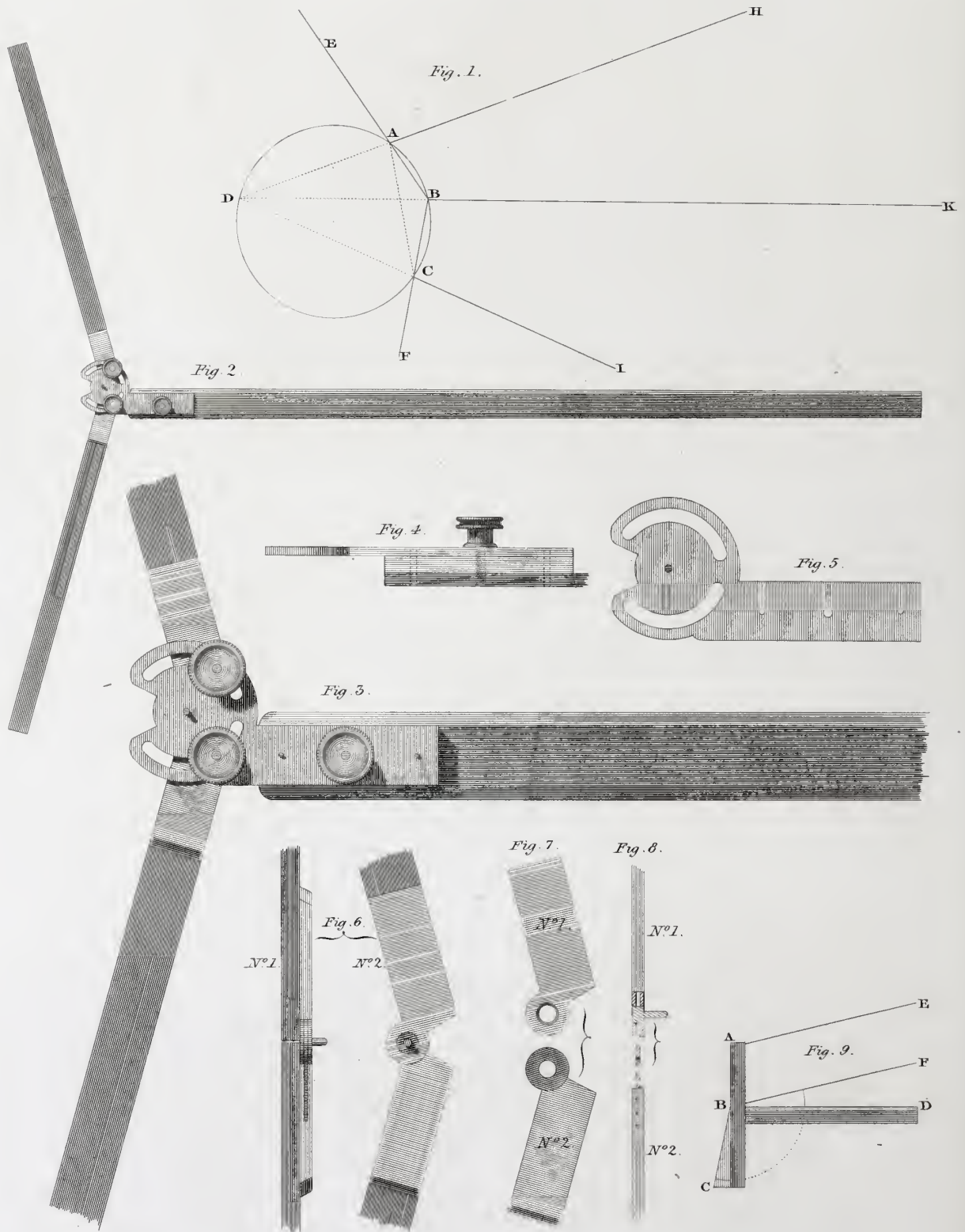
London, Published by John Day & Co Octo. 20 1822



PERSPECTIVE,

CENTROLLINEAD.

PLATE 1.



the object is found; for by drawing lines from all the angles of this section to the right hand vanishing point, we obtain all the perspective lines on the right hand; and by finding the middle points of these lines, we may draw the perspective lines to the other vanishing point on the left hand, and complete the object.

PLATE III.

This plate shows the outline completely finished.

The vanishing points are found by dividing half the line intercepted by those lines drawn from the extreme points of the plan towards the station point into seven equal parts, and from the middle point setting off one seventh, and drawing a line through the point of distance of that seventh part parallel to either line drawn through the extreme angle of the plan; then the point where it meets the middle line is the point through which the line parallel to each side of the plan must pass.

THE CENTROLINEAD

Is an instrument for drawing straight lines which will meet in the same point with any two straight lines given in position.

This instrument is of the greatest use in perspective delineation, when the vanishing points happen to extend beyond the reach of a long ruler, or to exceed the dimensions of the apartment in which the drawing is made, since lines may be drawn by it to a point at any given distance. The discovery is due to Mr. Peter Nicholson, who received a premium from the Society of Arts in 1815 for its invention. It is now made by Mr. Dollond, of St. Paul's Church-yard, whose fame in optical and mathematical instruments has been long and justly established.

We shall here exhibit the instrument and its parts, which will perhaps be easier understood by representation in drawing, than by verbal description, however minute.

Fig. 2 exhibits the centrolinead complete.

Fig. 3, a part of the same to a larger scale.

Fig. 4, the part represented in *fig. 3*, shown edgeways.

Fig. 5, brass plate, by which two running legs and the blade are connected.

Fig. 6, joint or hinge connecting the legs, *No. 1* showing the edge, and *No. 2* the top.

Fig. 7, *No. 1*, the two parts of the joint before united by the centre pin.

Fig. 8, edge of the parts shown in *fig. 7*.

Fig. 9 shows how a square may be converted into a centrolinead, in order to draw any number of lines which will converge to the same point with two given lines: thus, let *AE* and *BD* be the two given lines. Draw *BA* perpendicular to *BD*, and *BE* parallel to *AE*.

Suppose the edge DB of the blade of the square to meet the back of the stock at C; fasten a wedge with its vertex in B upon the back AB, so that the vertical angle of the wedge may be equal to the angle DBF; then if a pen be fixed in the point A, and another in the point B, and the instrument be moved so that the back AB of the square may be upon A, and the side BC of the wedge upon B; then, if the instrument be stopped at any place, a line drawn by the edge BD will tend to the same point to which the lines AB and BC converge.

In *fig. 1*, AH and CI are the two given lines, BE and BF represent the edges of the legs of the instrument, and BK the drawing edge of the blade; then all the lines will tend to the point D in the circumference of the circle ABCD.

FIVE ORDERS OF ARCHITECTURE.

THE ORDERS OF ARCHITECTURE are five pillars of specific character supporting a roof, of which the profile or face is ornamented with mouldings.

The pillars, being imitations of the trunks of trees, are made to taper upwards, and terminate in one end at least in a series of annular mouldings.

The pillar thus ornamented is called a *column*, of which the conic frustum is called the *shaft*, and the mouldings, if any, at the bottom are called the *base*.

Mouldings or other ornaments, terminating the column at the upper end of the shaft, are called the *capital*.

The ornamental part of the roof which is supported by the column, is called the *entablature*.

The entablature has in its height three distinct parts, of which the middle one is generally a plane vertical surface. The lowest of the three which rests immediately on the capital of the column is called the *architrave*, the middle one is called the *frieze*, and the uppermost part the *cornice*.

The mouldings of the architrave and cornice project more and more as they rise; but the projectures of the mouldings of the architrave are inconsiderable compared with the projectures of the cornice.

Three of the orders of architecture are of Grecian origin, and are named from the places where they were invented.

The first is called *Doric*, from the Dorians, a Grecian colony, who settled in Asia Minor. Its specific character is, that it has no base; the capital consists of mouldings, and the frieze is enriched with rectangular tablets, of which the angles are chamfered, and the intermediate surface grooved with two vertical channels.

The tablets are called *triglyphs* from the two channels, and the two champhers at the angles, which, if conceived to be set together, would form a third channel.

The spaces between the triglyphs which are nearly square, are called *metopes*.

The cornice has blocks called *mutules*, in imitation of the ends of the rafters supporting the eaves; the mutules are equidistant, and correspond not only to the triglyphs, but also to the middle of the metopes.

Each of the orders except the Doric has a base as well as a capital.

The second order is called *Ionic*, having been first executed in Ionia, a country in Asia Minor possessed by a Grecian colony. The specific character of this order is, that the capital is symmetrically ornamented to the right and left by spiral curves forming two bands or fillets, called *volute*s.

The third order is called *Corinthian*, from the city of Corinth, where it was first introduced. Its specific character is, that the capital consists of two rows of eight leaves each, with stalks springing up between them, and branching into volutes.

The other two orders are of Italian invention; the most simple of these (and indeed of all the five orders) is called the *Tuscan*, from Tuscany, the place where it was first used. Its character is similar to the Doric, but has both a base and capital, and the frieze is without triglyphs. This order is described by Vitruvius, but not the least vestige of any building where it had been used has reached our times. The restorers of antient architecture have differed considerably in their opinions concerning its figure and proportions.

The remaining order is called *composite*, from the capital, which consists of a composition of the Ionic and Corinthian; it is also called Roman, from its having been first used by the Romans.

DESCRIPTION of the PLATES.

The proportions of each order are regulated by the lower diameter of the shaft, which is divided into 60 equal parts, called minutes.

The projectures of the members are reckoned from the axis of the column, and are placed at the extremity of each moulding. The heights of the mouldings are not given individually, but in aggregate, as they rise to the entire height of every principal member, as base, capital, architrave, frieze, and cornice.

Plate 1, Tuscan order.

Plate 2, Doric order, from Sir William Chambers.

Plate 1, Ionic order, from the gate-way at Eleusis.

Plate 2, Ionic order, spiral line sections of the volute and base.

Plate 3, Ionic order, plan, profile, and sections of the capital.

Plate 1, Ionic order, spiral of volute, which may be drawn to any scale by the numbers or minutes affixed.

- Plate 2*, Ionic order, volute complete in the simple form.
Plate 3, Ionic order, volute complete in the complex form.
Plate 3, Corinthian order, capital and base from the portico of the Pantheon at Rome.
Plate 1, Corinthian order, entablature from the portico of the Pantheon at Rome.
Plate 1, Corinthian order, capital and base from the interior of the Pantheon at Rome.
Plate 2, Corinthian order, entablature from the interior of the Pantheon at Rome.
Plate 1, Corinthian order, modillion.
Plate 1, Composite order, base and capital, from the arch of Titus at Rome.
Plate 2, Composite order, entablature from the frontispiece of Nero at Rome.

THE CYCLOGRAPH, A DRAWING INSTRUMENT.

THIS is an instrument for drawing the arcs of large circles. In describing the arc, it moves between two pins, as in the centrolinead.

The instrument is so contrived, that the brass work may be entirely taken off and applied to any two rulers which may be found convenient for the description of smaller or larger arcs.

Fig. 1, *No. 1*, is the instrument completely fixed. The edge of the semicircular plate is intended to be graduated into 360° ; by this means the number of degrees in the arc to be described will be ascertained. *No. 2* is a view of the instrument set on its edge.

Fig. 2, the brass joint which connects the two legs, *No. 1* being a view of the face, and *No. 2* of the edge.

Fig. 3, *No. 1* and *2*, the parts separated.

Fig. 4, a section through the tube with the drawing pen.

Fig. 5, the same separated.

Fig. 6, brass plate for fastening the instrument.

Fig. 7, running legs of the instrument.

Fig. 8, screws for fastening the brass plate *fig. 6* to the joint *fig. 2*, and to the legs of the instrument.

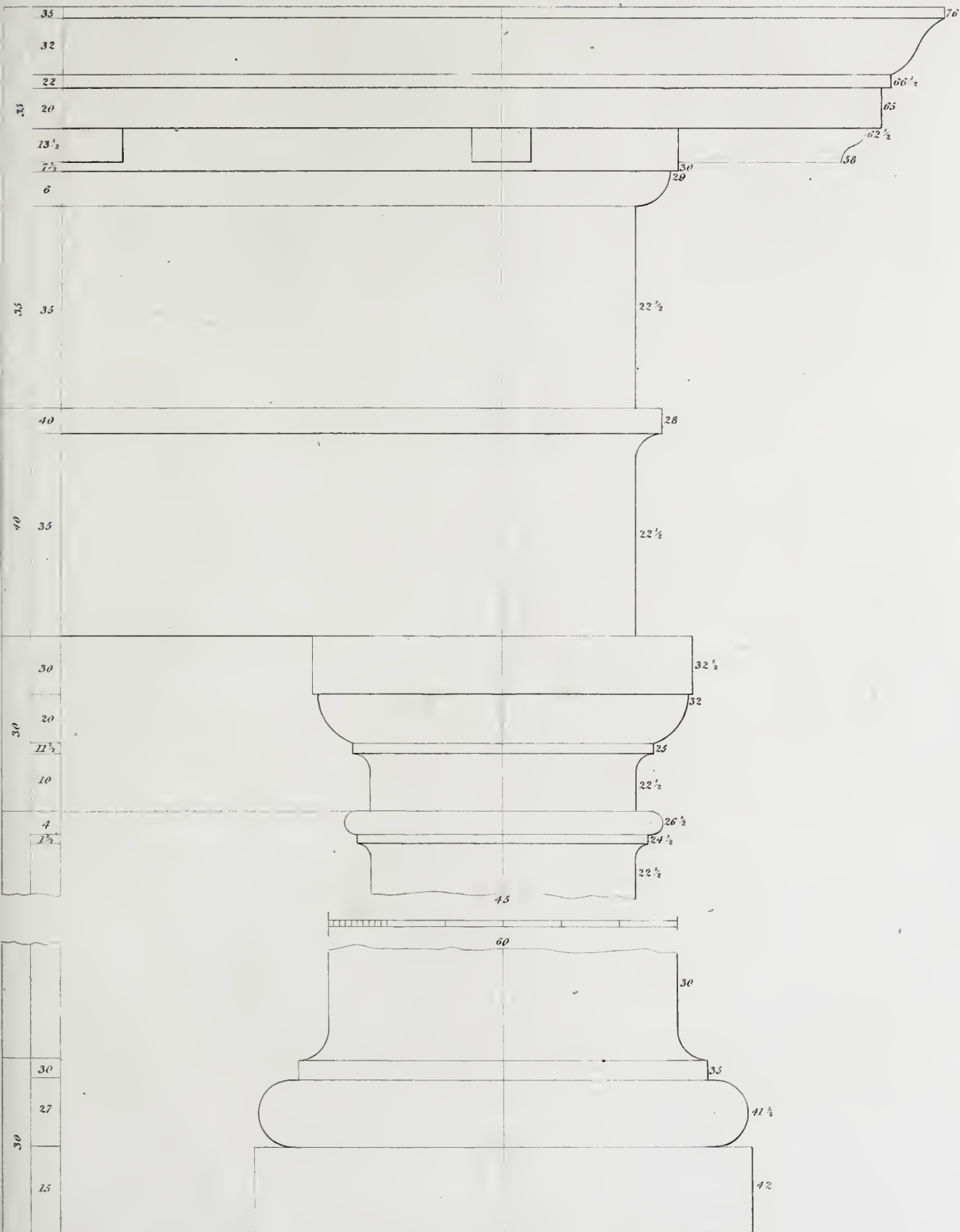
Plate 6, Plan and Elevation of a Country House, designed for a gentleman of moderate fortune.

THE END.

TUSCAN ORDER.

PLATE I.

ENTABLATURE BASE AND CAPITAL.

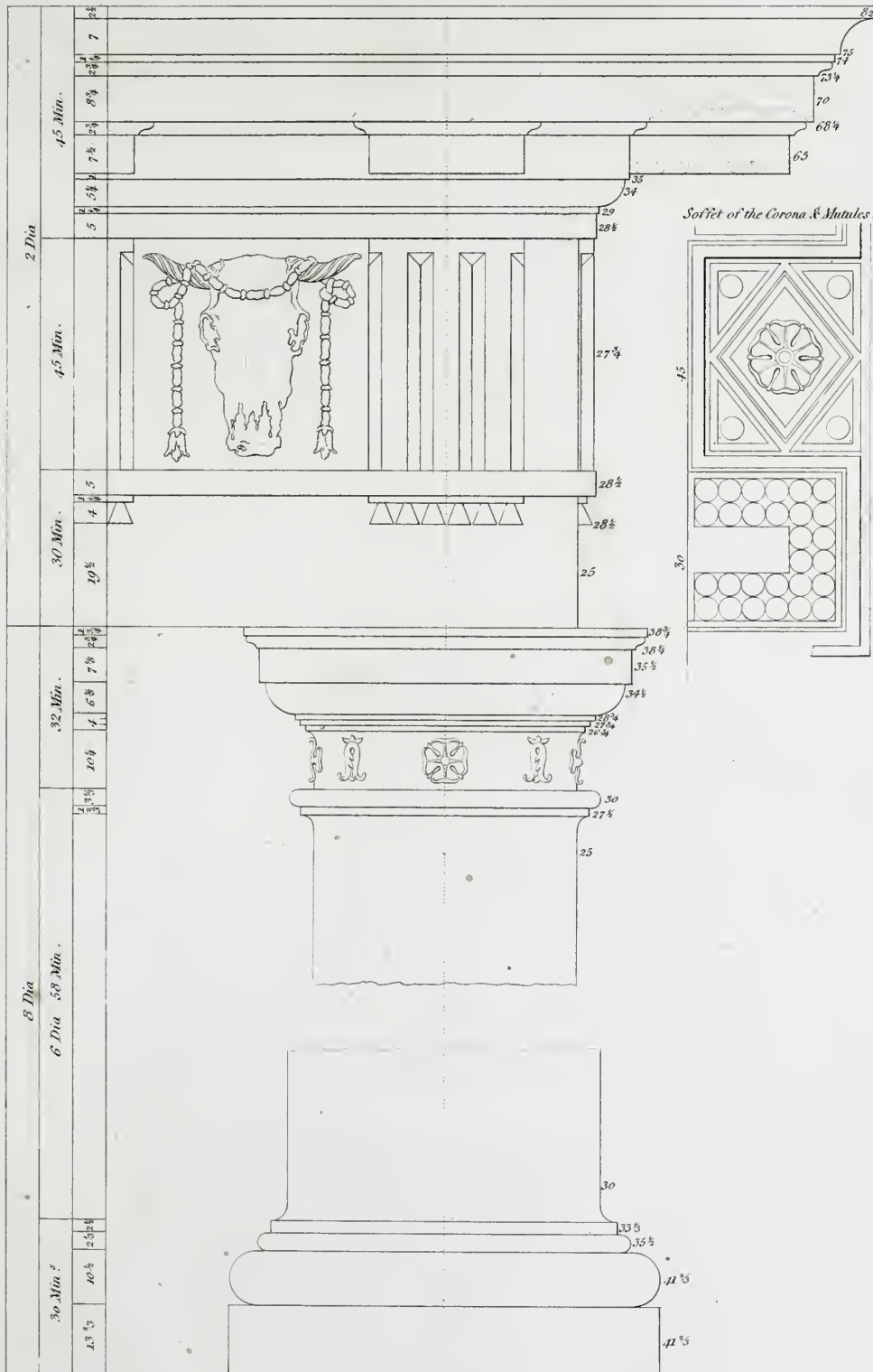




ROMAN ARCHITECTURE.

DORIC ORDER,

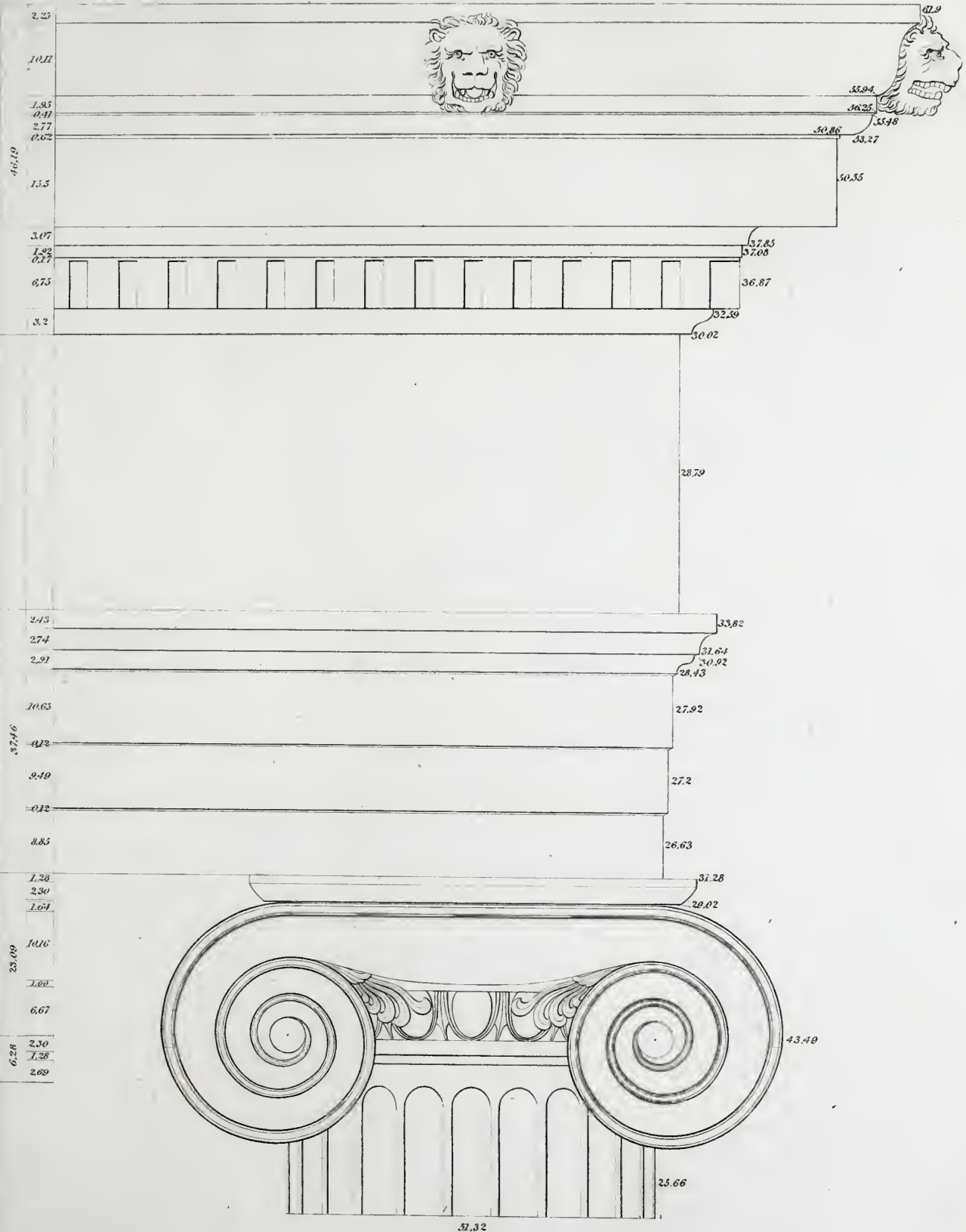
FROM SIR W^m CHAMBERS.



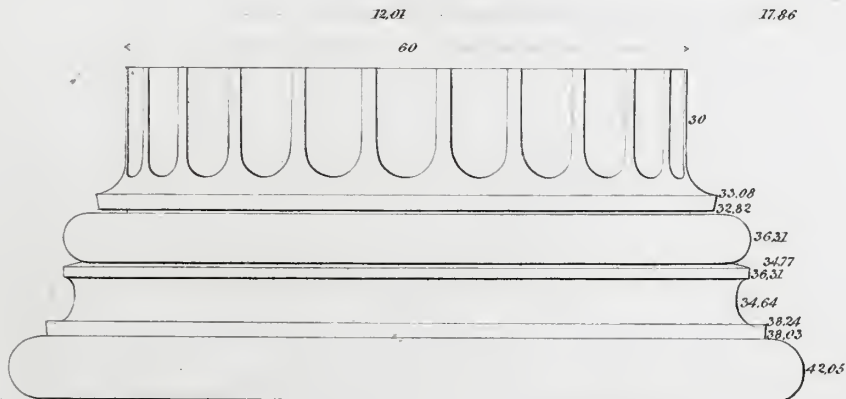
IONIC ORDER .

PLATE 1.

FROM GATE WAY AT ELEUSIS.



VOLUTE AND BASE OF COLUMN.



19,84	3,16	<u>1,41</u>	0,10
		<u>1,95</u>	
		5,73	0,26
		<u>1,38</u>	
		5,00	
		<u>1,41</u>	
		6,77	

23.86

IONIC ORDER .

PLATE 3.

PLAN, PROFILE, AND SECTIONS OF CAPITAL.

Fig. 1 .

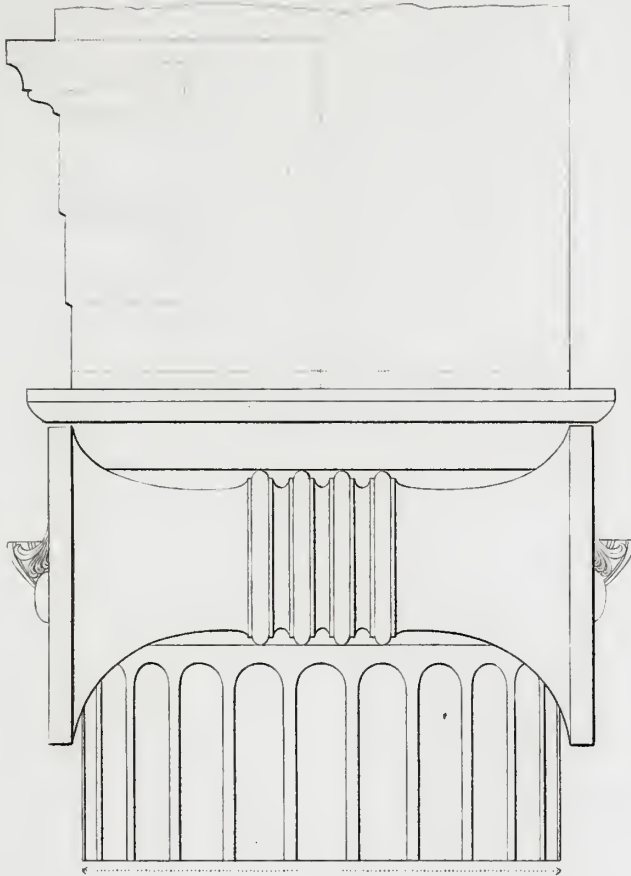


Fig. 2 .

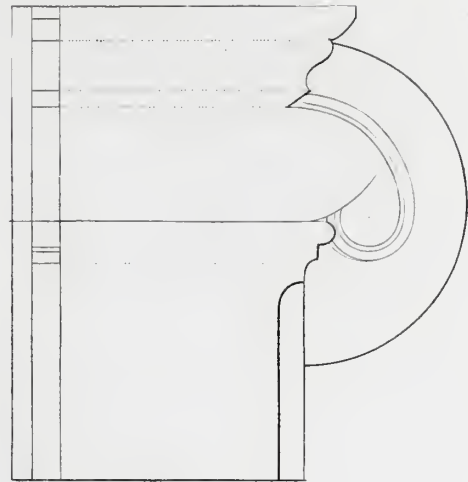


Fig. 4.

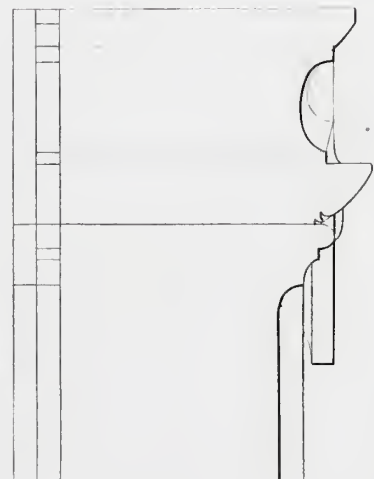
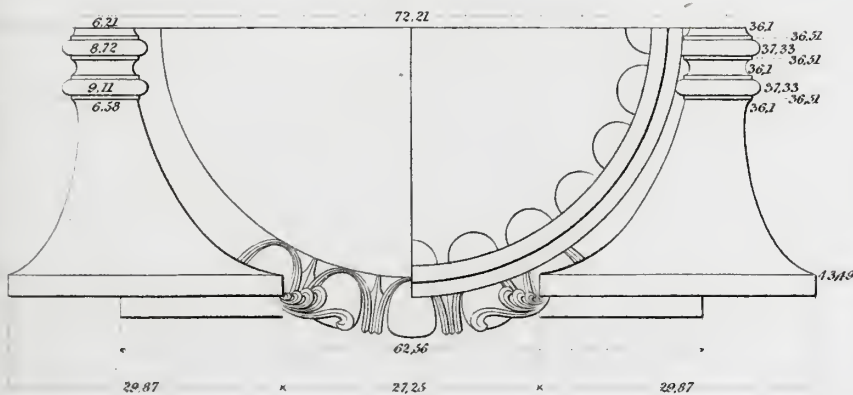


Fig. 3 .



Engraved by C. Armstrong.

London Published by John Day & Co. Octo 20 1822.

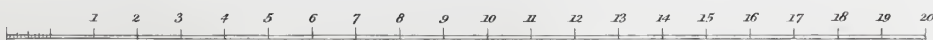
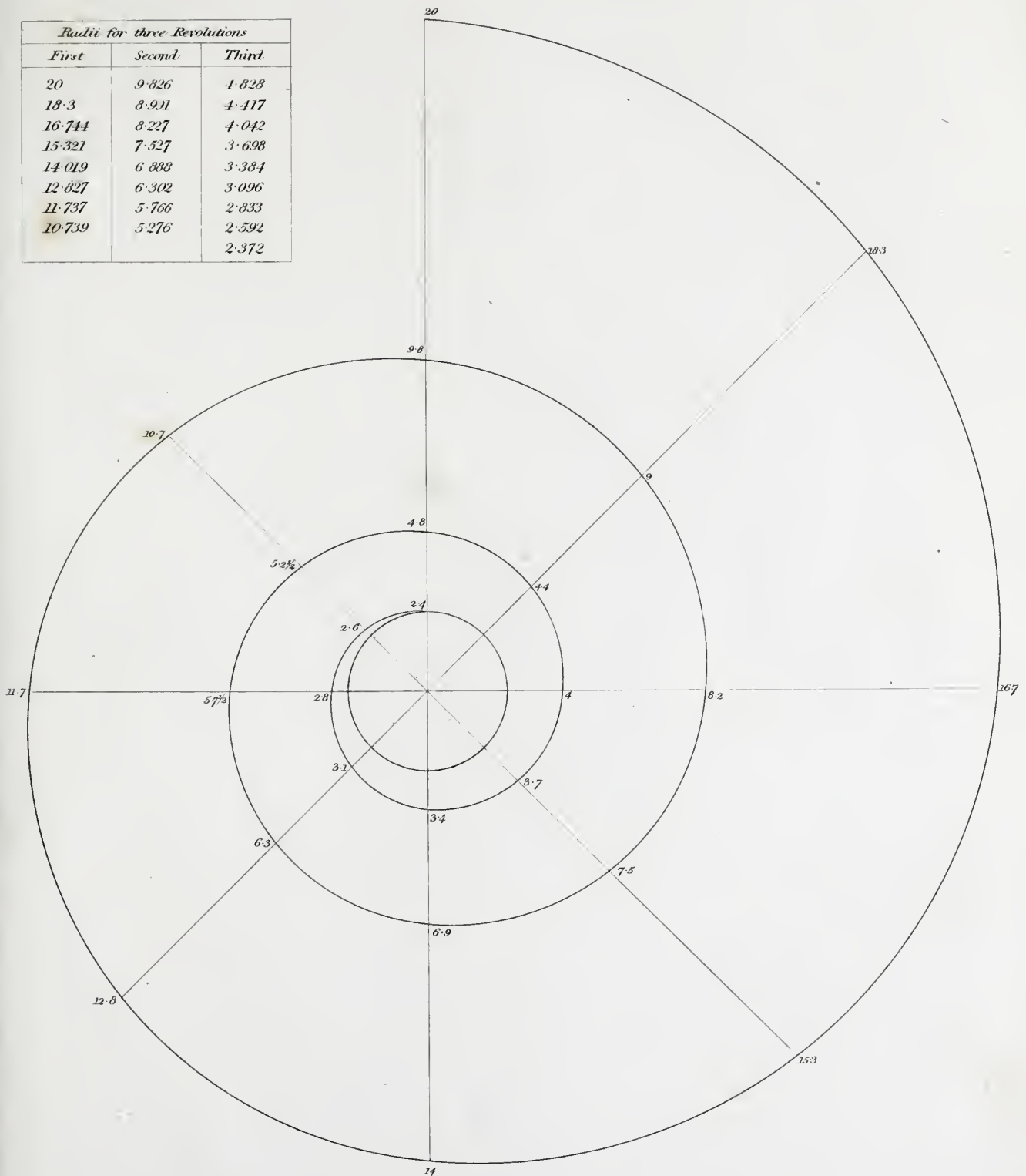
GRECIAN ARCHITECTURE.

IONIC ORDER.

VOLUTE.

PLATE I.

<i>Radii for three Revolutions</i>		
<i>First</i>	<i>Second</i>	<i>Third</i>
20	9.826	4.828
18.3	8.991	4.417
16.744	8.227	4.042
15.321	7.527	3.698
14.019	6.888	3.384
12.827	6.302	3.096
11.737	5.766	2.833
10.739	5.276	2.592
		2.372



GRECIAN ARCHITECTURE.

IONIC ORDER.

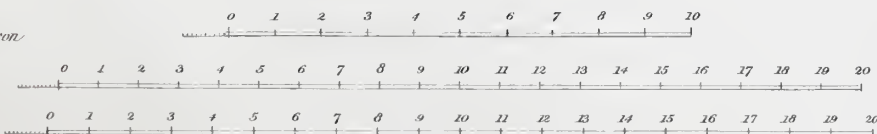
VOLUTE.

PLATE II.



Designed & Drawn by E. Nicholson

Engraved by W. Lowry.

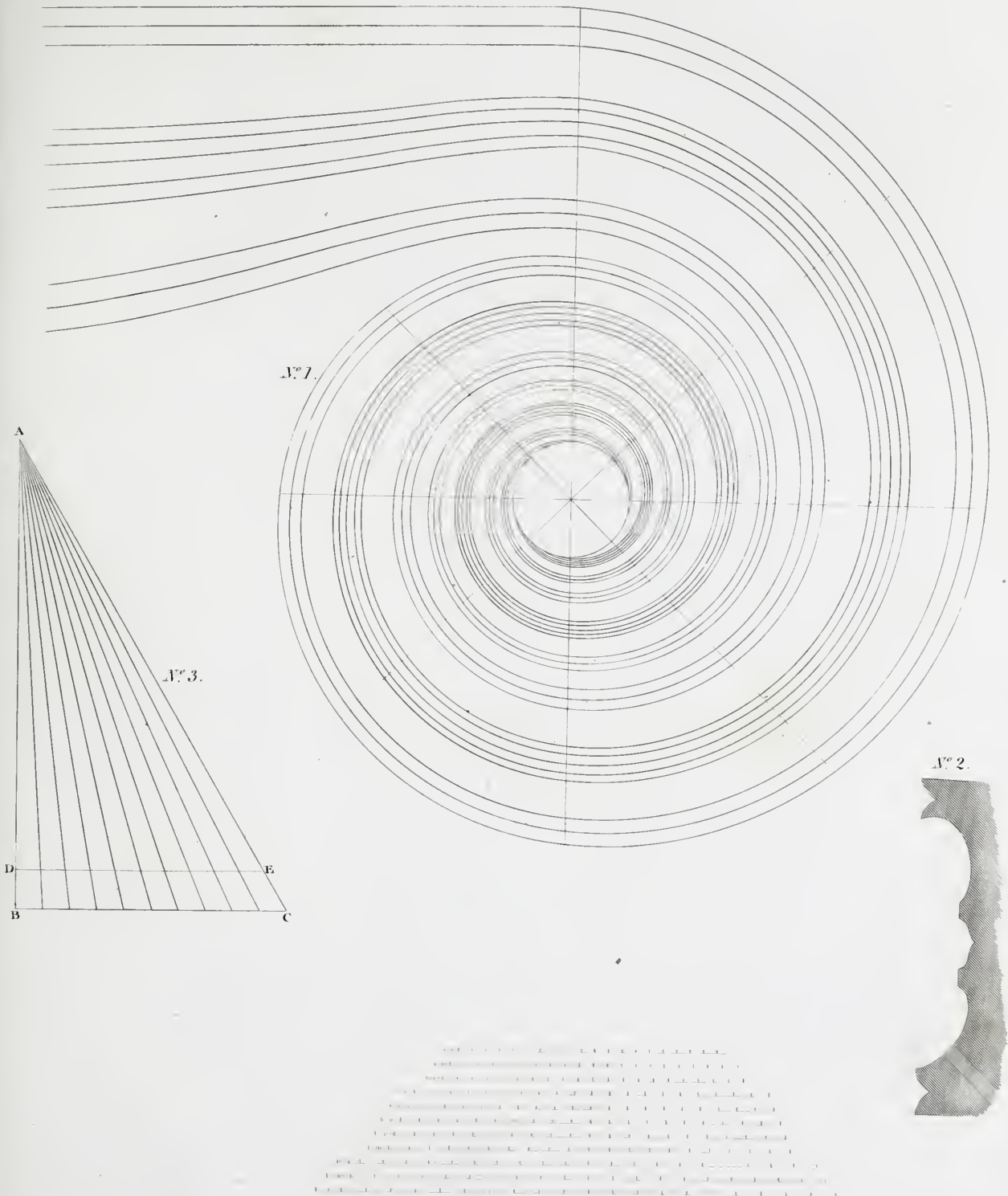


GRECIAN ARCHITECTURE.

IONIC ORDER.

VOLUTE.

PLATE III.





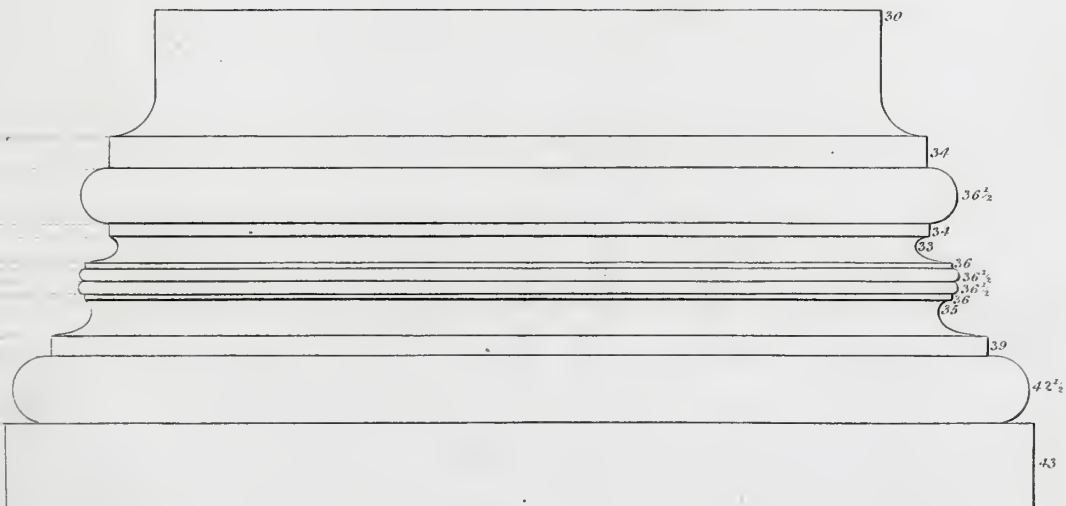
CORINTHIAN ORDER .

PLATE 3.

CAPITAL & BASE FROM THE PORTICO OF THE PANTHEON AT ROME.



5.59



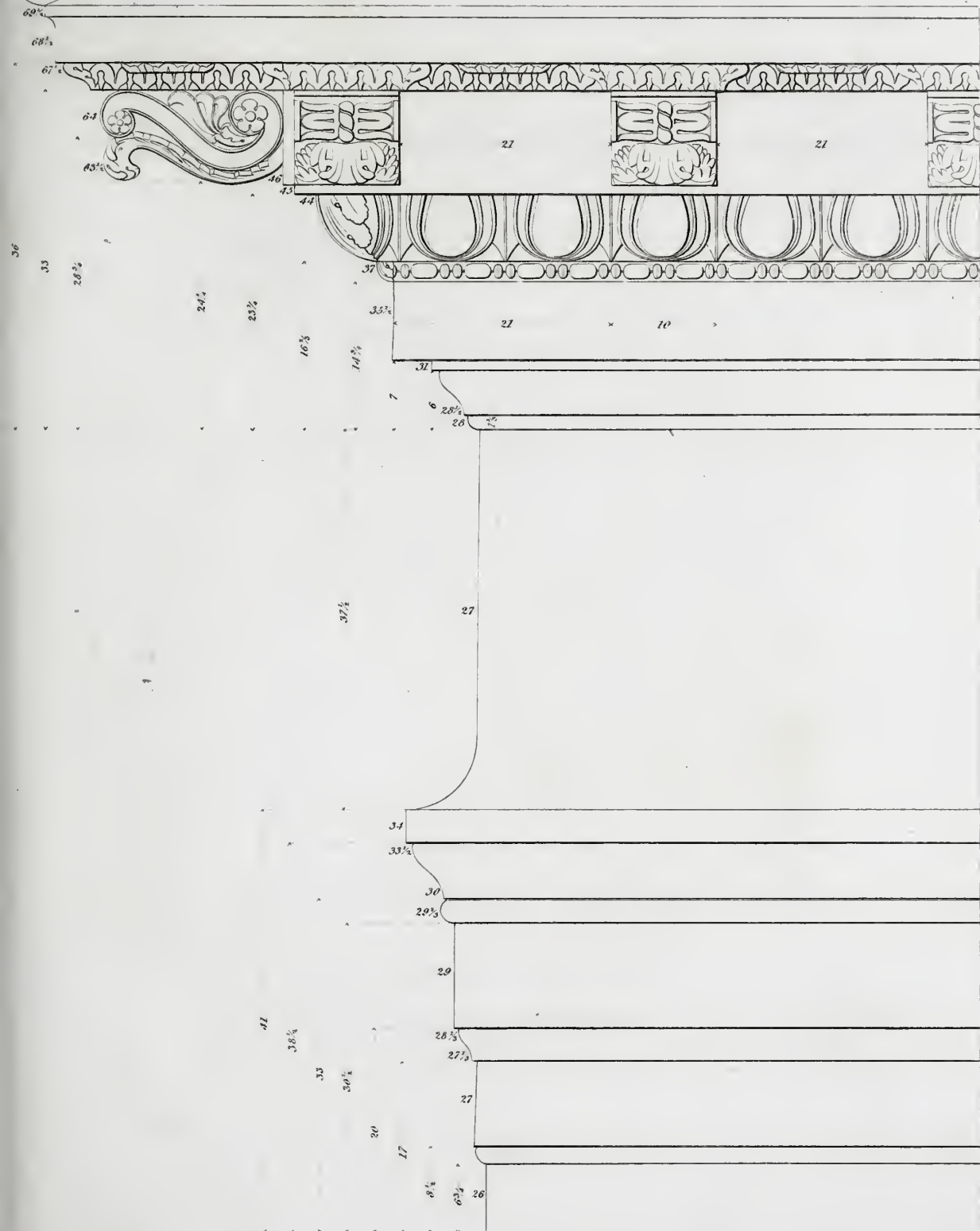
20
18
16
14
12
10
8
6
4
2



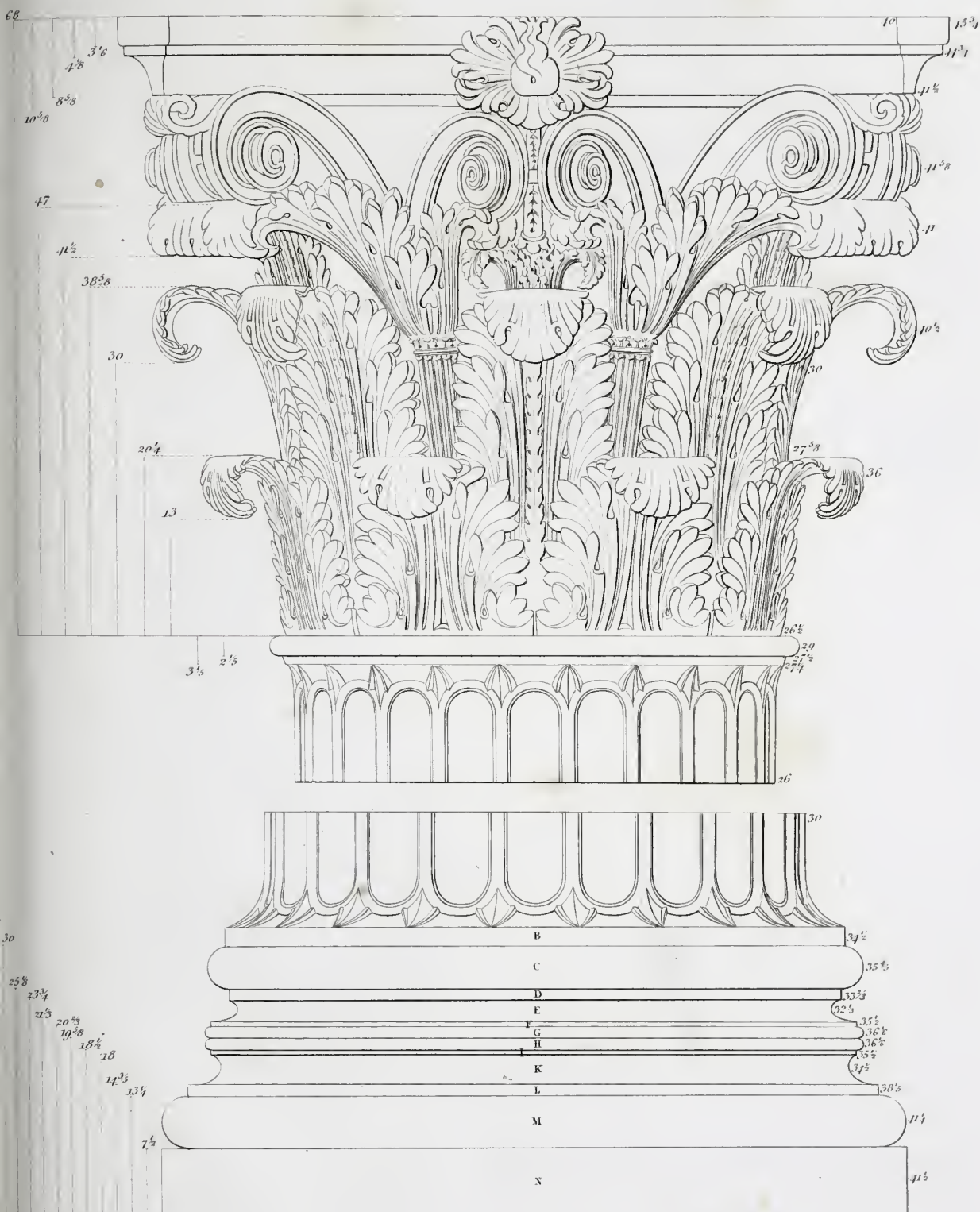
CORINTHIAN ORDER.

PLATE 1.

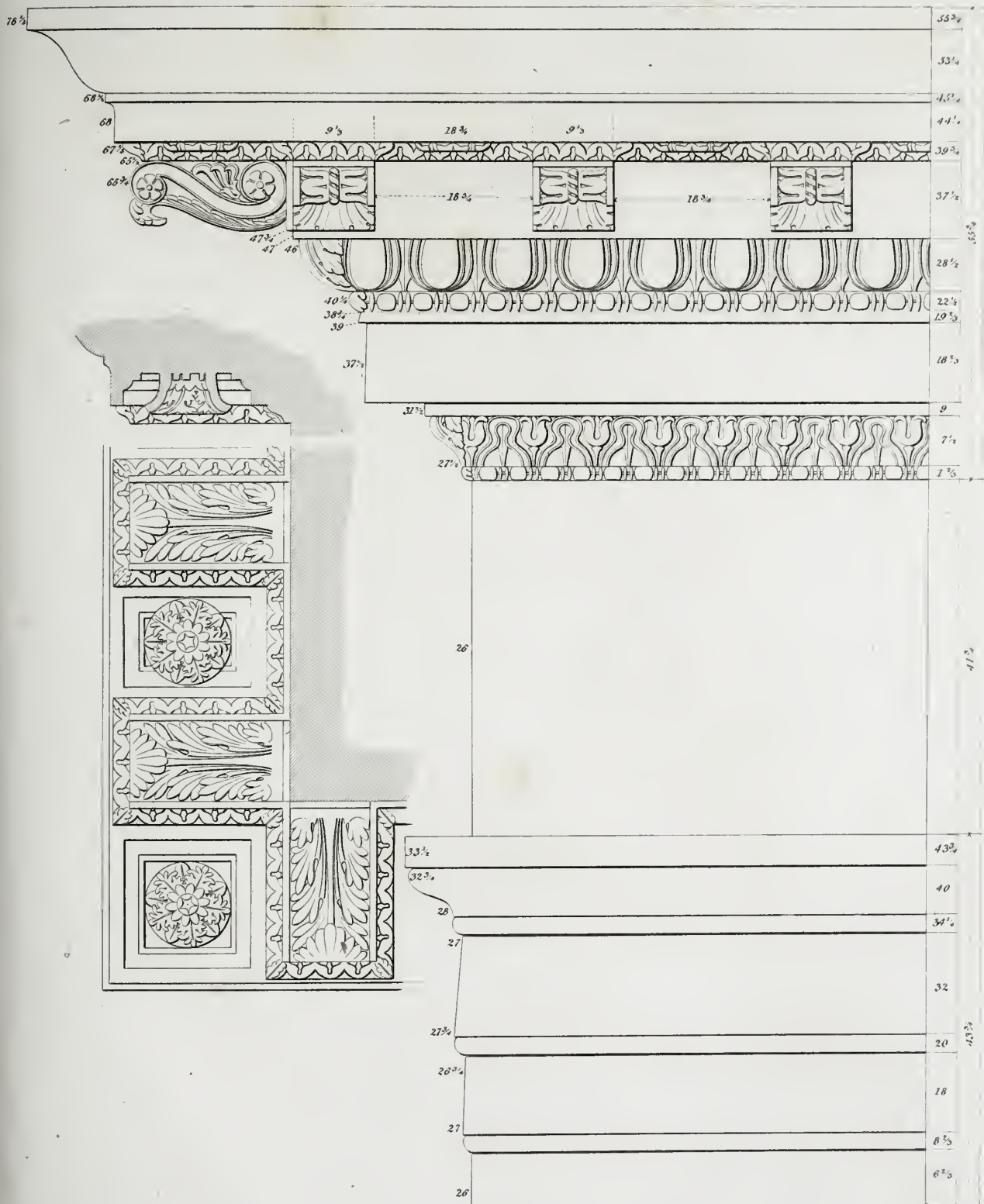
ENTABLATURE FROM THE PORTICO OF THE PANTHEON AT ROME.



CAPITAL & BASE FROM THE INTERIOR OF THE PANTHEON AT ROME.

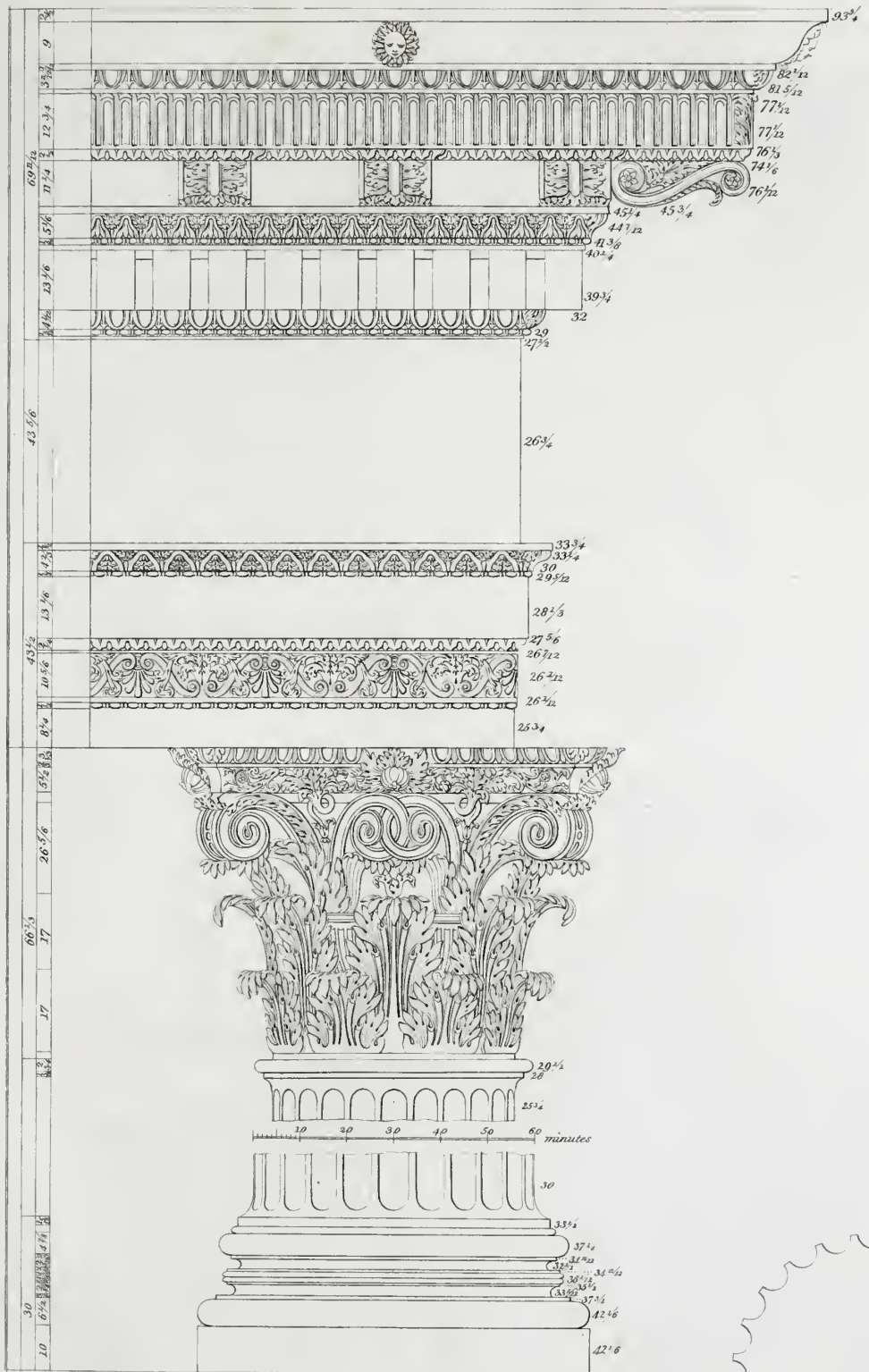


ENTABLATURE FROM THE INTERIOR OF THE PANTHEON AT ROME.



FROM THE TEMPLE OF JUPITOR STATOR AT ROME.

93 1/4



London. Published by John Day & Co. June 20 1822.



MODILLION.

Fig. 1.

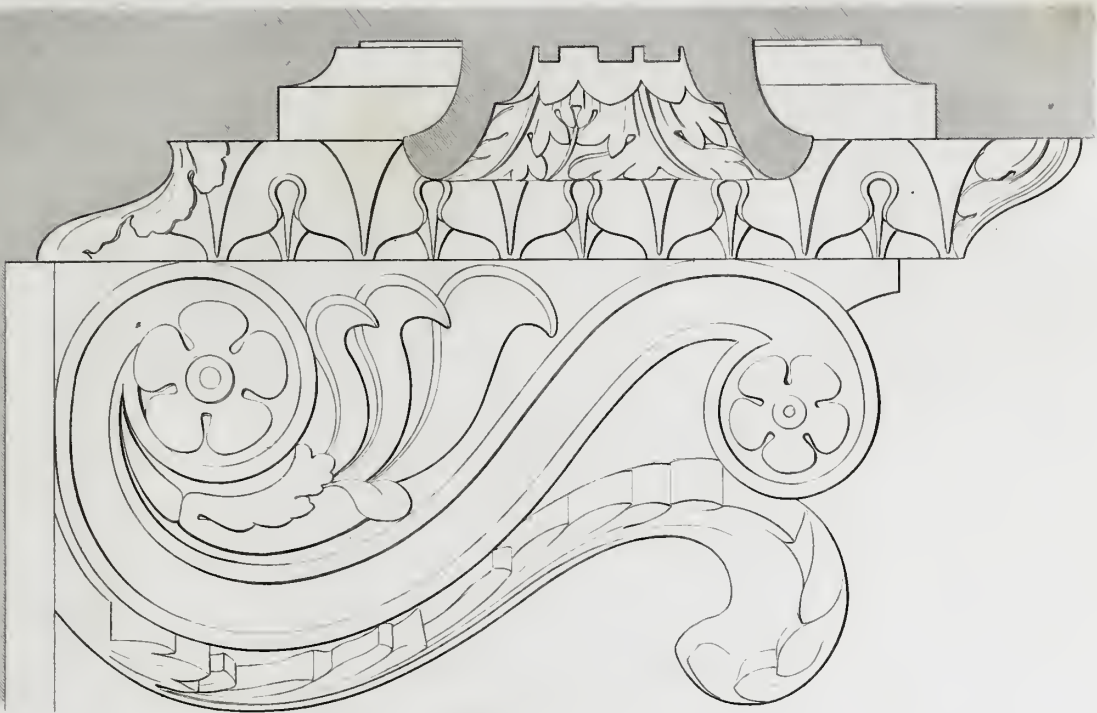
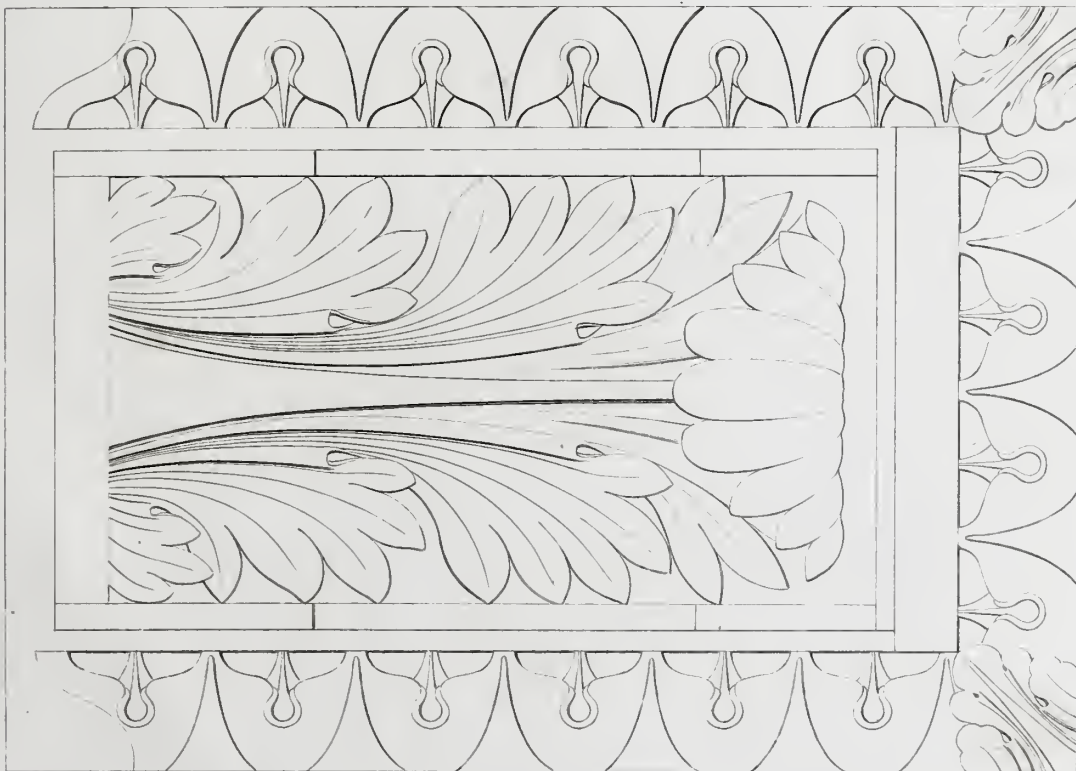


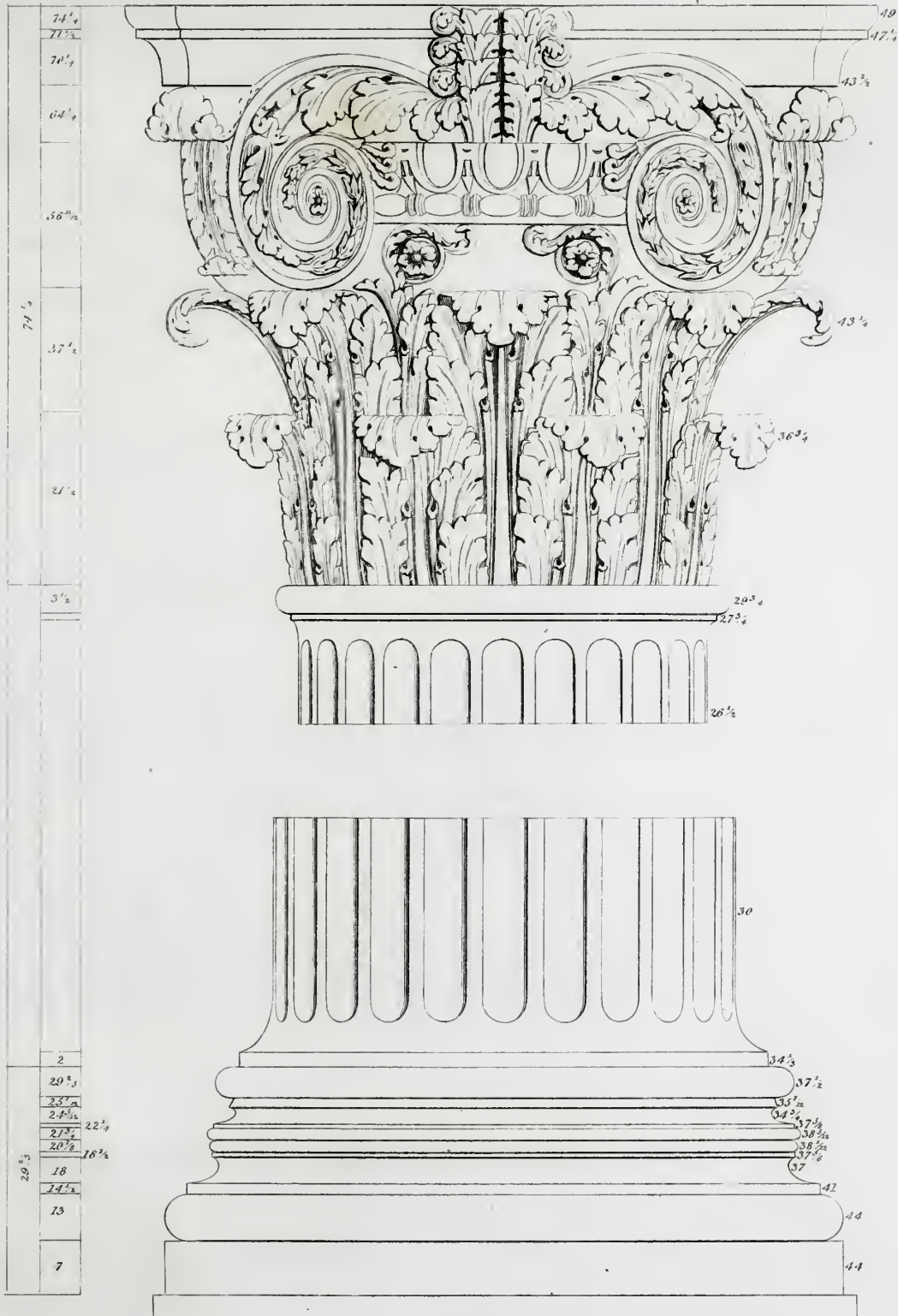
Fig. 2.



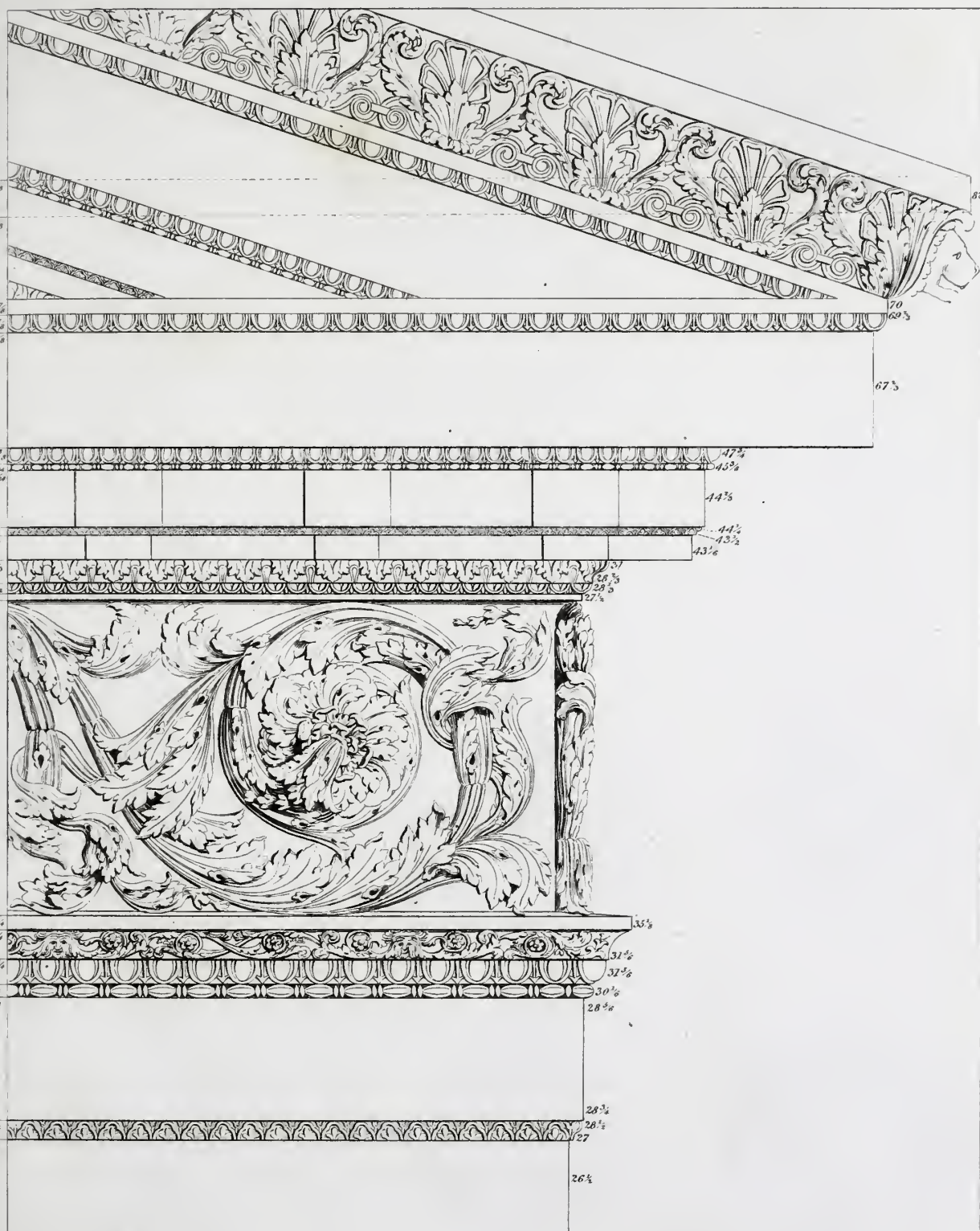
COMPOSITE ORDER.

PLATE 1.

CAPITAL & BASE FROM THE ARCH OF TITUS AT ROME.

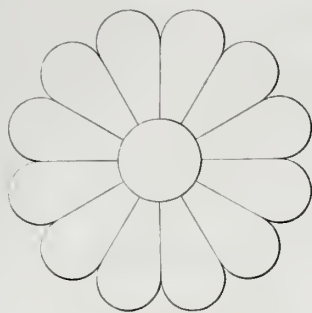


ENTABLATURE FROM THE FRONTISPIECE OF NERO AT ROME.





N^o 1



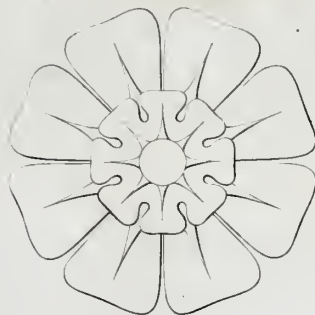
N^o 2



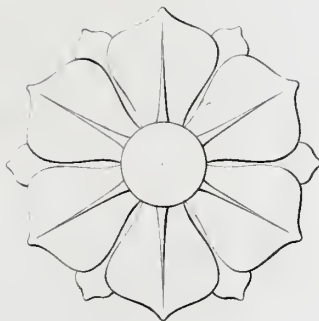
N^o 3



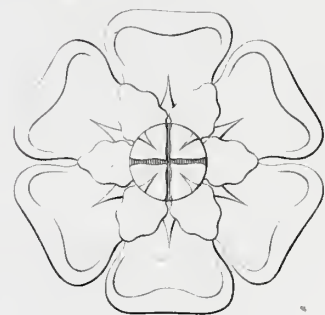
N^o 4



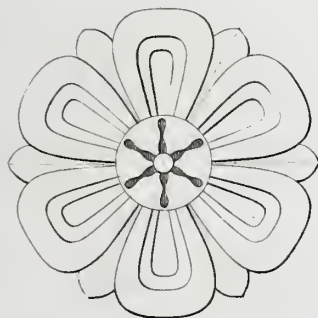
N^o 5



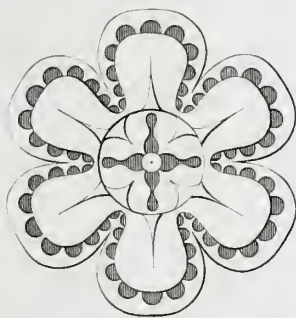
N^o 6



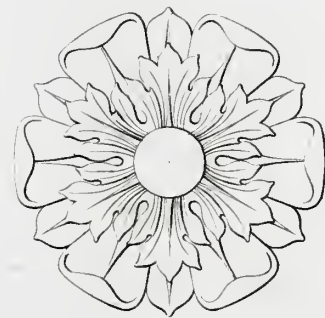
N^o 7



N^o 8



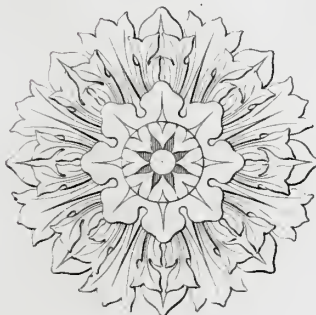
N^o 9



N^o 10



N^o 11



N^o 12

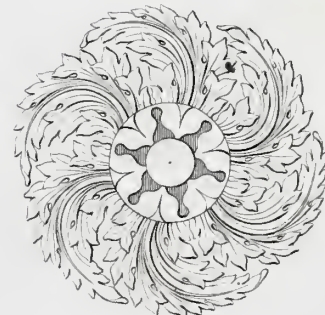


Fig. 2.

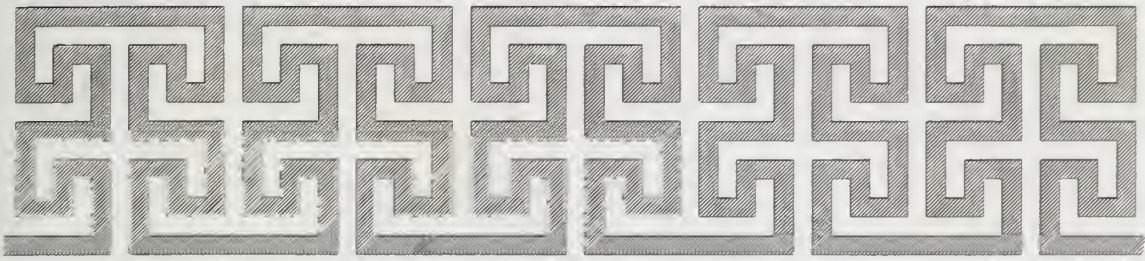


Fig. 1.



Fig. 3.

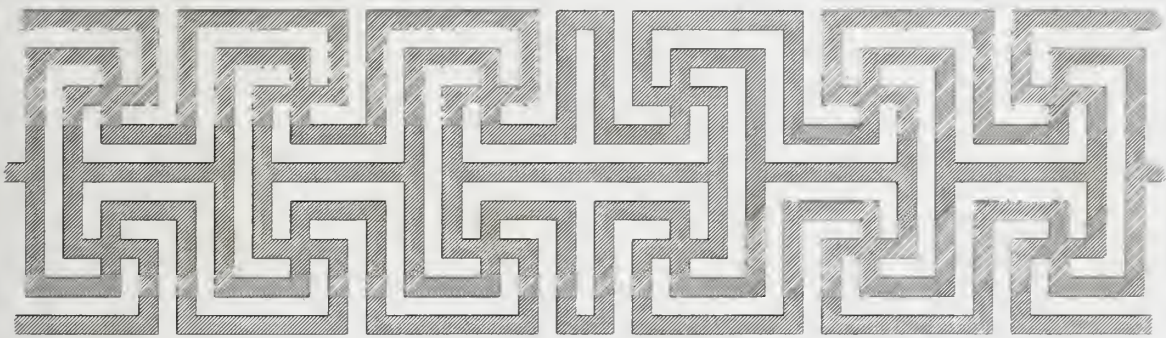
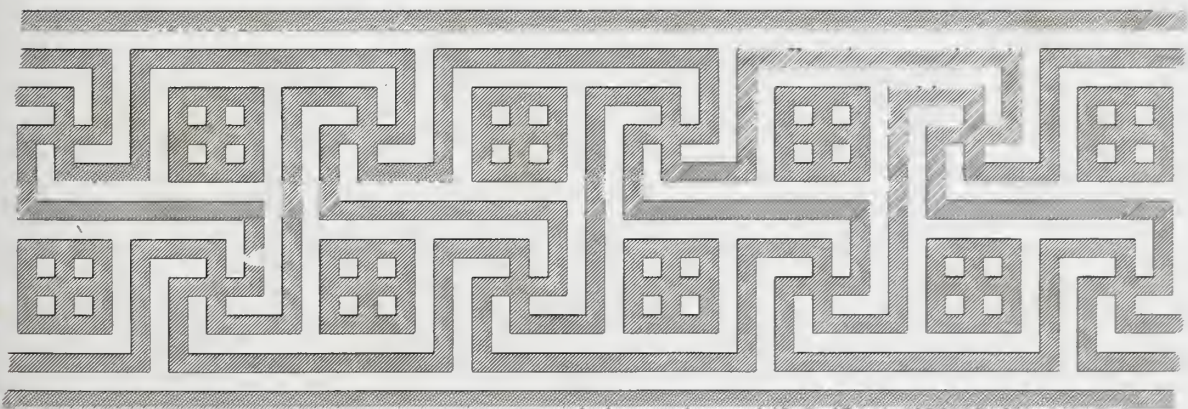


Fig. 4.





BUILDING;

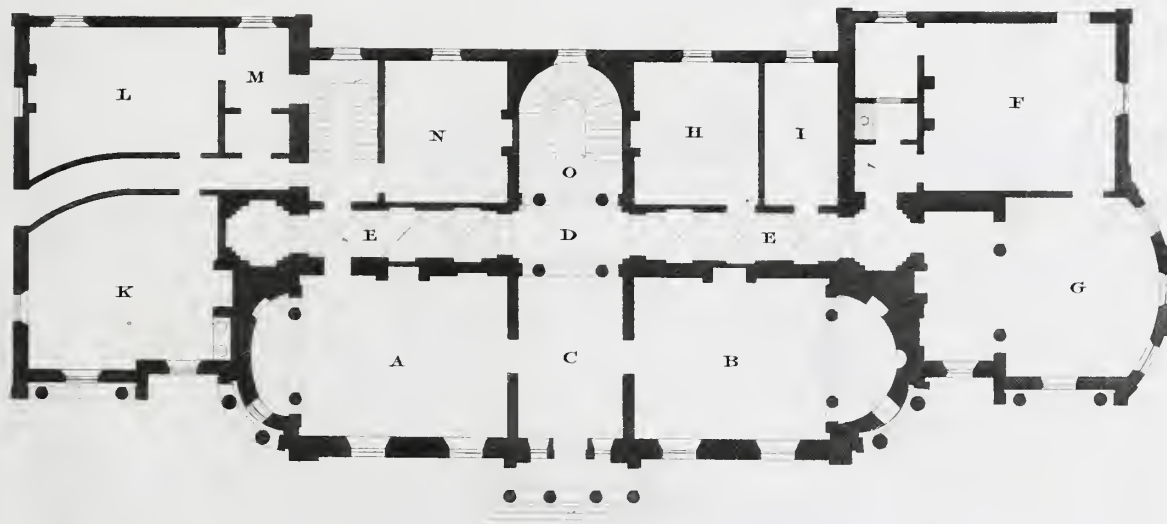
DESIGN.

PLATE VI.

Fig. 1.



Fig. 2.





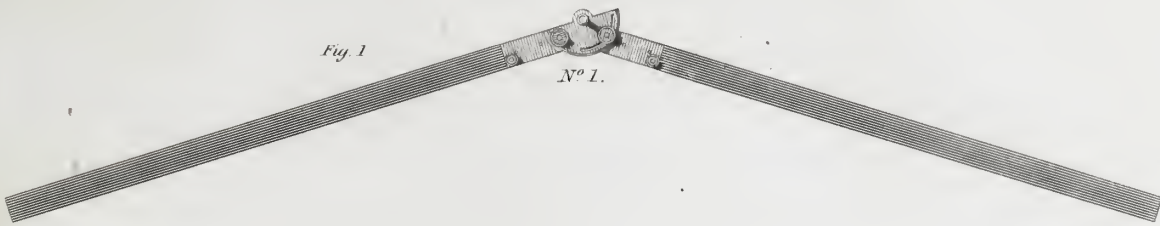
DRAWING INSTRUMENTS,

CYCLOGRAPH.

PLATE 1

Fig. 1.

N^o 1.



N^o 2.



Fig. 8.



Fig. 6.

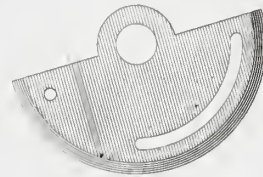
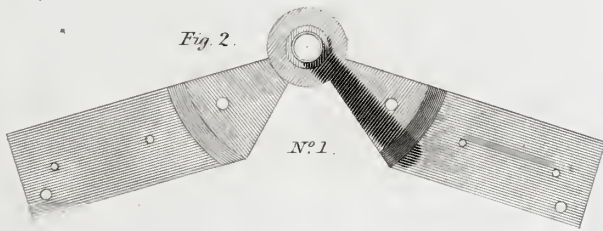


Fig. 2.

N^o 1.



N^o 2.

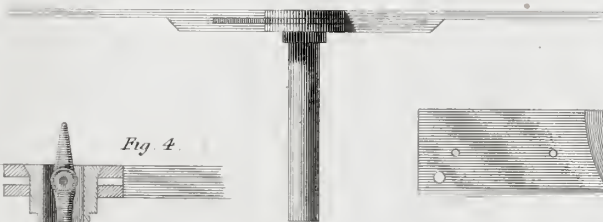


Fig. 4.



Fig. 3.

N^o 1.

N^o 2.

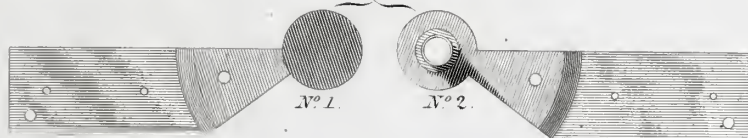


Fig. 5.

N^o 1.

N^o 2.

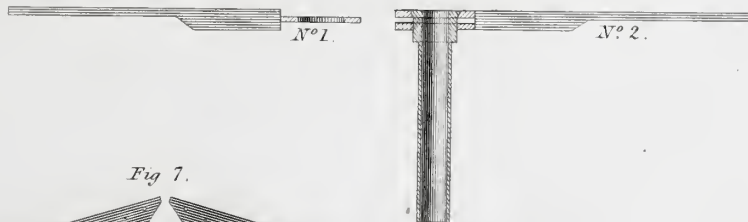


Fig. 7.



CURVE LINES.

ELLIPSE.

PLATE II.

Fig. 1.

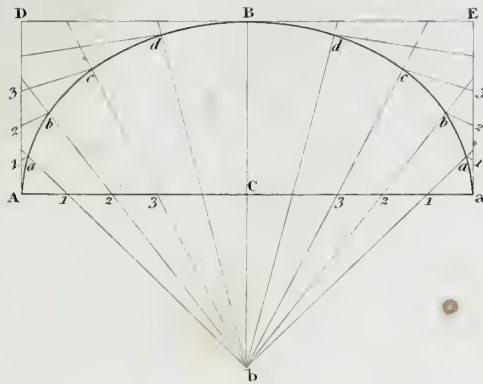


Fig. 2.

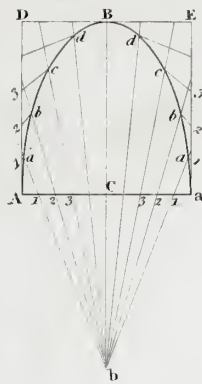


Fig. 3.

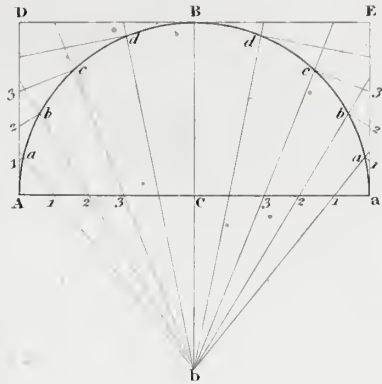


Fig. 5.

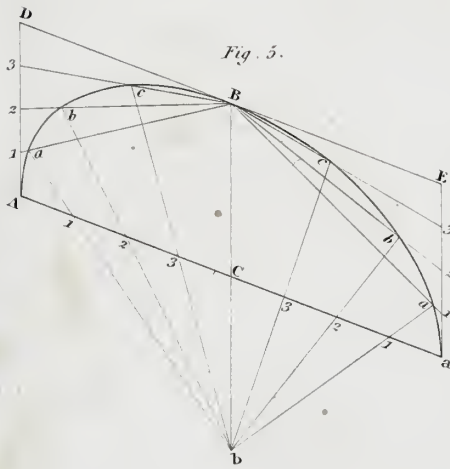


Fig. 6.

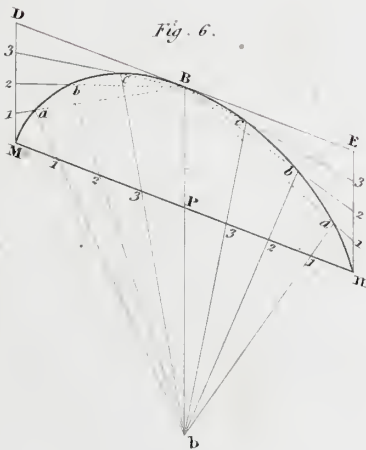


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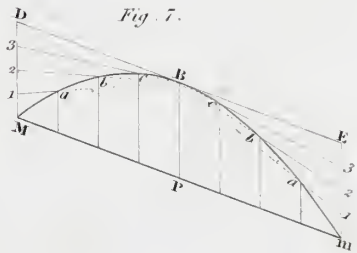


Fig. 7.

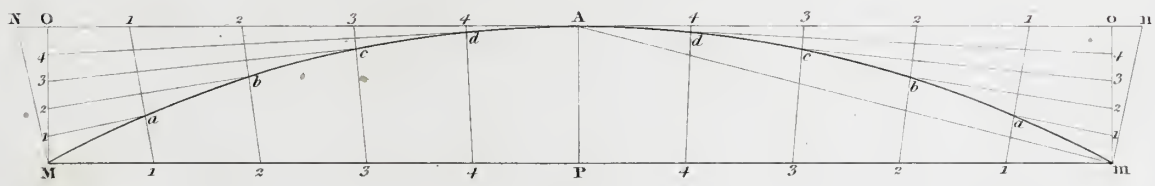


Fig. 8.





Fig. 1.

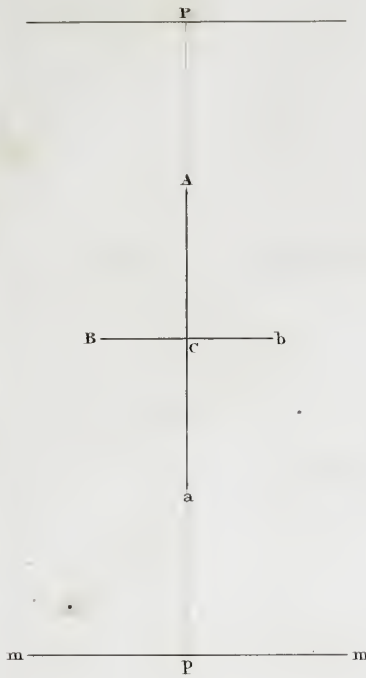


Fig. 2.

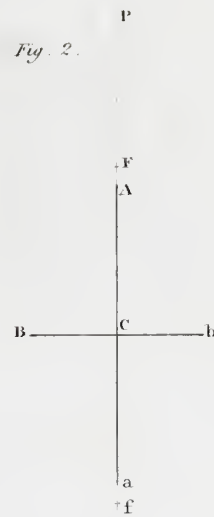
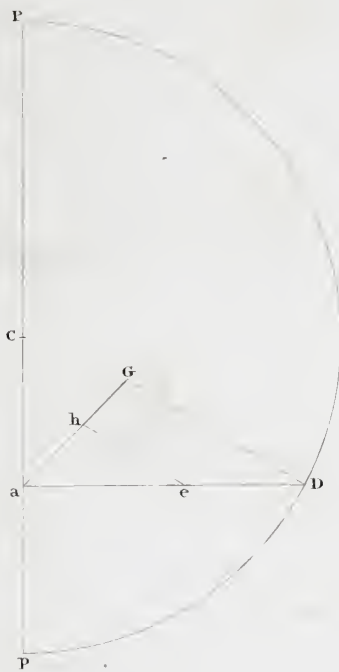


Fig. 3.

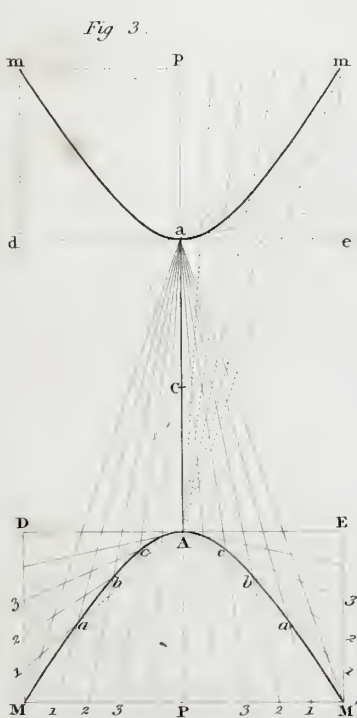


Fig. 4.

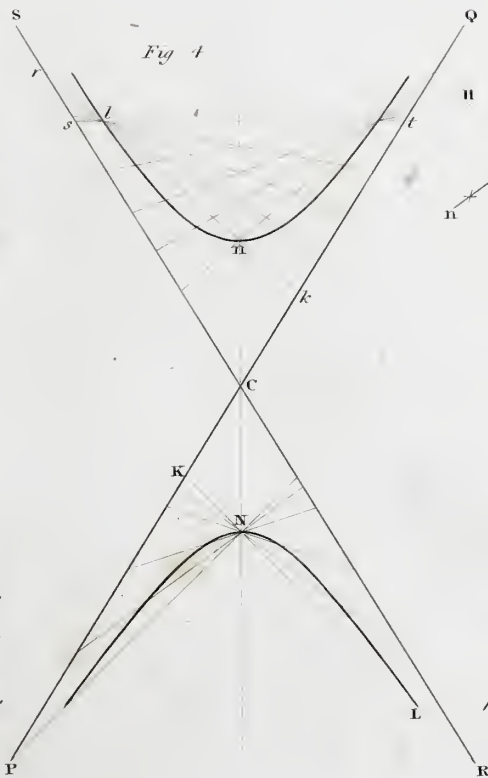
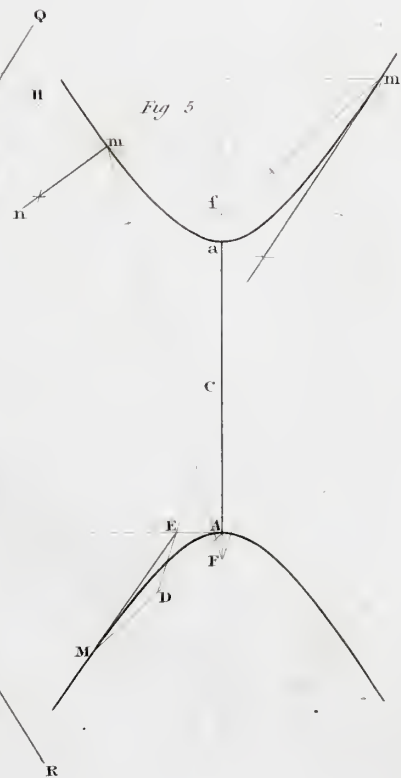


Fig. 5.





CURVE LINES.

PARABOLA.

PLATE I.

Fig. 1.

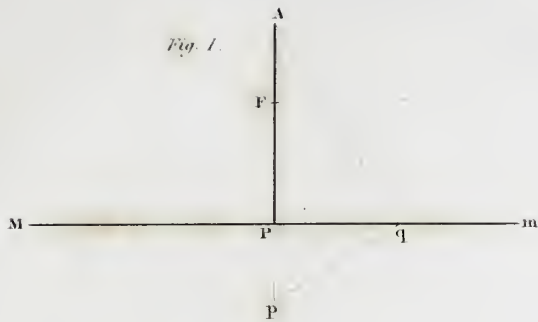


Fig. 2.

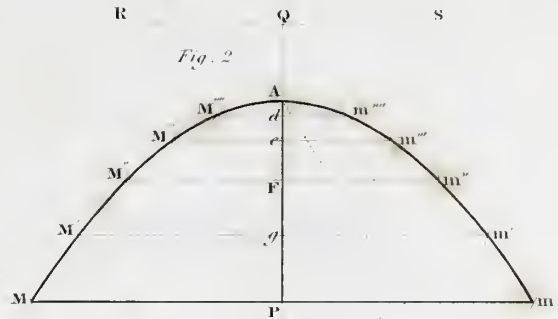


Fig. 3.

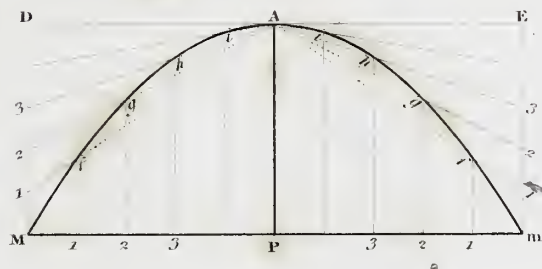


Fig. 4.

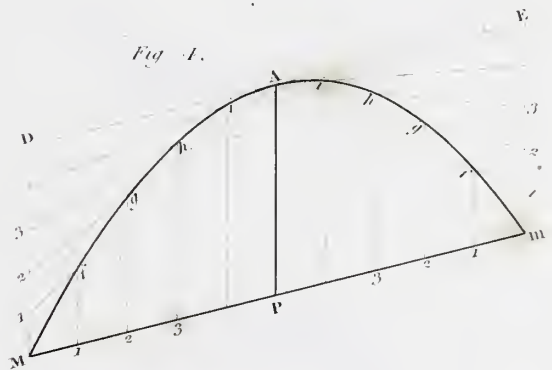


Fig. 5.

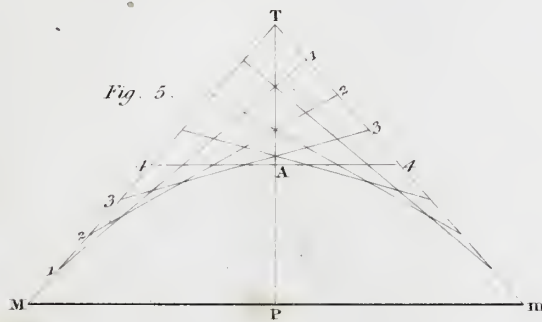


Fig. 6.

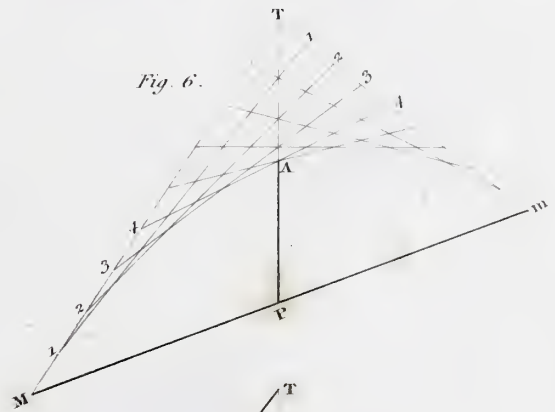


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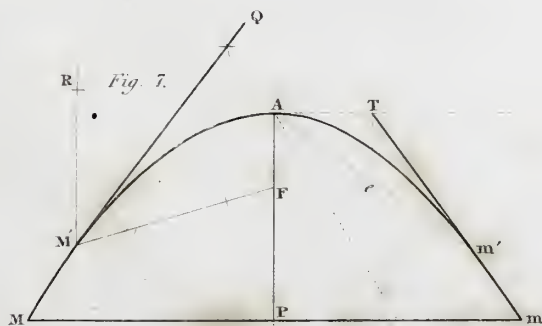
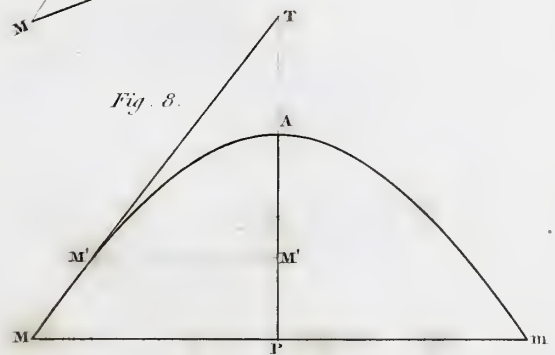
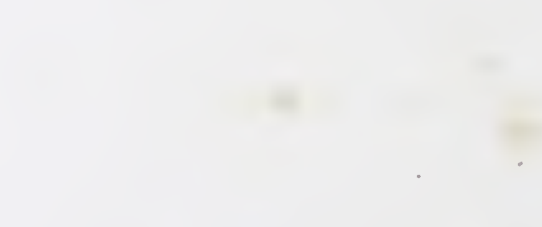


Fig. 8.

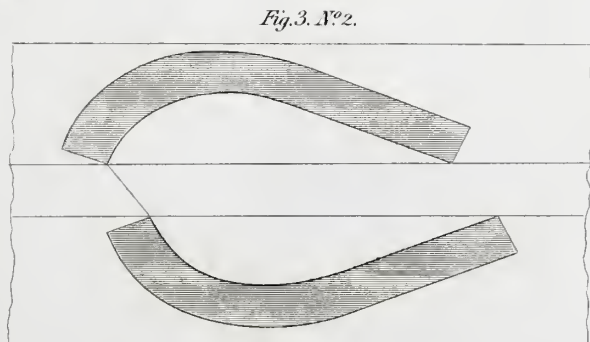
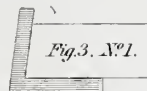
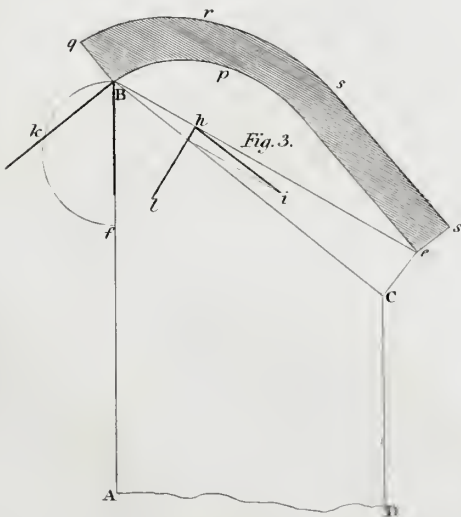
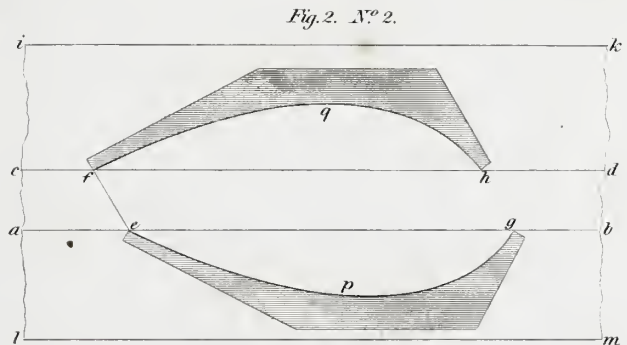
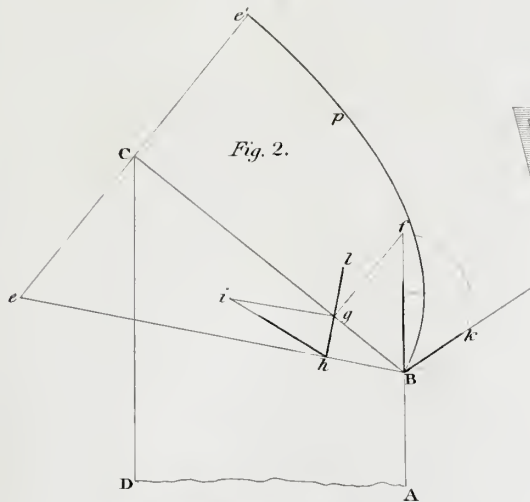
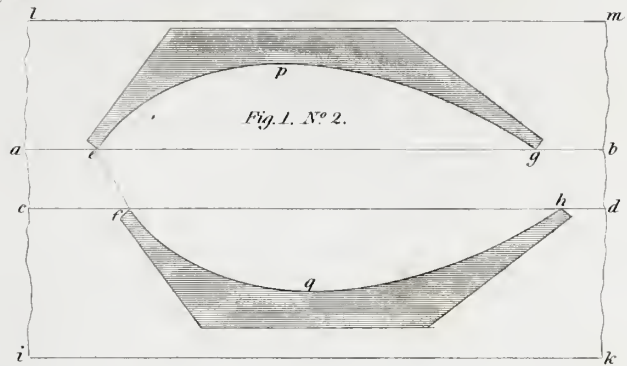
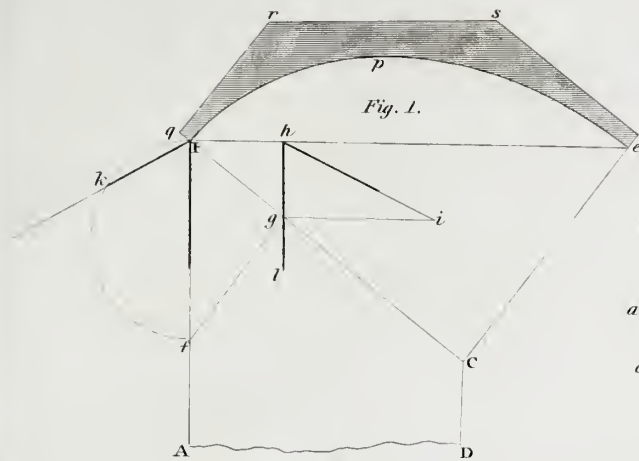




GENERAL PRINCIPLES.

FORMATION OF THE EDGES OF RIBS, BRACKETS OR HANDRAILS TO RANGE IN PRISMATIC OR CYLINDRIC SURFACES.

PLATE I.





EXERCICES IN SOLID TRIGONOMETRY.

OBLIQUE TRIHEDRAL.

PLATE I.

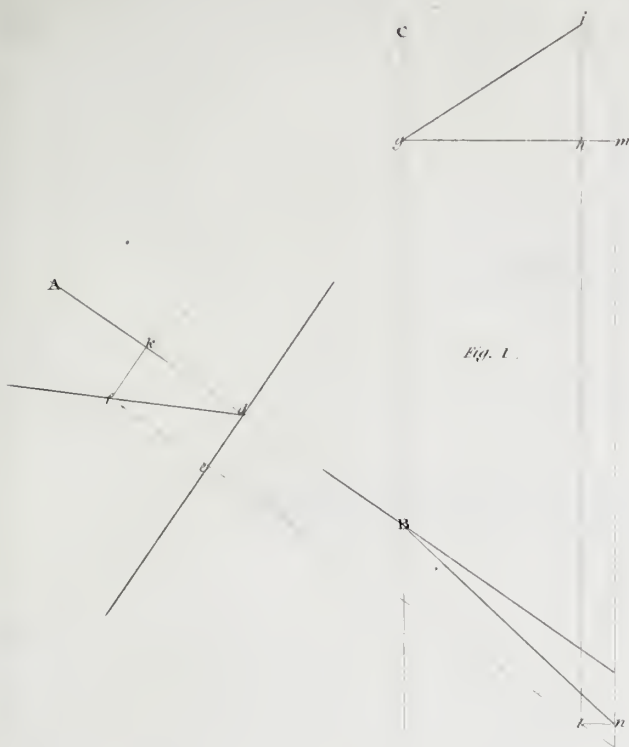


Fig. 1.



Fig. 3.

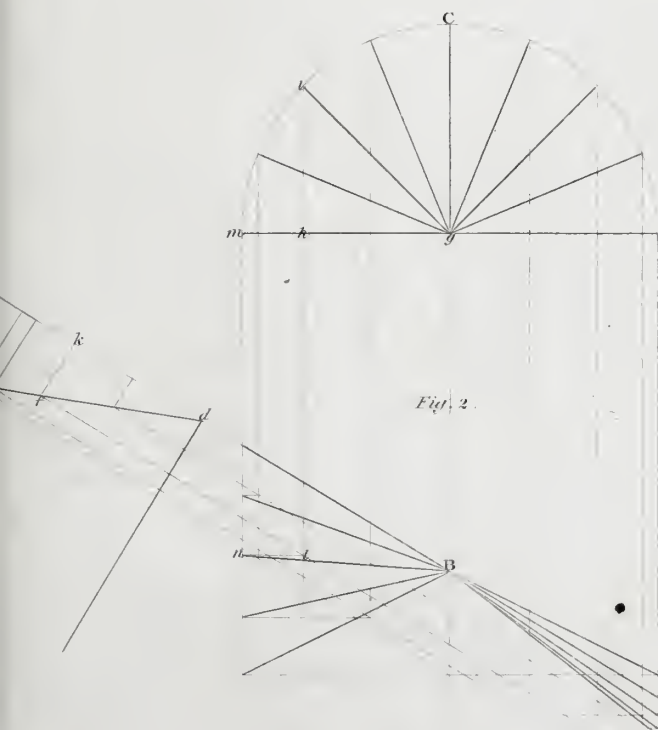


Fig. 2

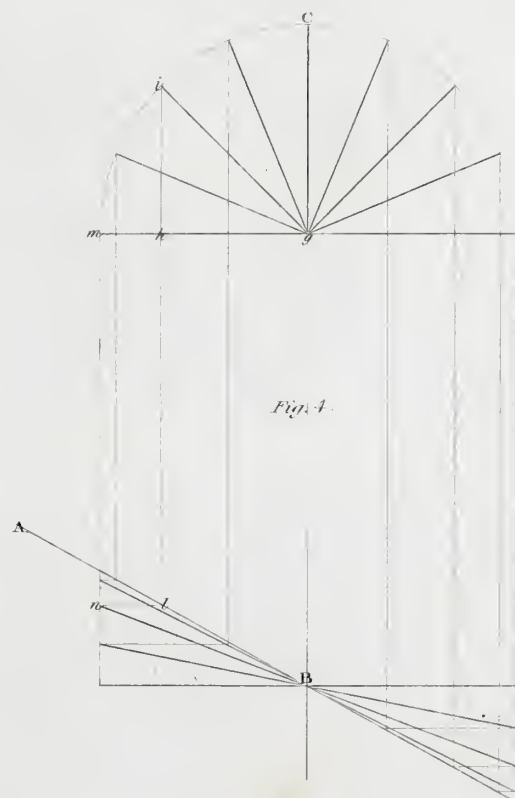


Fig. 4.



APPENDIX.

Elements of Geometry.

AXIOMS are such propositions in which the truth at once appears.

Demonstration is an argument either founded upon Axioms or other facts already known, in order to ascertain the truth of some assertion or thing to be performed.

When an operation is proposed to be done, the proposition is called a **Problem**; but when the truth of some assertion is proposed to be demonstrated, the proposition is called a **Theorem**.

Postulates are the petitions respecting the possibility of certain operations, that may be evidently performed without the aid of demonstration.

A **reductio ad absurdum** is a proposition in which the truth can only be elicited by indirect means with the assistance of Axioms only; and this method consists in showing that every other supposition besides that proposed is absurd.

A **Lemma** is when some premise is demonstrated in order to render the thing proposed more easy.

Scholium is when remarks or observations are made upon something going before.

Corollary is a consequent truth gained by a definition or some preceding truth in a demonstration.

An **Hypothesis** is a supposition or assumption made either in the enunciation of a proposition, or in the course of a demonstration.

SIGNIFICATION OF SIGNS.

The sign $=$ denotes that the quantities between which it is placed are equal; and is therefore read *equal to*, or *is equal to*, or *equals*.

The sign $>$ denotes that the quantity which precedes it is greater than that which follows it; and is therefore read *greater than* or *is greater than*.

The sign $<$ denotes that the quantity which precedes it is less than that which follows it, and is read *less than* or *is less than*.

The sign $+$ denotes that the quantity which follows it is to be added to that which goes before it.

The sign $-$ denotes that the quantity which follows it is to be subtracted from that which is placed before it.

The sign $\angle$ signifies an angle; and is therefore read *angle*.

The sign $\triangle$ signifies a triangle; and is therefore read *triangle*.

AXIOMS.

1. Things which are equal to the same thing are equal to one another.
2. The whole is greater than its part.
3. The whole is equal to the sum of all its parts.
4. If to equal things equal things be added, the wholes will be equal.

5. If from equal things equal things be taken away, the remainders will be equal.

6. If to unequal things equal things be added, the remainders will be unequal.

7. If from unequal things equal things be taken away, the remainders will be unequal.

8. All right angles are equal to one another.

POSTULATES.

1. A straight line may be drawn from one given point to another.

2. A straight line may be prolonged or continued at pleasure.

3. From any centre and with any given radius a circle may be described.

4. A straight line may be drawn perpendicular to another from any assigned point in that other.

5. A straight line may be made equal to another.

6. A straight line may be made equal to any part of another, or that any part of a straight line may be cut off.

7. A rectilincal angle may be made equal to any part of another, or any part of a given angle may be found.

The principal definitions are given under the head of PRACTICAL GEOMETRY.

The following definitions, in addition to what have been given under the head of PLANE GEOMETRY, are indispensably necessary to be attended to, in order to comprehend the Theorems demonstrated in the following Elements.

DEFINITIONS.

Def. 1. *Alternate angles* are those made on the contrary sides and at each end of a line meeting two others.

Def. 2. Figures which have their angles respectively equal to one another are said to be *equiangular*.

Def. 3. *Similar polygons* are those which may be divided into as many equiangular triangles as the figure has sides.

Def. 4. The *altitude* of a figure is a straight line drawn perpendicularly upon its base or upon the prolongation of its base.

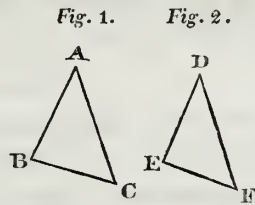
In order to produce the results with the greatest dispatch, the symbolical method is preferred to that of writing the whole in words, as the demonstration is at once brought under the notice of the eye in the form of equations; and by a single glance, every part may be instantly compared with another, and the consequences immediately drawn by taking away the common terms, the terms which stand in opposition to each other, and rejecting or dividing out common factors.

By this means conciseness and perspicuity are gained at the same time, without relaxing either from the strictness or elegance of the method of reasoning pursued by Euclid; and indeed the various equations, though symbolically expressed, may be read in the very same words as if the whole had been written; for it must be remembered, that though a symbol only occupies the place of a single letter, it conveys the meaning of one or more words.

THEOREM I.

If two triangles have two sides, and the included angle of the one respectively equal to two sides and the included angle of the other, then shall the third side of the one be equal to that of the other, the one triangle equal to the other, and the remaining angles of the one equal to those of the other, each to each which subtend the equal sides.

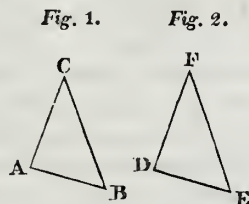
LET the two sides BA, AC of the $\triangle ABC$ be respectively equal to the two sides ED, DF of the triangle DEF, and the $\angle A = D$; then shall $BC = EF$, and the $\triangle ABC = DEF$, the $\angle B = E$, and the $\angle C = F$. For if the $\triangle ABC$ is applied to the $\triangle DEF$, so that the point A may coincide with the point D, and the straight line AB with DE, and because the $\angle A$ is equal D, the straight line AC will coincide with DF; and because AB coincides with DE, and AC with DF, and since $AB = DE$, and $AC = DF$, the point B will coincide with E, and C with F; therefore the base BC coincides with EF, the $\triangle ABC$ with DEF, the $\angle B$ with E, and the $\angle C$ with F.

**THEOREM II.**

If two angles of a triangle be equal to two angles of another, and the side of the one equal to the side of the other between the equal angles each to each, the two triangles shall be equal, and the corresponding sides and angles shall be equal.

Let the two triangles be ABC, DEF, having the $\angle A = D$, the $\angle B = E$, and the side $AB = DE$; the $\triangle ABC$ will be $=$ the $\triangle DEF$, the $\angle C = F$, the side AC to DF, and BC to EF.

For if the point A be applied upon D, so that the straight line AB may coincide with DE, the point B will coincide with E, and the angles at A and B being respectively equal to the angles at D and E, the straight line AC will coincide with DF, and BC with EF; therefore the point C will coincide with F; and because the three points A, B, C coincide with the



three points D, E, F, the $\triangle ABC$ will coincide with the $\triangle DEF$, the side AC with DF, BC with EF, and the $\angle C$ with F.

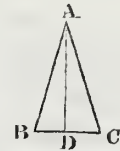
Coroll. Hence, if three angles of a triangle be equal, and a side of the one equal to a side of the other, each subtending an angle equal to that of the other, the two triangles will be equal, and their corresponding sides and angles equal.

THEOREM III.

The angles at the base of an isosceles triangle are equal to each other.

Let ABC be an isosceles $\triangle$ having the side $AB = AC$, the $\angle ABC$ is $=$ the $\angle ACB$.

For suppose the $\angle A$ to be bisected; then the side AB being $=$ AC and AD common, the two sides BA, AD will be $=$ the sides CA, AD; and the $\angle BAD$ being $=$ CAD, the $\triangle BAD$ will be $=$ CAD (Th. i); and the $\angle B$ will be $=$ C.

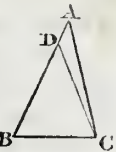
**THEOREM IV.**

If two angles of a triangle be equal, the sides which subtend the equal angles shall also be equal.

Let the $\triangle ABC$ have the $\angle B =$ the $\angle C$, the side AC shall also be $=$ the side AB.

For if AB be not $=$ AC, one of the 2 sides AB, AC is greater than the other: let AB be $>$ AC; therefore let $BD = AC$, and join DC.

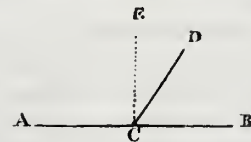
Then, because in the $\triangle s$ BDC, BAC, BD is $=$ AC, the two sides AC, CB respectively equal to the two sides DB, BC, and the $\angle ACB =$ the $\angle DBC$; therefore (Th. i) the $\triangle ACB$ is $=$ the $\triangle DBC$, the less to the greater, which is impossible.

**THEOREM V.**

The angles made by a line passing through one of the extremities of another, are together equal to two right angles.

Let the line AB pass through the extremity C of the line CD.

Then if the $\angle ACD$ be $=$ the $\angle BCD$, each of them will be a right angle; but if not, let CE make right $\angle s$ with AB then, the $\angle ACE + ECB = ACE + ECD + DCB = 2$ right $\angle s$; and since $\angle ACD = ACE + ECD$, therefore (Ax. 1) the $\angle ACD + DCB = 2$ right $\angle s$.



THEOREM VI.

The opposite angles formed by two straight lines cutting each other are equal.

Let the straight line AC cut the straight line BD in E, the $\angle AEB$ is $= DEC$, and the $\angle AED$ to BEC .

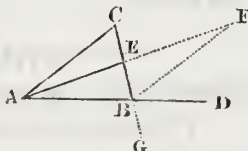
For the $\angle AED + AEB$ is $= 2$ right $\angle$ s, and the $\angle AEB + BEC$ is $= 2$ right $\angle$ s; therefore $\angle AED + AEB$ is $= AEB + BEC$; from each side of this equation take away the common $\angle AEB$, and there will remain the $\angle AED = BEC$.

**THEOREM VII.**

If one side of a triangle be produced, the exterior angle shall be greater than either of the two interior opposite angles.

Let the side AB of the $\triangle ABC$ be prolonged to D; then $\angle DBC$ is $\sqsupset$ either of the two $\angle$ s BAC, BCA.

For suppose BC to be bisected in E: join AE, and produce it to F. Make $EF = EA$, and join BF.



Then, because EC is $= EB$, $EF = EA$, and the $\angle BEF = \angle AEC$; therefore (Th. vi) the $\triangle AEC$ is $= \triangle BEF$, the $\angle ECA = \angle EBF$. But the $\angle EBD$ is $\sqsupset \angle EBF$; therefore also the $\angle EBD$ is $\sqsupset \angle ECA$.

In the same manner, if CB be prolonged to G, it may be demonstrated that the exterior $\angle GBA$ is $\sqsupset \angle BAC$; but the $\angle GBA$ is $= \angle DBC$ (Th. vi); therefore the $\angle DBC$ is $\sqsupset \angle BAC$.

THEOREM VIII.

The greatest side of every triangle is opposite the greatest angle.

Let AB be the greatest side of the $\triangle ABC$; then C is the greatest $\angle$.

In AB take $AD = AC$, and join DC; then (Th. iii) the $\angle ADC = \angle ACD$; but $\angle ADC$ is $\sqsupset B$; therefore the $\angle ACD$ is $\sqsupset B$ (Th. vii); therefore the whole $\angle ACB$ is $\sqsupset B$.

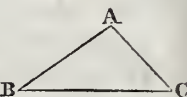
In the same manner it may be shown that $\angle ACB$ shall also be $\sqsupset A$.

**THEOREM IX.**

The greatest angle of every triangle is opposite to the greatest side.

Let BAC be the greatest $\angle$ of the $\triangle ABC$; then shall BC opposite be the greatest side.

For if BC is not $\sqsupset BA$, it must either be equal to it or less; but BC cannot be $= BA$, for then the $\angle A$ would be $= \angle C$ (Th. iii), but it is not; neither can BC be $\sqsupset BA$, for then the $\angle A$ would be $\sqsupset \angle C$ (Th. viii), which is contrary to the hypothesis: then, since the side BC is neither $= BA$ nor $\sqsupset BA$, it must necessarily be greater.

**THEOREM X.**

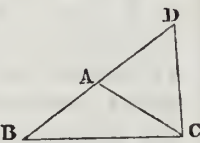
The sum of any two sides of a triangle is greater than the third side.

Let ABC be a $\triangle$; then $BA + AC$ is $\sqsupset BC$.

For produce BA to D, making $AD = AC$, and join DC.

Then, because AD is $= AC$ by construction, the $\angle ADC = \angle ACD$; but the $\angle BCD$ is $\sqsupset \angle ACD$; therefore also the $\angle BCD$ is $\sqsupset \angle ADC$; and since the greatest side of every $\triangle$ is opposite to the greatest $\angle$ (Th. viii), BD is $\sqsupset BC$; but BD is the sum of the sides BA, AC; wherefore the sum of the two sides BA, AC, is $\sqsupset BC$.

Coroll. Hence the shortest distance between two points is a straight line drawn from one point to the other.

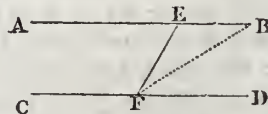
**THEOREM XI.**

The alternate angles made by a straight line meeting two parallel lines are equal.

Let EF meet the parallels AB, CD; then the $\angle AEF = \angle EFD$, and the $\angle BEF = \angle EFC$.

For if the angles AEF and EFD are not equal, one of them must be greater than the other: let EFD be $\sqsupset \angle AEF$, and make the $\angle EFB = \angle AEF$.

Then the exterior $\angle AEF$ of the $\triangle EFB$ will be $\sqsupset$ the interior $\angle EFB$ (Th. vii); but by construction it is also equal to it, which is impossible; therefore the $\angle AEF$ cannot be $\sqsupset \angle EFD$. In the same manner it may be proved that it cannot be $\sqsupset \angle EFD$; therefore as $\angle AEF$ can neither be less nor greater than the $\angle EFD$, the $\angle AEF$ must be $= \angle EFD$.



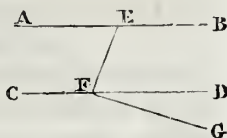
THEOREM XII.

Two straight lines passing through the extremities of another, and making the alternate angles equal, are parallel.

The two straight lines AB, CD passing through the two extremities E, F of the line EF, and making the $\angle AEF = \angle EFD$, are parallel.

For if they are not parallel, let some other line, as FG, be parallel to AB.

Then, because AB is parallel to FG, the $\angle AEF =$ the alternate $\angle EFG$ (Th. xi); but by hypothesis the $\angle AEF = \angle EFD$; therefore (Ax. 1) the $\angle EFD = \angle EFG$ the less to the greater, which is impossible.

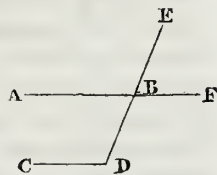
**THEOREM XIII.**

If a line pass through one extremity of two parallels, and meet the extremity of the other on the same side, the exterior angle is equal to the interior at the point of concurrence.

Let the line ED pass through B, and meet CD which is parallel to AB in D; then the $\angle EBA = \angle EDC$.

For produce AB to F.

Then (Th. xi), because AF and CD are parallel, the alternate $\angle$ s CDB and DBF are equal; but (Th. vi) the angle DBF = EBA or ABE being opposite angles, therefore the $\angle ABE = \angle CDB$ (Ax. 1).

**THEOREM XIV.**

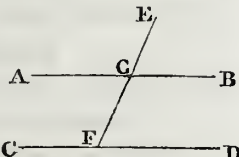
If a straight line passing through another, and terminating in a third, make the exterior angle at the intersection equal to the angle on the same side at the point of meeting, then the two straight lines thus intersected and met are parallel.

Let the straight line EF cutting AB in G, and meeting CD in F, make the $\angle AGE = \angle CFG$; then AB is parallel to CD.

Now, since (Th. v)

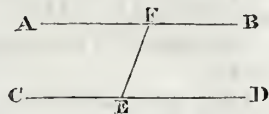
$$\begin{cases} \angle AGE + \angle AGF = 2 \text{ right } \angle\text{s} \\ \angle CFG + \angle DFG = 2 \text{ right } \angle\text{s} \end{cases}$$

whence $\angle AGE + \angle AGF = \angle CFG + \angle DFG$; but by hypothesis, the $\angle AGE = \angle CFG$; therefore $\angle AGF = \angle DFG$; wherefore (Th. xii) AB is parallel to CD.

**THEOREM XV.**

Two straight lines passing through the extremities of another, and making the angles on the same side of that other line equal to two right angles, are parallel.

Let the two straight lines AB, CD pass through the two extremities E, F of the line EF, making the $\angle AFE + \angle CEF = 2$ right $\angle$ s; then AB is parallel to CD.

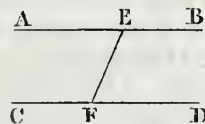


For by hypothesis, the $\angle AFE + \angle CEF = 2$ right $\angle$ s, and by (Th. v) - - - the $\angle AFE + \angle BFE = 2$ right $\angle$ s; therefore (Ax. 1) - - - the $\angle AFE + \angle CEF = \angle AFE + \angle BFE$; then, taking away the common $\angle AFE$, there will remain the $\angle CEF = \angle BFE$. But these are alternate $\angle$ s; therefore AB is parallel to CD (Th. xii).

THEOREM XVI.

The angles on the same side of a straight line meeting two parallels are equal to two right angles.

Let the straight line EF meet the two parallels AB, CD in the two points E, F; the $\angle AEF + \angle EFC = 2$ right $\angle$ s.



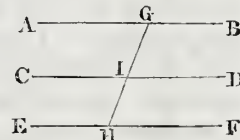
For (Th. v) - the $\angle CFE + \angle EFD = 2$ right $\angle$ s;
but (Th. xi) - - - - the $\angle AEF = \angle EFD$;
therefore - - - - $\angle CFE + \angle AEF = 2$ right $\angle$ s.

THEOREM XVII.

Straight lines which are parallel to the same straight line, are parallel to one another.

Let the two straight lines AB, EF be each parallel to CD; then AB and EF are parallel.

Draw GH, meeting AB in G and EF in H, and cutting CD in I.



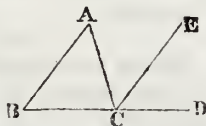
Then, because AB is parallel to CD, the $\angle AGI = \angle GID$ (Th. xi); and because EF is parallel to CD, the $\angle GID = \angle GHF$; therefore the $\angle AGI$ or $\angle AGH = \angle GHF$, and therefore (Th. xii) AB is parallel to CD.

THEOREM XVIII.

If one side of a triangle be produced, the exterior angle is equal to the sum of the two interior angles which are not contiguous or adjacent to the exterior angle.

Let ABC be a triangle, and let BC be produced to D; then the exterior $\angle ACD = CAB + ABC$.

For suppose CE to be drawn parallel to AB.



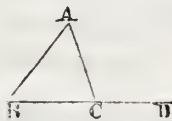
Since AB is parallel to CE, and AC joins them, the $\angle ECA = CAB$ (Th. xi); and since BD passes through the two extremities B, C of the parallels AB, EC, the $\angle ECD = ABC$ (Th. xiii); therefore, by adding these two equations together, the $\angle ECA + ECD = CAB + ABC$; but $\angle ECA + ECD = ACD$; therefore the $\angle ACD = CAB + ABC$ (Ax. 1).

THEOREM XIX.

The sum of the three angles of every triangle is equal to two right angles.

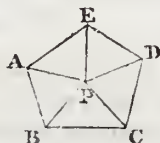
Let ABC be a $\triangle$; then the $\angle ABC + BCA + CAB = 2$ right $\angle$ s.

Produce BC to D; then the $\angle ACD = CAB + ABC$ (Th. xviii). To each side of this equation add the adjacent $\angle ACB$; then the $\angle ACD + ACB = CAB + ABC + BCA$; but the $\angle ACD + ACB = 2$ right $\angle$ s (Th. v); therefore the $\angle CAB + ABC + BCA = 2$ right $\angle$ s.

**THEOREM XX.**

The sum of all the angles made within any straight lined figure, by its sides, is equal to twice as many right angles, wanting four, as the number of the sides of the figure.

Let the figure be ABCDE. Take P any point within the figure, and join PA, PB, PC, PD, PE; then for every side of the figure there is a $\triangle$; and since the sum of the three angles of every $\triangle$ make 2 right angles, therefore the interior angles of the figure, together with 4 right $\angle$ s at the centre, make twice as many right angles as the figure has sides; and therefore the interior angles formed by the sides of the figure make twice as many right angles, wanting four, as the figure has sides.

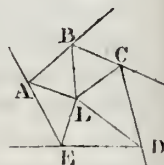


Coroll. Hence the sum of the interior angles of a quadrilateral make four right angles; for, deducting the 4 right angles at the centre from 8, which is twice the number of sides, there remains four for the interior $\angle$ s of the figure.

THEOREM XXI.

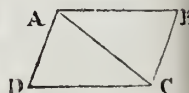
The sum of the exterior angles of any rectilineal figure is equal four right angles.

For when the sides are prolonged, for every side of the figure the exterior $\angle$ s, together with the interior $\angle$, make two right $\angle$ s; therefore the sum of the exterior and interior angles make twice as many right angles as the figure has sides; but the interior angles, together with the angles at the centre, also make twice as many right $\angle$ s as the figure has sides: from each of these equal sums take away the interior $\angle$ s, and there will remain the exterior $\angle$ s equal to the angles at the centre; that is, equal to four right $\angle$ s.

**THEOREM XXII.**

The opposite sides and also the opposite angles of a parallelogram are equal, and the diagonal divides the figure into two identical triangles.

Let ABCD be a parallelogram, the $\angle A = C$, and the $\angle B = D$, and likewise the side $AB = DC$, $BC = AD$: for join AC; and since AB is parallel to DC, the $\angle ABC + BCD = 2$ right $\angle$ s (Th. xvi); and because BC is parallel to AD, the $\angle BCD + CDA = 2$ right $\angle$ s (Th. xvi); therefore $\angle ABC + BCD = BCD + CDA$ (Ax. 5): take away the common $\angle BCD$, there will remain the $\angle ABC = CDA$. In the same manner the $\angle BAD$ may be proved to be equal BCD.



With respect to the sides,

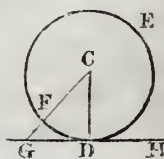
Because AC joins the two parallels AB, DC, the alternate $\angle$ s BAC, ACD are equal (Th. xi), and for the same reason the alternate $\angle$ s CAD, ACB are equal; therefore in the two $\triangle$ s ACD, ACB, the side AC is common, and the $\angle ACD = CAB$, and the $\angle CAD = ACB$; therefore $AB = CD$ (Th. ii), and $AD = BC$.

THEOREM XXIII.

The diameter of a circle passing through the point of contact is perpendicular to the tangent.

Let EDF be a circle; C its centre, GH = a tangent at the point D; CD, the semidiameter, is perpendicular to GH.

For if CD is not perpendicular to GH, let CG be perpendicular to GH: then



CGD will be a right angled Δ ; and consequently the $\angle$ CDG will be acute; therefore since, by hypothesis, the $\angle$ CGD $\sqsubset$ CDG, the side CD $\sqsubset$ CG. But CD = CF; wherefore CF $\sqsubset$ CG, a part greater than the whole, which is absurd; therefore CD is perpendicular to GH.

THEOREM XXIV.

The angle in a semicircle is a right angle.

Let DEF be a semicircle, the $\angle$ DEF is a right $\angle$. For draw the radius EC, then the $\angle$ CDE = $\angle$ CED } (Th. iii)
and the $\angle$ CFE = $\angle$ CEF }
therefore $\angle$ CDE + $\angle$ CFE = $\angle$ CED + $\angle$ CEF;
but the $\angle$ CED + $\angle$ CEF = $\angle$ DEF
therefore $\angle$ CDE + $\angle$ CFE = $\angle$ DEF (Ax. 1).

Now, since the three angles of every Δ are = 2 right $\angle$ s, the $\angle$ DEF is therefore the half of two right angles; that is, DEF is a right angle.

THEOREM XXV.

The angle at the centre of a circle is double to the angle at the circumference.

Let DEF be a circle, the $\angle$ ECF = twice EDF.

For in the first case, where one of the lines containing the $\angle$ at the circumference passes through the centre,

the $\angle$ ECF = $\angle$ CDF + $\angle$ CFE (Th. xviii);
but $\angle$ CDF = $\angle$ CFE (Th. iii);

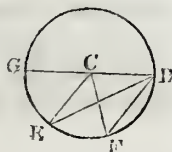
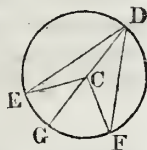
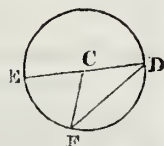
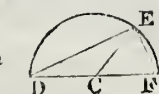
therefore $\angle$ ECF = 2 $\angle$ CDF or 2 $\angle$ EDF.

In the second case, where a line DG drawn from the point D of the $\angle$ at the circumference, and through the centre C, lies between the lines containing both $\angle$ s;

then $\angle$ ECG = 2 $\angle$ EDG } by case 1;
and $\angle$ FCG = 2 $\angle$ FDG }

therefore $\angle$ ECG + $\angle$ FCG = 2($\angle$ EDG + $\angle$ FDG) (Ax. 4);
but $\angle$ ECG + $\angle$ FCG = $\angle$ ECF and $\angle$ EDG + $\angle$ FDG = $\angle$ EDF;
therefore $\angle$ ECF = 2 $\angle$ EDF.

In the third case, when the line DG falls without both $\angle$ s,



then $\angle$ GCF = 2 $\angle$ GDF } by case 1;
and $\angle$ GCE = 2 $\angle$ GDE }

therefore the $\angle$ GCF - $\angle$ GCE = 2($\angle$ GDF - $\angle$ GDE) (Ax. 5);
but $\angle$ GCF - $\angle$ GCE = $\angle$ ECF, & the $\angle$ GDF - $\angle$ GDE = $\angle$ EDF;
therefore the $\angle$ ECF = 2 $\angle$ EDF.

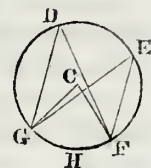
THEOREM XXVI.

All angles in the same segment of a circle are equal to one another.

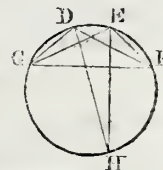
Let DGHFE be a circle, and GDF, GEF $\angle$ s in the same segment; then the $\angle$ GDF = $\angle$ GEF.

In the first case, where the $\angle$ s stand on a segment less than a semicircle, the case is evident.

For then each of the $\angle$ s GDF, GEF is half the $\angle$ GCF at the centre: but things which are the half of the same thing are equal; therefore the $\angle$ GDF = $\angle$ GEF.



In the second case, where the angles stand upon a segment greater than a semicircle, draw DH, EH, so that the arcs GH or HE may each be less than a semicircle;



then since $\angle$ GDH = $\angle$ GEH } by case 1,
and since $\angle$ FDH = $\angle$ FEH }
therefore $\angle$ GDH + $\angle$ FDH = $\angle$ GEH + $\angle$ FEH (Ax. 4)
therefore $\angle$ GDF = $\angle$ GEF.

THEOREM XXVII.

The angle formed between a tangent to a circle, and a chord drawn through the point of contact, is equal to the angle in the alternate segment standing on that chord.

Let AB be a tangent at C to the circle CDEF, CD a chord, BCD the $\angle$ contained by the chord and the tangent, and CFD the $\angle$ in the alternate segment; then the $\angle$ BCD = $\angle$ CFD.

For draw the diameter CE, and join ED.

Then, because EC is perpendicular to AB, the $\angle$ s ACE, BCE are each a right $\angle$, and, the $\angle$ CDE being in a semicircle, is also a right $\angle$;



therefore - - - - - $\angle DCE + DEC = \text{a right } \angle$ (Th. xxiv);
 and since - - - - - $\angle DCE + DCB = \text{a right } \angle$
 therefore - - - - - $\angle DCE + DEC = DCE + DCB$ (Ax. 1)

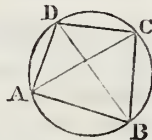
Take from these equals the common $\angle DCE$, and
 there will remain the - - - - $\angle DEC = DCB$; but
 the $\angle DEC$ or $CED = CFD$; therefore the $\angle CFD = BCD$.

THEOREM XXVIII.

The opposite angles of any quadrilateral described in a circle, are equal to two right angles.

Let ABCD be a quadrilateral described in the circle ABCD,

the $\angle ADC + ABC = 2 \text{ right } \angle$ s.



For since - $\angle CAB + ABC + BCA = 2 \text{ right } \angle$ s,
 and since - - - - - the $\angle CAB = CDB$ }
 and the - - - - - $\angle ACB = ADB$ }
 therefore - - - $\angle CAB + ACB = ADC$ (Ax. 1);
 therefore $\angle CAB + ACB + ABC = ADC + ABC$ (Ax. 4).
 But - - $\angle CAB + ACB + ABC = 2 \text{ right } \angle$ s (Th. xix);
 therefore - - - $\angle ADC + ABC = 2 \text{ right } \angle$ s (Ax. 1).

THEOREM XXIX.

Parallelograms upon the same base and between the same parallels are equal.

Let ABCD, EBCF be two parallelograms upon the same base BC, and between the same parallels BC, AF, the parallelogram ABCD is equal to the parallelogram EBCF.

Fig. 1.

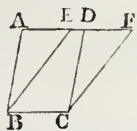
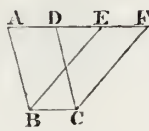


Fig. 2.



For since the line AF meets the parallels AB, DC, the two $\angle$ s FDC, FAB are equal; and because $AD = BC$, $BC = EF$; therefore $AD = EF$.

Then in the case (Fig. 1) where the opposite sides have a common segment, take away ED, and there will remain $AE = DF$.

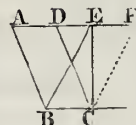
In the other case, where the sides opposite to the base have no common segment, since $AD = EF$, add DE to each; then AE is equal to DF.

Then, since in both cases $AE = DF$, and $DC = AB$, therefore the two sides AE, AB are respectively equal to the two sides FD, DC, and the $\angle EAB = FDC$; therefore the $\triangle AEB$ is $\equiv$ the $\triangle DFC$: from the whole figure take away the $\triangle FDC$, and there will remain the parallelogram ABCD: again, from the whole figure take away the equal triangle EAB, and there will remain the parallelogram EBCF equal to ABCD.

THEOREM XXX.

If a parallelogram and a triangle be upon the same base and between the same parallels, the parallelogram is double of the triangle.

Let ABCD be a parallelogram, and EBC a triangle both upon the same base BC, and between the same parallels BC and AF, the parallelogram ABCD is double of the $\triangle$ EBC.



For draw CF parallel to BE; then BCFE is a parallelogram, and the diagonal EC divides the figure into two equal triangles (Th. xxii); therefore the parallelogram BCFE is double of the triangle EBC: but the parallelogram ABCD is equal to the parallelogram EBCF (Th. xxix); wherefore the parallelogram ABCD is also double to the triangle EBC.

THEOREM XXXI.

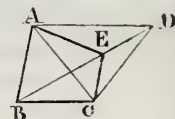
Triangles upon the same base and between the same parallels are equal.

This is evident, since the parallelogram upon the same base and between the same parallels is equal, and since each parallelogram is double of the triangle.

THEOREM XXXII.

Equal triangles upon the same base and on the same side of it are between the same parallels.

Let the equal $\triangle$ s ABC, DBC be upon the same base BC, and upon the same side of it; then if AD be joined, AD is parallel to BC.



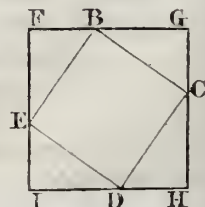
For if AD is not parallel to BC, draw AE parallel to BC, and join EC; then will the $\triangle ABC$ be $\equiv$ EBC (Th. xxxi): but the $\triangle ABC$ is $\equiv$ DBC; therefore the $\triangle DBC$ is $\equiv$ EBC, the greater to the less, which is impossible.

THEOREM XXXIII.

A rectangle circumscribed about a square is a square also.

Let BCDE be a square, and let FGHI be a rectangle circumscribing it, the rectangle FGHI is a square.

For since the 4 $\triangle$ s BGC, CHD, DIE, EFB, are right $\angle$ d;



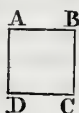
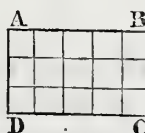
and since (Th. v) $\angle CBG + EBC + EBF = 2$ right $\angle$ s ;
 therefore $\angle CBG + EBF =$ a right $\angle$;
 but since $\angle BGC$ is a right $\angle$, $\angle BCG + CBG =$ a right $\angle$;
 therefore (Ax. 1) $\angle CBG + EBF = BCG + CBG$;
 wherefore (Ax. 5) $\angle EBF = BCG$;
 therefore the two triangles EFB , BGC are equiangular ; and
 since the hypotenuses of these Δ s are equal, the Δ s EFB ,
 BGC are equal and similar ; consequently the four Δ s EFB ,
 BGC , CHD , DIE are equal and similar ; wherefore the four
 distances EF , BG , CH , DE are all equal, as also the four
 distances FB , GC , HD , IE are all equal, and consequently
 the rectangle $FGHI$ is a square.

THEOREM XXXIV.

The area of every rectangle may be expressed by the product of two of its adjoining sides.

For let the length AB be divided into a equal parts, and the breadth AD into b equal parts, so that each of the equal parts of AD may be equal to each of the equal parts of AB . Then if, through each point of section in the line AD , straight lines be drawn parallel to AB to meet the line BC , the whole figure will be divided into b rectangles, each containing a squares ; therefore the area of the whole rectangle $ABCD$ will contain b times a squares $= ab$, which is the analytical expression for the area when one side is denoted by a , and the adjoining side by b .

Therefore the rectangle $ABCD$ is expressed by $AB \times BC$, or more shortly by $AB \cdot BC$. Similarly, when the figure is a square $ABCD$, instead of writing $AB \cdot CD$, we shall write AB^2 or CD^2 . — [See Notes.]



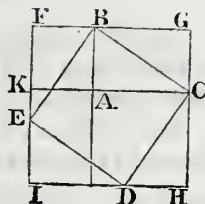
THEOREM XXXV.

In any right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Let BAC be a Δ , having a right $\angle$ BAC , $BC^2 = AB^2 + AC^2$.

For on BC describe the square $BCDE$, and through the points B , D , E , C draw FG , IH , FI , GH respectively parallel to AC , AB , and produce CA to meet FI in K .

Then (Th. xxxiii) the figure $FGHI$ is a square, and the four Δ s BGC , CHD , DIE , EFB are all equal ; but the rectangle $ABGC$ is $=$ twice the Δ ABC (Th. xxx) ;



wherefore the Δ $BGC + CHD + DIE + EFB =$ twice the rectangle $ABGC$; and therefore the circumscribing square $FGHI$ is $= BC^2 + 2 \cdot AB \cdot AC$; and because $AB = CG = BF = AK$, AB is $= AK$; whence $AB + AC = AK + AC = KC = FG$; let, therefore, $AB = a$, $AC = b$, and $BC = h$; Then (Algebra) since $FG = a + b$, $FG^2 = a^2 + 2ab + b^2$; and since $FG^2 = BC^2 + 2 \cdot AB \cdot BC$, $FG^2 = h^2 + 2ab$; wherefore (Ax. 1) $h^2 + 2ab = a^2 + 2ab + b^2$; therefore (Ax. 5) $h^2 = a^2 + b^2$.

Coroll. Whence $a^2 = h^2 - b^2$, or $b^2 = h^2 - a^2$.

THEOREM XXXVI.

In any obtuse angled triangle, the square of the side subtending the obtuse angle is equal to the sum of the squares of the other two sides plus twice the rectangle of the base and its prolongation between the obtuse angle and a perpendicular.

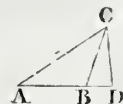
Let ABC be a Δ having the obtuse $\angle ABC$.

Produce AB to D , and draw CD perpendicular to AD , $AC^2 = AB^2 + BC^2 + 2AB \cdot BD$.

Let $AC = a$, $AB = b$, $BC = c$, $BD = d$, and $CD = p$; then will $AD = AB + BD = b + d$.

$$\text{Then (Th. xxxv)} \begin{cases} a^2 = p^2 + (b + d)^2 \\ d^2 + p^2 = c^2 \end{cases}$$

Adding these two equations together, and squaring $b + d$, we have $a^2 + d^2 = b^2 + 2bd + d^2 + c^2$; then, by rejecting d^2 common to both sides, we have $a^2 = b^2 + c^2 + 2bd$.



THEOREM XXXVII.

The square of the side subtending one of the acute angles of a triangle is equal to the sum of the squares of the other two sides, minus twice the rectangle under either of the sides, and that part of the straight line of that side between the perpendicular upon it and the point of the acute angle.

Let ABC be a triangle, and B the acute angle, and let CD be perpendicular to AB ; $AC^2 = AB^2 + BC^2 - 2(AB \cdot BD)$.

For let $AC = a$, $AB = b$, $BC = c$, $BD = d$, $CD = p$.

Then, in the case where the perpendicular falls within the base, $AD = AB - BD = b - d$.

Fig. 1.

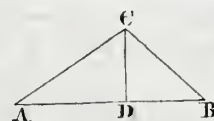
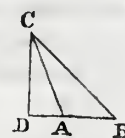


Fig. 2.



$$\text{By (Th. xxxv)} \begin{cases} a^2 = p^2 + (b-d)^2 \\ p^2 + d^2 = c^2 \end{cases}$$

Therefore, adding these two equations together, squaring $b-d$, and taking away the common term p^2 , we have

$$a^2 + d^2 = c^2 + b^2 - 2bd + d^2;$$

and again, taking away d^2 common to both sides of the equation, we have

$$a^2 = b^2 + c^2 - 2bd.$$

In the other case, where d falls without the side AB , we shall have $AD = DB - AB = d - b$.

$$\text{By (Th. xxxv)} \begin{cases} a^2 = p^2 + (d-b)^2 \\ d^2 + p^2 = c^2 \end{cases}$$

adding these equations together, squaring $d-b$, and taking away the common term p^2 , we have

$$a^2 + d^2 = c^2 + d^2 - 2bd + b^2;$$

and again, taking away d^2 common, we have

$$a^2 = b^2 + c^2 - 2bd.$$

THEOREM XXXVIII.

The difference of the squares of the two sides of a triangle is equal to the difference of the squares of the parts of the straight line of the base between each extremity of the base and a perpendicular from the opposite angle.

Let ABC be a triangle, and let CD be drawn perpendicular to AB ; then $AC^2 - BC^2 = AD^2 - DB^2$.

Fig. 1.

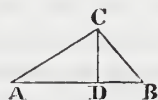
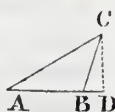


Fig. 2.



$$\text{Then (Cor. Th. xxxv)} \begin{cases} AC^2 - AD^2 = CD^2 \\ CD^2 = BC^2 - BD^2 \end{cases}$$

Adding these two equations, $AC^2 - AD^2 = BC^2 - BD^2$ and by transposition $AC^2 - BC^2 = AD^2 - BD^2$ or, in symbols, let $AC = a$, $BC = b$, $AD = u$, $BD = v$, and $CD = p$.

$$\text{Then (Cor. Th. xxxv)} \begin{cases} a^2 - u^2 = p^2 \\ p^2 = b^2 - v^2 \end{cases}$$

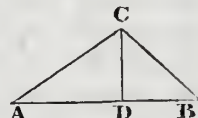
adding these equations, $a^2 - u^2 = b^2 - v^2$

and by transposition, $a^2 - b^2 = u^2 - v^2$

THEOREM XXXIX.

The rectangle under the sum and difference of the sides of a triangle is equal to the rectangle under the sum and difference of two lines in the same straight line with the base contained between a perpendicular and each extremity of the base.

That is $(AC + BC)(AC - BC) = (AD + BD)(AD - BD)$. For the difference of the squares of any two quantities is equal to the rectangle under their sum and difference; and since (Th. xxxviii) we have $AC^2 - BC^2 = AD^2 - BD^2$, we shall therefore have $(AC + BC)(AC - BC) = (AD + BD)(AD - BD)$.



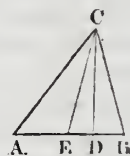
THEOREM XL.

The sum of the squares of any two sides of a triangle is equal to twice the sum of the squares of half the remaining side and of the line drawn from the middle of that side to the opposite angle.

In the triangle ABC , if the side AB be bisected in E , then $AC^2 + BC^2 = 2(AE^2 + CE^2)$.

For draw CD perpendicular to AB , and let $AE = EB = a$, $EC = b$, $AC = c$, $BC = d$, $ED = e$, and $CD = p$; then will $DB = EB - ED = a - e$, $AD = AE + ED = a + e$. Then (Th. xxxvi) $c^2 = a^2 + b^2 + 2ae$ and (Th. xxxvii) $d^2 = a^2 + b^2 - 2ae$

whence $c^2 + d^2 = 2(a^2 + b^2)$.

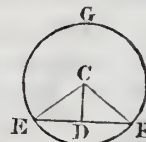


THEOREM XLI.

A line drawn perpendicular to the chord of a circle will bisect that chord.

Let EFG be a circle, C its centre, and EF a chord; then, if CD is perpendicular to EF , EF is bisected in the point D .

For let $CE = CF = r$, $ED = a$, $DF = b$, and let CD which is common $= p$.



Then (Th. xxxv) by the right $\angle \triangle \begin{cases} CDE, a^2 + p^2 = r^2 \\ CDF, b^2 + p^2 = r^2 \end{cases}$

Therefore, eliminating p and r , we have $a^2 = b^2$, and consequently $a = b$; wherefore EF is bisected in the point D .

THEOREM XLII.

If two secants or chords intersect each other, the rectangle of the two distances on the one, from the point of concurrence to each point where it meets the circumference, is equal to the rectangle of the two corresponding distances on the other.

Let ABHCDI (Fig. 1), or ABHIDC (Fig. 2), be a circle, and let the two lines AFC, HFI (Fig. 1), or the two lines ACF, HIF (Fig. 2) cut one another in the point F; then $AF \times FC = HF \times FI$ in each case, whether the point F is within or without the circle.

Draw EG perpendicular to AF, and join EA; then BF is the sum of the two sides AE, EF of the triangle AEF, and FD is the difference; likewise AF is the sum of the two segments AG, GF of the base, and CF is their difference;

Therefore (Th. xxxix) - - $AF \times FC = BF \times FD$
and by the same argument $HF \times FI = BF \times FD$
therefore (Ax. 1) - - - - $AF \times FC = HF \times FI$.

Coroll. 1. Hence, if one of the lines become a tangent, the rectangle on the secant, from the intersection to each point where it meets the circumference, is equal to the square of the tangent.

Coroll. 2. If both lines cut one another at right angles within the circle, and one of the lines be a diameter, in this case the other will be bisected; therefore if a chord cut a diameter at right angles, the rectangle of the two parts of that diameter is equal to the square of the half chord.

THEOREM XLIII.

The ratio of any two triangles of equal or of the same altitude is equal to that of their bases.

Let ABH and ABF be two triangles of which the bases are in the same straight line, and of which the altitude AP is the same; $\frac{\Delta ABH}{\Delta ABF} = \frac{BH}{BF}$.

For suppose the bases BH, BF to be



divided, each into equal parts, so that each of the equal parts of BH may be equal to each of the equal parts of BF; then, drawing lines from each point of section to the vertex A, the triangles ABC, ACD, &c. will be all equal. Suppose BH to contain m of the equal parts, and BF to contain n equal parts; then the ΔABH will contain m times the ΔABC , and the ΔABF will contain n times the ΔABC .

$$\begin{aligned} \text{Therefore} \quad & \frac{BH}{BF} = \frac{m}{n} \\ \text{and} \quad & \frac{\Delta ABH}{\Delta ABF} = \frac{m}{n} \\ \text{wherefore by equality} \quad & \frac{\Delta ABH}{\Delta ABF} = \frac{BH}{BF} \end{aligned}$$

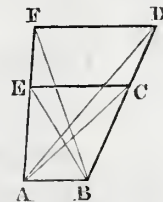
that is, the ratio of the triangles is equal to that of the corresponding bases.

THEOREM XLIV.

Triangles upon the same or equal bases, have a ratio equal to that of their altitudes.

Let ABC, ABD be two Δ s, and let the straight line AEF be perpendicular to the base AB, and let CE and DF be drawn parallel to AB; then the ratio of the two Δ s ABD, ABC is equal to that of their altitudes AF, AE.

For join BF and BE,



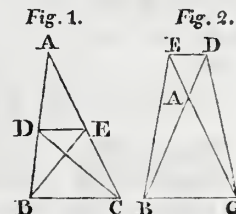
$$\begin{aligned} \text{then (Th. xliii)} \quad & \frac{\text{the } \Delta ABF}{\text{the } \Delta ABE} = \frac{AF}{AE} \\ \text{but the } \Delta ABF &= \Delta DBF, \text{ and the } \Delta ABE = \Delta CBE; \\ \text{therefore, by substitution,} \quad & \frac{\text{the } \Delta ABD}{\text{the } \Delta ABC} = \frac{AF}{AE} \end{aligned}$$

THEOREM XLV.

If a straight line be drawn parallel to one of the sides of a triangle to cut the other two sides or the other two sides produced, the ratio of the two segments of the one side will be equal to that of the other two.

Let ABC be a Δ , and let DE be drawn parallel to BC, the ratio of AD and DB is equal to that of AE and EC.

For join BE and CD.



$$\text{Then (Th. xliii)} \begin{cases} \frac{\text{the } \triangle ADE}{\text{the } \triangle BDE} = \frac{AD}{DB} \\ \frac{\text{the } \triangle ADE}{\text{the } \triangle CDE} = \frac{AE}{EC} \end{cases}$$

and since the $\triangle BDE$ is equal to the $\triangle CDE$ (Th. xxxi),

$$\text{therefore (Ax. 1)} \quad \frac{AD}{DB} = \frac{AE}{EC}$$

THEOREM XLVI.

If the sides of a triangle or the sides produced be cut in equal ratios, the straight line joining the points of section will be parallel to the remaining side.

Let ABC be a triangle; then if the ratio of AB and AD in the same straight line be equal to the ratio of AC and AE in the straight line of the other side, and if DE be joined, DE is parallel to BC .

Fig. 1.



Fig. 2.



$$\text{For (Th. xliii)} \begin{cases} \frac{DB}{AD} = \frac{\text{the } \triangle BDE}{\text{the } \triangle ADE} \\ \frac{\text{the } \triangle CDE}{\text{the } \triangle ADE} = \frac{EC}{AE} \end{cases}$$

$$\text{and (Th. xlv)} \quad \frac{EC}{AE} = \frac{DB}{AD}$$

Multiplying these equations and dividing out the common factors, we find $\triangle CDE = \triangle BDE$

But (Th. xxxii) equal triangles upon the same base and on the same side of it are between the same parallels; therefore DE is parallel to BC .

THEOREM XLVII.

The ratios of the sides about the equal angles of equiangular triangles are equal.

Fig. 1.

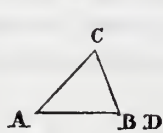
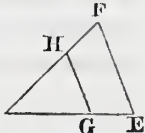


Fig. 2.



Let ABC, DEF be two triangles, having the $\angle A = D$, $B = E$, and $C = F$.

$$\text{Then } \frac{AC}{AB} = \frac{DF}{DE}, \text{ or } \frac{AC}{CB} = \frac{DF}{FE}$$

For make $DG = AB$, and $DH = AC$.

Then (Th. i) the $\triangle DGH = ABC$, and the corresponding $\angle$ s opposite to the equal sides are equal; therefore, because

in the $\triangle$ s ABC, DEF , the $\angle B = E$, and since the $\angle DGH = B$; therefore (Ax. 1) the $\angle DGH = E$; wherefore (Th. xiv) GH is parallel to EF .

$$\text{Therefore (Th. xlvii)} \quad \frac{DH}{HF} = \frac{DG}{GE}$$

$$\text{consequently } \frac{DH}{DH+HF} = \frac{DG}{DG+GE}$$

But $AC = DH$, $AB = DG$, $DH + HF = DF$, and $DG + GE = DE$;

$$\text{therefore } \frac{AC}{DF} = \frac{AB}{DE}$$

$$\text{consequently } \frac{AC}{AB} = \frac{DF}{DE}$$

THEOREM XLVIII.

If the ratio of the two sides of a triangle be equal to the ratio of two sides of another, and the angles contained by these sides equal, the two triangles shall be equiangular.

Let the ratio of the sides AB, BC of the $\triangle ABC$ be equal to the ratio of the sides DE, EF of the $\triangle DEF$, and the $\angle B = E$; then will the $\angle C = F$, and the $\angle A = D$.

Fig. 1.

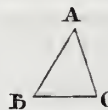
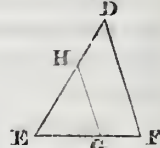


Fig. 2.



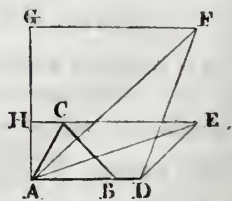
For in EF make $EG = BC$, and in ED make $EH = BA$, and join GH ; then (Th. i) the $\angle EGH = C$, and the $\angle EHG = A$: but, by hypothesis, $\frac{AB}{BC} = \frac{DE}{EF}$; and since $EG = BC$, and $EH = BA$, $\frac{EH}{EG} = \frac{DE}{EF}$ (Ax. 1); therefore (Th. xlvii) GH is parallel to FD ; wherefore (Th. xliii) the $\angle EGH = F$, and the $\angle EHG = D$: but the $\angle EGH = C$, and the $\angle EHG = A$; therefore the $\angle A = D$, and the $\angle C = F$; consequently the $\triangle$ s ABC and DEF are equiangular.

THEOREM XLIX.

Triangles have a ratio equal to that of the rectangle of their bases and altitudes.

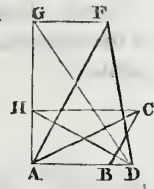
Let ABC, ADF be two $\triangle$ s having their bases AB, AD in the same straight line. Let AG be drawn perpendicular to AD , and FG, CH parallel to BA , cutting AG in H ; then

$$\frac{\text{the } \triangle ADF}{\text{the } \triangle ABC} = \frac{AD \times AG}{AB \times AH}$$



Join DH and DG; then (Th. xxxi) the $\triangle$ AFD = AGD;

and since (Th. xlv) $\left\{ \begin{array}{l} \text{the } \triangle \text{ AHD} \\ \text{the } \triangle \text{ ACB} \end{array} \right. = \frac{AD}{AB}$
 $\left\{ \begin{array}{l} \text{the } \triangle \text{ AGD} \\ \text{the } \triangle \text{ AHD} \end{array} \right. = \frac{AG}{AH}$
 therefore, $\dots \dots \frac{\text{the } \triangle \text{ AGD}}{\text{the } \triangle \text{ ACB}} = \frac{AD \times AG}{AB \times AH}$



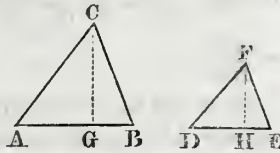
THEOREM L.

The ratio of similar triangles is equal to that of the squares of their homologous sides.

Let ABC, DEF be similar $\triangle$ s, having the $\angle A = D$, the $\angle B = E$, and the $\angle C = F$;

$$\text{then } \frac{\text{the } \triangle \text{ ABC}}{\text{the } \triangle \text{ DEF}} = \frac{AB^2}{DE^2}$$

For draw CG perpendicular to AB, cutting AB in G, and draw FH perpendicular to DE, cutting DE in H; then, since the $\angle A = D$, and the $\angle$ s CGA, FHD are right $\angle$ s, therefore the $\angle ACG = DFH$; consequently the $\triangle$ s ACG, DFH are equiangular;



$$\text{therefore (Th. xlvii) } \dots \dots \left\{ \begin{array}{l} \frac{AC}{DF} = \frac{CG}{FH} \\ \frac{AB}{DE} = \frac{AC}{DF} \end{array} \right.$$

$$\text{and since (Th. xlix) } \frac{AB \times CG}{DE \times FH} = \frac{\text{the } \triangle \text{ ABC}}{\text{the } \triangle \text{ DEF}}$$

$$\text{therefore, by multiplication, } \frac{AB^2}{DE^2} = \frac{\text{the } \triangle \text{ ABC}}{\text{the } \triangle \text{ DEF}}$$

THEOREM LI.

The ratio of similar polygons is equal to that of the squares of their homologous sides.

Let ABCDE, FGHIK be similar polygons; then

$$\frac{\text{the figure ABCDE}}{\text{the figure FGHIK}} = \frac{AB^2}{FG^2}$$

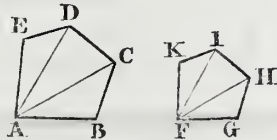
for since (Th. l)

$$\frac{AB^2}{FG^2} = \frac{\triangle \text{ ABC}}{\triangle \text{ FGH}} = \frac{AC^2}{FH^2} = \frac{\triangle \text{ ACD}}{\triangle \text{ FHI}} = \frac{AD^2}{FI^2} = \frac{\triangle \text{ ADE}}{\triangle \text{ FIK}}$$

therefore, by equality,

$$\frac{AB^2}{FG^2} = \frac{\triangle \text{ ABC}}{\triangle \text{ FGH}} = \frac{\triangle \text{ ACD}}{\triangle \text{ FHI}} = \frac{\triangle \text{ ADE}}{\triangle \text{ FIK}}; \text{ therefore,}$$

$$\frac{AB^2}{FG^2} = \frac{\triangle \text{ ABC} + \triangle \text{ ACD} + \triangle \text{ ADE}}{\triangle \text{ FGH} + \triangle \text{ FHI} + \triangle \text{ FIK}} = \frac{\text{polygon ABCDE}}{\text{polygon FGHIK}}$$

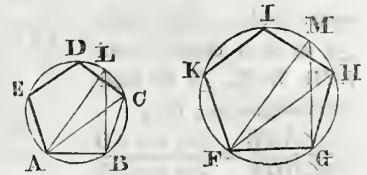


THEOREM LII.

The ratio of similar polygons inscribed in circles is equal to that of the squares of their diameters.

Let ABCDE, FGHIK

be similar polygons inscribed in the circles ABCDE, FGHIK, and let AL, FM be the diameters of the circles; then



$$\frac{\text{the polygon ABCDE}}{\text{the polygon FGHIK}} = \frac{AL^2}{FM^2}$$

For join AC, BL, FH, GM; since the $\angle ACB = ALB$, and the $\angle FHG = FMG$ (Th. xxvi); therefore the $\angle ALB = FMG$. Again, because ABCL, FGHM are semicircles, the $\angle$ s ABL, FGM are right angles (Th. xxiv); therefore the triangles ABL, FGM are equiangular; wherefore

$$\frac{AB}{FG} = \frac{AL}{FM}$$

$$\text{whence (Algebra) } \dots \dots \frac{AB^2}{FG^2} = \frac{AL^2}{FM^2}$$

$$\text{and since (Th. li) } \dots \dots \frac{\text{the polygon ABCDE}}{\text{the polygon FGHIK}} = \frac{AB^2}{FG^2}$$

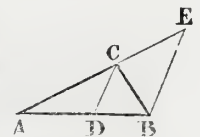
$$\text{therefore, by multiplication, } \frac{\text{the polygon ABCDE}}{\text{the polygon FGHIK}} = \frac{AL^2}{FM^2}$$

THEOREM LIII.

If any angle of a triangle be bisected, the ratio of the segments of the opposite side cut by the bisecting line is equal to that of the other two sides of the triangle.

Let ABC be a $\triangle$; and if the $\angle ACB$ be bisected by the straight line CD cutting the opposite side AB in D, then

$$\frac{AD}{DB} = \frac{AC}{CB}$$



For produce AC to E, and draw BE parallel to CD; and since BE is parallel to CD, and BC joins them, the alternate $\angle$ s DCB and CBE are equal (Th. xi), and the $\angle$ BEC = DCA (Th. xiii); but the $\angle$ DCA = DCB; therefore the $\angle$ CEB = CBE; wherefore (Th. iv) CB = CE;

$$\text{and since CD is parallel to BE, } \frac{AC}{CE} = \frac{AD}{DB}$$

$$\text{therefore, since CB = CE, } \dots \dots \frac{AC}{CB} = \frac{AD}{DB}$$

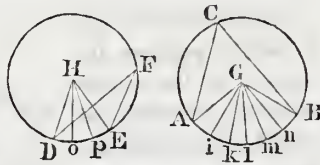
THEOREM LIV.

The angles in equal circles, whether at the centres or circumferences, have a ratio equal to that of the arcs on which they stand.

Let ABC, DEF, be equal circles, AGB, DHE $\angle$ s at their centres, and ACB, DFE, $\angle$ s at their circumferences; then

$$\frac{\text{the } \angle \text{AGB}}{\text{the } \angle \text{DHE}} = \frac{\text{the arc AB}}{\text{the arc DE}}$$

$$\text{and } \frac{\text{the } \angle \text{ACB}}{\text{the } \angle \text{DFE}} = \frac{\text{the arc AB}}{\text{the arc DE}}.$$



For let the arc AB be divided into m equal parts, Ai, ik, kl, &c., and the arc DE into n equal parts, Do, op, &c., so that each of the equal parts in the arc AB may be equal to each of the equal parts in the arc DE: draw the straight lines Gi, Gk, Gl, &c., as also Ho, Hp, &c.; then the $\angle$ AGB will contain the $\angle$ AGi as often as the arc AB contains Ai, that is m times, and the $\angle$ DHE will contain the $\angle$ DHo as often as the arc DE contains Do, that is, n times: therefore the arc DE will contain the same parts of the arc AB that the $\angle$ DHE contains of the $\angle$ AGB.

$$\text{Now, since } \frac{\text{the arc AB}}{\text{the arc DE}} = \frac{m}{n},$$

$$\text{and since } \frac{\text{the } \angle \text{AGB}}{\text{the } \angle \text{DHE}} = \frac{m}{n};$$

$$\text{therefore, by equality, } \frac{\text{the } \angle \text{AGB}}{\text{the } \angle \text{DHE}} = \frac{\text{the arc AB}}{\text{the arc DE}};$$

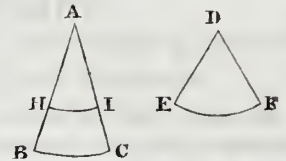
$$\text{therefore also (Th. xxv) } \frac{\text{the } \angle \text{ACB}}{\text{the } \angle \text{DFH}} = \frac{\text{the arc AB}}{\text{the arc DE}}.$$

Coroll. Hence angles are measured by the arcs of circles of the same radii described between their legs from the angular points.

THEOREM LV.

The ratio of the angles contained by any two equal arcs is equal to the reciprocal ratio of their radii.

Let BAC, EDF be any two $\angle$ s, and let the arc BC described from the centre A be equal to the arc EF described from the centre D, $\frac{\text{the } \angle \text{D}}{\text{the } \angle \text{A}}$



$$= \frac{\text{AB}}{\text{DE}}.$$

For in AB take AH = DE, and from the centre A with the distance AH describe an arc cutting AC at I.

Let AB = R, AH = DE = r , the arc BC = EF = A, and the arc HI = a .

$$\text{Then, by similar sectors, ABC, AHI, } \frac{A}{a} = \frac{R}{r},$$

$$\text{and (Th. liv) } \frac{A}{a} = \frac{\angle \text{D}}{\angle \text{A}};$$

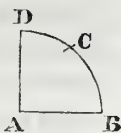
$$\text{therefore, by equality, } \frac{\angle \text{D}}{\angle \text{A}} = \frac{R}{r} = \frac{\text{AB}}{\text{DE}}.$$

Elements of Plane Trigonometry.

DEFINITIONS.

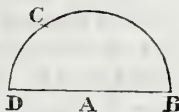
Def. 1. Complement of an arc is another arc which is the difference between that arc and a quadrant.

Let BC be an arc, and BCD a quadrant; then CD is the complement of that arc BC .

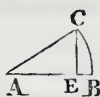


Def. 2. Supplement of an arc is another arc which is the difference between the first arc and a semicircle.

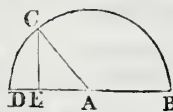
Let BC be an arc, and BCD a semicircle; then CD is the supplement of the arc BC .



Def. 3. Sine of an arc is a straight line drawn from one extremity of the arc upon the diameter passing through the other extremity.

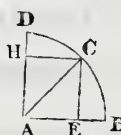


Let BC be the arc of a circle described from the point A ; then CE , drawn perpendicularly upon the radius AB or the diameter DB , is the sine of the arc BC or CD .

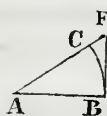


Def. 4. Cosine of an arc is the sine of the complement of that arc.

Let BC be an arc, and BCD a quadrant; then CH or AE is the cosine of the arc BC .



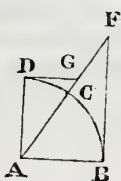
Def. 5. Tangent of an arc is a straight line intercepted between one of its extremities where it touches the arc, and a straight line drawn from the centre through the other extremity of that arc.



Let BC be an arc; then BF , intercepted between the point of contact B and the line AF , is the tangent of the arc BC .

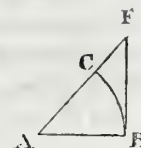
Def. 6. Cotangent of an arc is the tangent of the complement of that arc.

Let BC be an arc, and BCD a quadrant; then DG , the tangent of the complementary arc CD , is the cotangent of the arc BC .



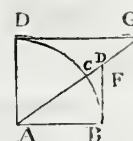
Def. 7. Secant of an arc is a line drawn from the centre through one extremity of an arc to meet a tangent from the other extremity.

AF is the secant of the arc BC .



Def. 8. Cosecant of an arc is the secant of the complement of that arc.

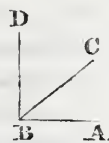
Let BC be an arc, and BCD a quadrant; then the cosecant of the arc BC is the line AG .



The sine, cosine, tangent, cotangent, secant, cosecant of an angle, is the sine, cosine, tangent, cotangent, secant, cosecant of an arc described from the angular point, and terminated by the legs of that angle.

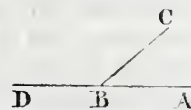
Def. 9. Complement of an angle is an angle which is equal to the difference between that angle and a right angle.

Let ABC be an angle, and ABD a right angle; then CBD is the complement of the angle ABC .



Def. 10. Supplement of an angle is an angle equal to the difference between that angle and right angles.

Let AD be a straight line, and BC another meeting it at the point B ; then the angle CBD is the supplement of the angle ABC .



CONTRACTIONS.

Sin. for sine, cos. for cosine, tan. for tangent, cot. for cotangent, sec. for secant, cosec. for cosecant, and rad. for radius.

THEOREM I.

The sine and cosine of an arc is the same as the sine and cosine of its supplement.

Let ACD be a semicircle, and AC an arc less than a semicircle; then (by *Def. 3*) CE is the sine not only of the

Fig. 1.

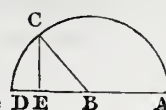
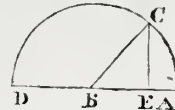


Fig. 2.

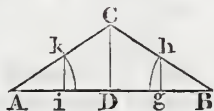


are AC, but of the arc CD, and BE is the cosine; but with regard to *Fig. 1*, the cosine of the arc AC being greater than a quadrant is negative, while in *Fig. 2* it is affirmative, being less than a quadrant.

THEOREM II.

The ratio of the sides of a triangle is equal to that of the sines of their opposite angles.

Draw CD perpendicular to AB, cutting AB in D. On AC and BC with any distance cut off Ak, Bh, equal to each other, and draw ki and hg parallel to CD, cutting AB in i and g.



Let $BC = a$, $AC = b$, $CD = p$, $Ak = Bh = r$.

$$\text{Then, by sim. } \triangle s \left\{ \begin{array}{l} \text{Aik, ADC, } \frac{\sin. A}{r} = \frac{p}{b} \\ \text{Bgh, BDC, } \frac{\sin. B}{r} = \frac{p}{a} \end{array} \right.$$

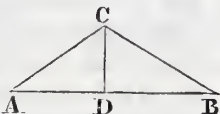
$$\text{therefore, eliminating } r, p, \quad \frac{\sin. A}{\sin. B} = \frac{a}{b}$$

The elimination is made by dividing the higher equation by the lower.

THEOREM III.

The cosine of any angle is equal to the sum of the squares of the sides containing that angle, minus the square of the remaining side multiplied into the reciprocal of twice the product of the containing sides, radius being considered unity.

For draw CD perpendicular to AB, cutting AB in D, and let the sides be denoted by the corresponding small letters to the capitals at the opposite angles, and let AD = the distance of the perpendicular CD from the $\angle A$ be called d :



Then (Geom. Th. xxxvii) we have $a^2 = b^2 + c^2 - 2cd$ and therefore $d = \frac{b^2 + c^2 - a^2}{2c}$

but since $\frac{b}{d} = \frac{1}{\cos. A}$, the $(\cos. A) b = d$;

therefore, mult. the two eqs. } $\cos. A = \frac{b^2 + c^2 - a^2}{2bc}$
and dividing by b , we find

THEOREM IV.

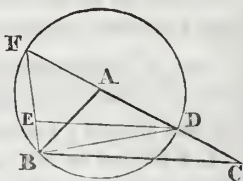
The ratio of the sum and difference of the sides of a triangle is equal to the ratio of the tangents of the half sum and half difference of the angles at the base.

Let ABC be a triangle.

From A, with the distance AB, describe the circle FBD cutting AC in D, or AC produced in D. Produce CA to F; join DB and FB: draw DE parallel to BC, cutting FB in E, or FB produced in E; then the $\angle ABD = ADB$ is the half sum of the $\angle s$ ABC, ACB, and the $\angle DBC = BDE$ half their difference; and since in the $\triangle s$ ABC, ABD, the $\angle DAB + ABC + ACB = ADB + ABD + DAB$; therefore (Ax. 5) $ABC + ACB = ADB + ABD = 2ADB$ therefore $\frac{1}{2}(ABC + ACB) = ADB = ADE + BDE$; therefore $\frac{1}{2}(ABC + ACB) = ACB + BDE$; whence (Ax. 5) $\frac{1}{2}(ABC - ACB) = BDE$.

Now, in the diagram, FC is the sum of the two sides AB, AC, and CD is their difference; also BF is the tangent of the $\angle ADB$ or BDF, and BE is the tangent of the $\angle BDE$;

$$\text{therefore, by parallels, } \frac{FC}{CD} = \frac{BF}{BE} = \frac{\tan. BDF}{\tan. BDE}$$



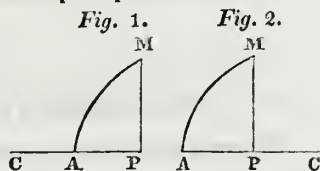
Besides the characters already employed, we shall introduce $\perp$ for *perpendicular to*, and $\parallel$ for *parallel to*; also, in the subsequent part of the work, the letters expressing the sides and angles of similar triangles will be written exactly in the order of the lines of the figure, as has been strictly observed in the preceding part. By this means, we may take the product of the extremes and means at once, or their ratios; thus, in the triangles ABC, DEF we may have $AB \cdot EF = BC \cdot DE$, or $\frac{AB}{BC} = \frac{DE}{EF}$, or $\frac{AB}{AC} = \frac{DE}{DF}$, or $\frac{AB}{DE} = \frac{BC}{EF}$, &c. according as the one or the other may be the more convenient.

Curve Lines.

DEFINITIONS.

Def. 1. A *CURVE* is a line, such that, if any two points be taken in it, a straight line passing through these two points will not coincide with the intercepted portion of that line.

Def. 2. If PM be considered to move parallel to itself at the same angle with the line AC or AC produced, and if we have an equation expressed in terms of the constant quantity AC, and the two variable quantities CP, PM, the point M will trace out a line AM, called the *locus of the equation*.



Def. 3. The variable line CP is called the *abscissa*, and the other variable line PM the *ordinate*; also the fixed point C is called the *origin of the abscissa*, and the other fixed point A the *vertex of the curve*.

Def. 4. The straight line on which CP is in, is called the *line of the abscissa*.

Def. 5. A *tangential line* is a straight line which touches a curve, and which cannot cut it, though produced from either of its extremities.

Def. 6. The point where a tangential line touches a curve is called the *point of contact*.

Def. 7. The portion of a tangential line, intercepted between the point of contact at the extremity of an ordinate and the line of the abscissa, is called the *tangent of the curve*.

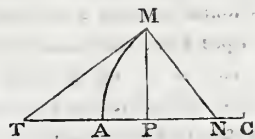
Def. 8. That portion of the line of the abscissa intercepted between the ordinate and a tangent to the curve at the other extremity of that ordinate, is called the *subtangent*.

Def. 9. A line drawn from the extremity of an ordinate perpendicular to a tangent at the same point to meet the line of the abscissa, is called the *normal*.

Def. 10. The intercepted portion of the line of the abscissa between the ordinate and the normal is called the *subnormal*.

NOTATION.

In the figure here annexed, AC is the abscissa, MT the tangent, PT the subtangent, MN the normal, and PN the subnormal; also AT is the complement of the subtangent.



In the Algebraic Notation, let $CA = a$, $CP = x$, $PM = y$, $MT = t$, $PT = s$.

CURVES OF THE FIRST ORDER.

DEFINITIONS.

Def. 1. A *Curve of the first Order* is the locus of a point of which the sum or difference of its distances from two fixed points is a given straight line.

Def. 2. Each of the two fixed points is called a *focus*.

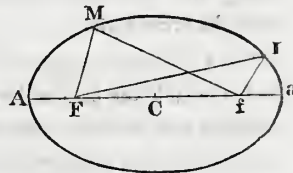
Def. 3. Each of the two straight lines is called a *radius vector*.

Def. 4. A straight line passing through each focus, if terminated at both ends by the curve or opposite branches of it, is called the *axis major* or *transverse axis*.

Def. 5. The middle point of the straight line between the two foci is called the *centre of the figure*.

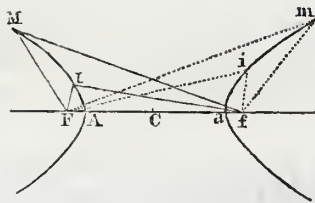
Def. 6. The curve described by the sum of the two radius vectors, when it is equal to a given line, is called an *ellipse*.

Thus, if FM, fM be any two radius vectors, and if FI, fI be any other two radius vectors; then if $FM + fM = FI + fI$, the curve is an ellipse, the straight line Aa is the axis major, and the point C in the middle of Ff is the centre.



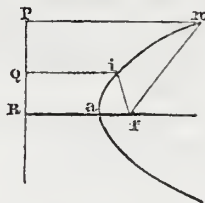
Def. 7. The curve described by the difference of the two radius vectors, when it is equal to a given line, is called an *hyperbola*.

Thus if FM , fM be any two radius vectors, and FI , fi any other two; then if $fM - FM = fi - FI$, the curve AIM is an hyperbola, and the equal and similar curve aim is called the opposite hyperbola.



Coroll. Hence, if one of the focal points be infinitely distant, any radius vector drawn from that point will be parallel to the axis major or line passing through the focal points; whence we shall have another species of curve between the ellipse and hyperbola, called a *parabola*.

Let R be any point in the axis; draw $RP \perp Rf$, and if Qi and Pm be any two lines parallel to Rf , and if $Qi - if = Pm - mf$, the curve aim is a parabola.



Hence if $Pm = mf$, Qi must be $= if$, as also $Ra = af$.

CURVES OF THE FIRST ORDER.

OF THE ELLIPSE.

From the definition given of the ellipse, it is evident that the curve may be united so as to form one continued line, which must be understood in the following

DEFINITIONS (continued).

Def. 8. A straight line passing through the centre at right angles to the axis major, and terminated at each end by the curve, is called the *axis minor*.

Def. 9. A straight line passing through either focus parallel to the axis minor, and terminated by the curve, is called the *latus rectum* or *parameter*.

Def. 10. The distance from the centre to either focus is called the *excentricity*.

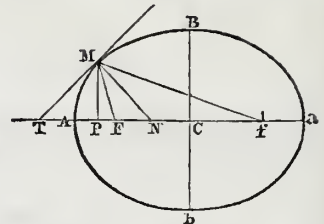
Def. 11. The straight line drawn through the centre perpendicularly to the axis major, and terminated by the curve, is called the *axis minor* or *conjugate axis*.

Def. 12. Any straight line drawn through the centre, and terminated at each end by the curve, is called a *diameter*.

Def. 13. A diameter which is parallel to a tangent at one extremity of another diameter, is called the *conjugate diameter* of that other diameter.

NOTATION OF THE FIGURE.

In the figure, Aa is the axis major or transverse axis, Bb the axis minor or conjugate axis, C the centre, CP the abscissa commencing at the centre, PM the ordinate, F, f the two foci, CF or Cf the excentricity, FM, fM the radius vectors, MT the tangent, PT the subtangent, MN the normal, and PN the subnormal.



In the Algebraic Notation, besides what has already been defined of curves in general, let $CF = Cf = e$.

PROPOSITION I, THEOREM.

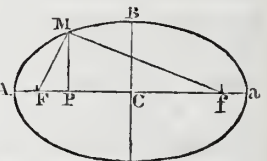
The sum of the two radius vectors is equal to the axis major.

For (Def. 6)

$$\begin{cases} FM + fM = Ff + 2AF \\ FM + fM = Ff + 2af \end{cases}$$

therefore - - - $AF = af$

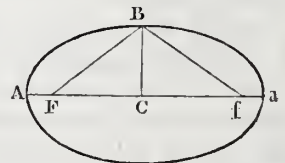
wherefore $FM + fM = Ff + AF + af = Aa = 2 \cdot CA$.



PROPOSITION II, THEOREM.

The distance of either focus from one extremity of the axis minor is equal to the semi-axis major.

For the $\triangle BCF$ is equal and similar to the $\triangle BCF$; and consequently $BF = Bf$, and since (Prop. i) $FB + Bf = 2 \cdot AC$ therefore - - - $2 \cdot FB = 2 \cdot AC$ wherefore - - - $FB = AC$.

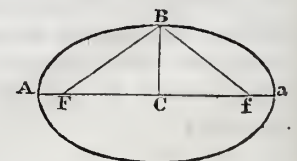


PROPOSITION III, THEOREM.

The difference of the squares of the semi-axis major and of the excentricity is equal to the square of the semi-axis minor.

Here $FB = AC$ (Prop. i); therefore $FB = a$ (see Notation, p. 1 & 2). Since $BC^2 = FB^2 - FC^2$ (Ge. Th. 35, Cor.), $b^2 = a^2 - e^2$.

Coroll. Hence $e^2 = a^2 - b^2$.

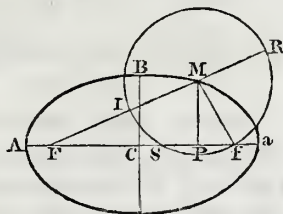


PROPOSITION IV, THEOREM.

Half the difference of the two radius vectors is equal to the product of the excentricity and the abscissa divided by the semi-axis major.

$$\frac{1}{2}(MF - Mf) = \frac{ex}{a}$$

From M with the radius MF describe the circle ISfR, cutting FM at I, and Aa at S, and produce FM to R:



Then - - - SF = 2Cf + 2Pf = 2CP = 2x,

and since - Aa = FM + Mf = FM + MR = FR;

therefore - Aa = FR = 2a.

Now let FI = v, and since - - - - - Ff = 2e,

then (Ge. Th. 42) FR·FI = FfFS - - - - - 2av = 4ex;

therefore, dividing by 4a, - - - - - $\frac{v}{2} = \frac{ex}{a}$

Coroll. FM = a + $\frac{ex}{a}$, and fM = a - $\frac{ex}{a}$.

PROPOSITION V, THEOREM.

The rectangle of the square of the semi-axis major and the square of an ordinate is equal to the rectangle of the square of the semi-axis minor and of the difference of the squares of the semi-axis major and the abscissa.

That is,
CA²·PM² = BC²(CA² - CP²)
Here FP = e + x

and (Pr. 4, Cor.) - FM = a + $\frac{ex}{a}$

and (Ge. Th. 35, Cor.) y² = (a + $\frac{ex}{a}$)² - (e + x)²

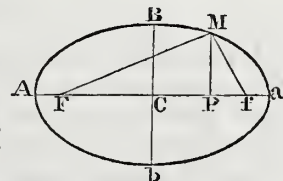
and (Pr. 3, Cor.) - - e² = a² - b²:

whence, eliminating e, and by reduction, we obtain

$$a^2y^2 = b^2(a^2 - x^2)$$

which is the equation of the co-ordinates of the axis.

By actual involution of the two terms on the second side of the second conditional equation, and by reduction, we obtain a²y² = a⁴ + e²(x² - a²) - a²x²; then, by substituting a² - b² for e², we have a²y² = a⁴ + (a² - b²)(x² - a²) - a²x², which, by multiplying the factors of the second term, and by reduction, becomes a²y² = b²(a² - x²).

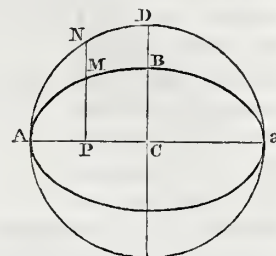


PROPOSITION VI, THEOREM.

The ratio of two ordinates to either axis is equal to the ratio of the two ordinates in the same lines of a semicircle described upon that axis.

$$\text{That is, } \frac{CB}{PM} = \frac{CD}{PN}$$

For upon Aa, as a diameter, describe the semicircle ANDa, and produce the ordinate CB, PM of the ellipse to meet the circle in D and M.



Let PN = γ.

By the co-ordinates CA, CB, PM of the ellipse a²y² = b²(a² - x²) and by the co-ordinates CA, CD, PN of the circle γ² = a² - x²;

therefore, eliminating x, - - - - - $\frac{a^2y^2}{\gamma^2} = b^2$;

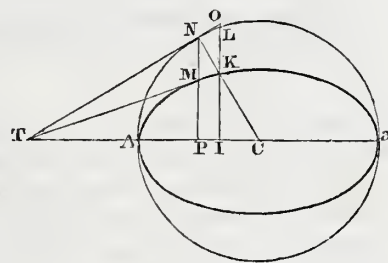
wherefore, dividing by y² and extracting, - - - $\frac{a}{\gamma} = \frac{b}{y}$

The elimination of x is found by 'dividing the first equation by the second.

PROPOSITION VII, PROBLEM.

To draw a tangent through any point in the prolongation of the axis.

On the axis major Aa describe the circle ANOa. Through M draw PN ⊥ Aa, meeting the axis in P and the circle in N; join CN, and draw NT ⊥ CN, and join TM; then TM is a tangent to the curve at M.



For if TM is not a tangent at M, it will cut the curve in some other point K. Through K draw IK || PM, and prolong IK to meet the arc of the circle in L, and the tangent TN in O; then IO will be greater than IL.

By similar triangles, { TPM, TIK - - - TP·IK = PM·TI
TPN, TIO - - - PN·TI TP·IO
and by corresponding ordinates (Pr. 6) - - IL·PM = PN·IK
whence, eliminating PN, PM, PT, IK, IT, we have IL = IO,
which is impossible, since IO is greater than IL.

The elimination is found by simply multiplying the three conditional equations in the order in which they stand, rejecting the common factors.

PROPOSITION VIII, PROBLEM.

The curve, the axis major, and any diameter of an ellipse, being given, to find a conjugate to that given diameter.

The construction of the figure (Pr. 7) remaining, draw $CE \perp NC$, cutting the circle in E , and $ER \perp Aa$, cutting the ellipse in D , and Aa in R , and draw the diameter Dd ; then Mm and Dd are conjugate diameters.

Because NT , CE are each $\perp NC$, the radius CE is $\parallel NT$; and since RE is $\parallel PN$, the Δ s TPN and CRE are similar;

Therefore, by similar Δ s TPN , CRE , $TP \cdot RE = PN \cdot CR$ and since (Pr. 6) $\frac{PM}{RD} = \frac{PN}{RE}$ $PM \cdot RE = PN \cdot RD$

wherefore, eliminating PN , RE $\frac{TP}{PM} = \frac{CR}{RD}$, consequently the two Δ s TPM and CRD are similar, and therefore the diameter Dd is parallel to MT ; wherefore the diameter Dd is conjugate to Mm .

PROPOSITION IX, THEOREM.

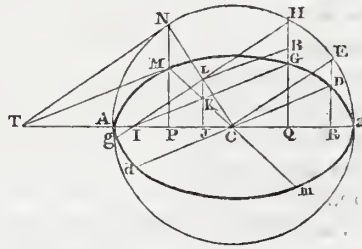
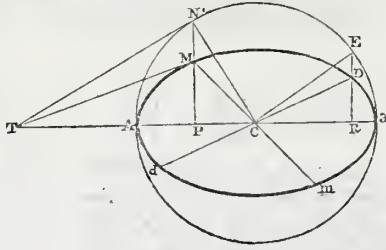
The rectangle of the squares of any semidiameter, and of an ordinate to it, is equal to the rectangle of the square of the semiconjugate, and of the difference of the squares of the semidiameter of the abscissa and the abscissa itself.

The construction of the figure (Pr. 8) remaining, let KG be an ordinate to the diameter Mm at K , and prolong GK till it meet Aa in I and the curve in g , and through the point G draw $QG \perp Aa$, and prolong QG to meet the arc of the circle in H . Draw $HI \perp NC$, cutting NC in L , and Aa in I , and through K draw $JL \perp Aa$, meeting CN and IH in L . Lastly, draw $LB \parallel IG$, cutting QH in B ; then shall $CM^2 \cdot KG^2 = CD^2(CM^2 - CK^2)$.

For let $CM = a$, $CD = b$, $CK = x$, $LB = KG = y$, $CN = r$, $CL = u$, and $LH = v$;

Then by similar Δ s $\begin{cases} HLB, ECD, & \dots r^2 y^2 = b^2 v^2 \\ MCN, KCL, & \dots a^2 u^2 = r^2 x^2 \end{cases}$

and by the co-ordinates of the circle $v^2 = r^2 - u^2$; whence, by eliminating r , u , v , we obtain $a^2 y^2 = b^2(a^2 - x^2)$



In the third equation, for u^2 substitute its equal $\frac{r^2 x^2}{a^2}$ from the second, and we have $v^2 = \frac{r^2}{a^2}(a^2 - x^2)$. In the first equation, for v^2 substitute its equal $\frac{r^2}{a^2}(a^2 - x^2)$ now found, and we have $y^2 = \frac{b^2}{a^2}(a^2 - x^2)$, or, multiplying by a^2 , $a^2 y^2 = b^2(a^2 - x^2)$.

Coroll. 1. Hence, also, $b^2 x^2 = a^2(b^2 - y^2)$, where b is the semidiameter of the abscissa, y the abscissa itself, x the ordinate, and a the semiconjugate diameter.

For $a^2 y^2 = a^2 b^2 - b^2 x^2$; therefore, by transposition, $b^2 x^2 = a^2 b^2 - a^2 y^2$. That is, $b^2 x^2 = a^2(b^2 - y^2)$, and this is the same as substituting a for b , b for a , x for y , and y for x in the original equation $a^2 y^2 = b^2(a^2 - x^2)$.

Since the relation is the same taken upon either semidiameter, the equation, which is the locus of the figure, is properly called the equation of the co-ordinates.

Coroll. 2. Hence either diameter is conjugate to the other, since the same curve is produced between every two semiconjugate diameters, whether the abscissa is taken on the one or the other.

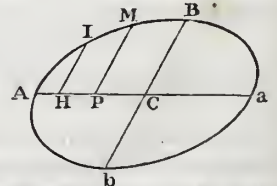
Coroll. 3. Hence, in any equation expressed in terms of a , b , x , y , if the constants a , b , are exchanged for each other, and the variables x , y , for each other, the relations will be the same.

NOTATION.

We have just seen, that any two conjugate diameters have the same property as the two axes. We are now to understand in the sequel, that, in all conjugate diameters, the *abscissal* diameter is marked Aa , and its semiconjugate by CB or Cb , the abscissa by CP , the ordinate by PM ; and if any other ordinate be necessary on the same diameter, the abscissa of that ordinate is indicated by CH , and the ordinate itself by HI .

In the algebraic data and demonstrations, $CA = Ca$ is denoted by a , CB by b , CP by x , PM by y , as has been adopted in respect of the two axes.

Moreover, we shall denote the abscissa CH by z , and its corresponding ordinate HI by z , so that the equation of the co-ordinates as refers to PM is $a^2 y^2 = b^2(a^2 - x^2)$, and the equation of the co-ordinates as refers to HI is $a^2 z^2 = b^2(a^2 - z^2)$.



PROPOSITION X, THEOREM.

The ratio of the two rectangles contained each by the two segments of a diameter cut by an ordinate, is equal to that of the squares of two ordinates themselves.

That is, $\frac{AP \cdot Pa}{AH \cdot Ha} = \frac{PM^2}{HI^2}$ (see Fig. in Notation)

For (Pr. 9) $----- a^2 y^2 = b^2 (a^2 - x^2)$

Again (Pr. 9) $----- a^2 y^2 = b^2 (a^2 - z^2)$

wherefore, eliminating b , $----- \frac{y^2}{z^2} = \frac{a^2 - x^2}{a^2 - z^2}$

consequently $----- \frac{y^2}{z^2} = \frac{(a+x)(a-x)}{(a+z)(a-z)}$

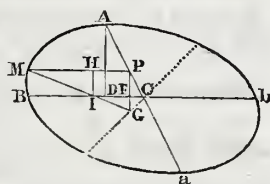
For dividing the first by the second of the given equations, we obtain $\frac{y^2}{z^2} = \frac{a^2 - x^2}{a^2 - z^2}$; and by resolving each term of the fraction on the second side of this equation into its factors, we obtain $\frac{y^2}{z^2} = \frac{(a+x)(a-x)}{(a+z)(a-z)}$.

Coroll. Hence $y^2(a^2 - z^2) = z^2(a^2 - x^2)$ is the equation of the co-ordinates CA, PM, HI, or $y^2(a+z)(a-z) = z^2(a+x)(a-x)$.

PROPOSITION XI, THEOREM.

Given any diameter and an ordinate, to describe the curve of the ellipse by continued motion.

Draw AD $\perp$ Bb, cutting Bb in D. From M, with the radius AD, describe an arc cutting Bb in I; join MI, and prolong MI to G: draw PG $\perp$ PM, and join CG, which produce both ways as far as necessary.



Then, in moving the straight line GIM so that the point I may always be in the line Bb, and the point G in the line CG, or its prolongation, the point M will trace out the curve.

For draw IH $\parallel$ PG, cutting PM in H, and let AD = MI = c, HI = PF = u, and GP = v, and we may observe, that since MG = BC, MG as well as BC will be denoted by b:

Then, because of the right $\angle$ d $\triangle$ MPG, $----- y^2 = b^2 - v^2$

By similar triangles, $----- \begin{cases} DAC, FPC, & - c^2 x^2 = a^2 u^2 \\ MGP, MIH, & - b^2 u^2 = c^2 v^2 \end{cases}$

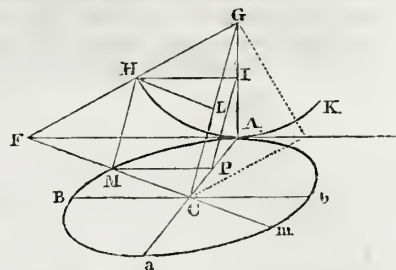
wherefore, eliminating c, u, v, we have $----- a^2 y^2 = b^2 (a^2 - x^2)$

For multiplying the second and third equations together, we have $a^2 v^2 = b^2 x^2$; then, substituting $\frac{b^2 x^2}{a^2}$ from this equation in the first for v^2 , and we have $a^2 y^2 = b^2 (a^2 - x^2)$.

PROPOSITION XII, PROBLEM.

Given any two conjugate diameters and the position of any other diameter, to find the extremities of that diameter.

Let Fm be any line given in position to the two conjugate diameters Aa, Bb; it is required to find the portion of Fm which is a diameter of the ellipse.



Draw FA $\parallel$ Bb, and AG $\perp$ FA. Make AG = BC; from G, with the radius GA, describe the arc HAK: draw FG, cutting the arc in H, and join GC, and draw HM $\parallel$ GC, M is a point in the curve.

For let CF = m, CM = LH = n, GF = p, GH = q, and GI = v; and because AG = BC, AG as well as BC will = b.

By similar triangles $----- \begin{cases} ACF, PCM & - a^2 n^2 = m^2 x^2 \\ CFG, LHG & - m^2 q^2 = p^2 n^2 \\ FGA, HGI & - p^2 v^2 = b^2 q^2 \end{cases}$

and, by the co-ordinates of the circle, $----- b^2 - v^2 = y^2$; therefore, eliminating m, n, p, q, v, we have $----- a^2 y^2 = b^2 (a^2 - x^2)$

For multiplying the three first equations together, we have $a^2 v^2 = b^2 x^2$; then, substituting $\frac{b^2 x^2}{a^2}$ from this equation for v^2 in the fourth, and we have $a^2 y^2 = b^2 (a^2 - x^2)$.

PROPOSITION XIII, PROBLEM.

Any diameter and an ordinate being given, to find any point in the curve.

Draw FI $\parallel$ Aa, and AF $\parallel$ HI: divide HI, FI each into two such parts in g and h, that the ratio of HI, Hg may be equal to the ratio of FI, Fh: join hA, and through g draw aM, cutting hA in M, the point M is in the curve.

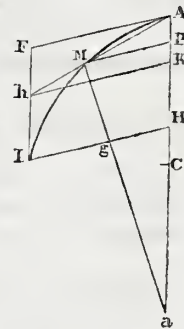
For draw hK $\parallel$ FA, cutting AH in K; and let HI = Kh = r, Hg = m, Fh = AK = n, FI = AH = g, Ha = h, AP = v, and Pa = w.

By similar triangles $----- \begin{cases} APM, AKh, & v r = n y \\ aPM, aHg, & m w = h y \end{cases}$

and since, by construction, $\frac{r}{m} = \frac{g}{n} ----- n r = m g$

whence, eliminating m, n, $----- y^2 v w = y^2 g h$, which result is agreeable to the equation of the co-ordinates (Prop. 10, Cor.).

This result is simply obtained by multiplying the three given equations.



PROPOSITION XIV, THEOREM.

If through any given point two straight lines be drawn to cut the curve in two points each, the ratio of the rectangles of the segments of each line, from the given point to each point in the curve, is equal to the ratio of the squares of the semidiameters parallel thereto.

Fig. 1.

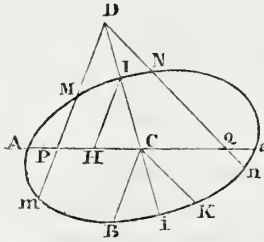
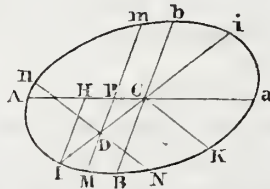


Fig. 2.



Let Dm, Dn be two straight lines drawn from D, so that Dm may meet the curve in M, m and Dn in N, n; then will $\frac{DM \cdot Dm}{DN \cdot Dn} = \frac{CB^2}{CK^2}$.

For let us first suppose that one of the lines Di passes through the centre, and that it meets the curve in I, i; also let CI = Ci = d, PD = v, and CD = w.

Then, by similar triangles, $\begin{cases} HCI, PCD, d^2x^2 = w^2z^2 \\ CHI, CPD, v^2z^2 = x^2y^2 \end{cases}$
and (Pr. 9) ----- $a^2y^2 = b^2(a^2 - x^2)$
again (Pr. 9) ----- $a^2y^2 = b^2(a^2 - z^2)$

therefore, eliminating* a, x, y, z, we have $\frac{b^2}{y^2 - v^2} = \frac{d^2}{w^2}$

that is, ----- $\frac{BC^2}{mD \cdot DM} = \frac{CI^2}{iD \cdot DI}$

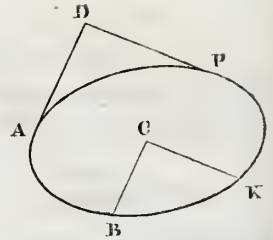
therefore also, if CK be parallel to Dn - $\frac{CK^2}{nD \cdot DN} = \frac{CI^2}{iD \cdot DI}$

and consequently $\frac{BC^2}{mD \cdot DM} = \frac{CK^2}{nD \cdot DN}$, or $\frac{mD \cdot DM}{nD \cdot DN} = \frac{BC^2}{CK^2}$

Coroll. 1. Hence the ratio of two tangents meeting each other is equal to that of the semidiameters parallel thereto.

* Substitute $\frac{d^2x^2}{w^2}$ found from the first equation, and $\frac{d^2v^2}{w^2}$ found from the product of the first and second equations respectively, for z^2 , y^2 in the fourth equation, and we have $\frac{d^2v^2}{w^2} = b^2 - \frac{b^2d^2x^2}{a^2w^2}$ from this equation, and also from the third equation find the value of $\frac{x^2}{a^2}$, and we at last obtain $\frac{b^2}{y^2 - v^2} = \frac{d^2}{w^2}$, which is equivalent to $\frac{BC^2}{mD \cdot DM} = \frac{CK^2}{nD \cdot DN}$, and consequently $\frac{mD \cdot DI}{nD \cdot DN} = \frac{BC^2}{CK^2}$.

For suppose Dm to touch the curve at A, and Dn at P, the rectangle DM·Dm will become DA², and the rectangle DN·Dn will become DP²; therefore the equation $\frac{DM \cdot Dm}{DN \cdot Dn} = \frac{CB^2}{CK^2}$ will become $\frac{DA^2}{DP^2} = \frac{CB^2}{CK^2}$; wherefore $\frac{DA}{DP} = \frac{CB}{CK}$, or DA·CK = DP·CB.



PROPOSITION XV, THEOREM.

The rectangle under that part of the line of a diameter, from the centre to the intersection of a tangent at the extremity of an ordinate and the abscissa, is equal to the square of the semidiameter of that abscissa.

That is, CT·CP = CA².

Produce the tangent TM to H, and draw AK, aH tangents at A, a, meeting TH at K as well as H: draw the ordinate PM, and parallel to Aa draw KL, MN, cutting PM in L, and aH in N: draw the semidiameter CD parallel to KH, and the semidiameter CE parallel to AK or aH.

Let CD = f, CE = g, aH = m, AK = u, HM = v, MK = w, and CT = u; then will aP = MN = a + x, KL = AP = a - x, Ta = u + a, and AT = u - a.

Now, since (Cor. Pr. 14) $\begin{cases} MK \cdot CE = AK \cdot CD, gw = nf \\ aH \cdot CD = MH \cdot CE, mf = vg \end{cases}$
and by similar Δ s - $\begin{cases} TAK, TaH, n(u + a) = m(n - a) \\ HMN, MKL, v(a - x) = w(a + x) \end{cases}$
therefore, eliminating m, n, v, w, we have - $ux = a^2$.

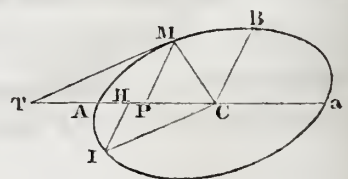
Multiply the four equations together, and we have $(u + a)(a - x) = (u - a)(a + x)$; and by actual multiplication, and rejecting the common terms, we have $ux = a^2$.

PROPOSITION XVI, THEOREM.

If any semidiameter is situate between two semiconjugate diameters, and if ordinates be drawn from each extremity of these semiconjugate diameters to the intermediate semidiameter, the rectangle of the intermediate semidiameter and one of its ordinates is equal to the rectangle of the semiconjugate to the intermediate semidiameter and the abscissa of the other ordinate.

Let AC be a semidiameter between the two semiconjugate diameters IC, MC, then will CA·PM = CB·CH.

For draw the semicon-



jugate diameter CB to CA, and prolong CA to T, and draw MT a tangent at M. We have here $PT = u - x$.

Then (Prop. 15) ----- $ux = a^2$
and by similar triangles, MPT, IHC, $\gamma^2(u - x)^2 = z^2y^2$,
and (Pr. 10, Cor.) ----- $y^2(a^2 - z^2) = \gamma^2(a^2 - x^2)$
again (Pr. 9) ----- $a^2y^2 = b^2(a^2 - x^2)$
wherefore, eliminating u, x, γ , we have ----- $ay = bz$.

In the second equation, for u substitute $\frac{a^2}{x}$ from the first, and we have $\gamma^2(a^2 - x^2)^2 = x^2y^2z^2$; multiply this and the third equation, and we have $(a^2 - z^2)(a^2 - x^2) = x^2z^2$, which by actual multiplication and reduction becomes $a^2 - x^2 = z^2$; and this being multiplied by the fourth equation, becomes $ay = bz$.

Coroll. 1. Hence $ay = bz$.

Coroll. 2. Hence $z^2 = a^2 - x^2$, $x^2 = a^2 - z^2$, $y^2 = b^2 - z^2$, and $\gamma^2 = b^2 - y^2$.

PROPOSITION XVII, THEOREM.

The rectangle under the two contiguous parts of a tangent, limited on each side of the point of contact by the prolongation of two conjugate diameters, is equal to the square of the semidiameter parallel to that tangent.

Let CI, CM be two semiconjugate diameters, meeting the tangent in T and N, which passes through A, the extremity of the semidiameter CA, then $AT \cdot AN = CB^2$.

For let $AT = t$, $AN = u$.

By similar triangles $\begin{cases} \text{CPM, CAN,} & \text{-----} & ux = ay \\ \text{CHI, CAT,} & \text{-----} & tz = ay \end{cases}$
and (Prop. 16) ----- $ay = bz$
and (Pr. 16, Cor. 1) ----- $ay = bx$
therefore, eliminating a, x, z, y, γ , ----- $tu = b^2$

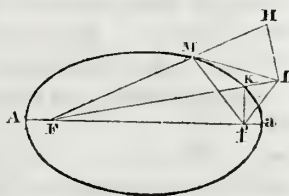
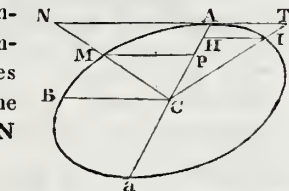
The result is obtained by multiplying the four conditional equations as they stand, and expunging the common quantities.

PROPOSITION XVIII, PROBLEM.

If the angle contained by one of the radius vectors and the prolongation of the other be bisected, the bisecting line will be a tangent to the curve.

Let FM be produced to H, and let the $\angle fMH$ be bisected; the line MI which bisects the $\angle fMH$ is a tangent to the curve at M.

For make $MH = Mf$, and from any point I in MI draw IH and If, and join FI and Kf, and let FI meet the curve in K.



Then, because $MH = Mf$ and MI common, and the $\angle HMI = fMI$, therefore $IH = If$; and because $FI + If = FI + IH$, $FI + If < FH$, and consequently greater than $FM + Mf$. Hence the point I is without the curve; and since every point in MI except M is without the curve, MI is therefore a tangent to the curve at M.

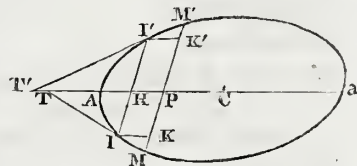
PROPOSITION XIX, THEOREM.

If two straight lines be tangents at the extremities of a chord ordinately applied to a diameter, they will meet each other in the prolongation of that diameter.

Let the chord M, M' cut the diameter Aa in P . Draw $II' \parallel MM'$, and join $MI, M'I'$. Prolong aA to T' , and let $MI, M'I'$ prolonged meet aT' in T and T' . Draw $IK, I'K'$, each parallel to Aa , cutting MM' in K, K' ; the tangents $MT, M'T'$ cannot meet the prolongation of the diameter aA in two separate points T, T' .

For let $IK = I'K' = u$, $KM = K'M' = v$, $PT = s$, and $PT' = s$.

Then, by similar triangles $\begin{cases} MKI, MPT & \text{-----} & sv = uy \\ M'K'I', M'P'T' & \text{-----} & Sv = uy \end{cases}$
wherefore, eliminating u, v, y , we have ----- $S = s$;
therefore $PT = PT'$; consequently the points T, T' coincide, whatever may be the distance of MM', II' .



PROPOSITION XX, THEOREM.

The rectangle under the semi-axis major and half the parameter is equal to the square of the semi-axis minor.

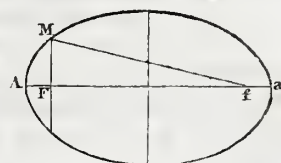
Because $FM + fM = 2a$, therefore, when $FM \perp Aa$, FM will be $= y$; therefore $y + fM = 2a$, and by transposition $fM = 2a - y$; and since $Ff = 2e$,

(Ge. Th. 35, Cor.) - - $y^2 = (2a - y)^2 - (2e)^2$,

and since (Pr. 3) - - - $b^2 = a^2 - e^2$;

then, eliminating e , - $ay = b^2$.

Therefore y is here a constant quantity. If we now put $y = \frac{1}{2}p$, we shall have $\frac{1}{2}ap = b^2$.



PROPOSITION XXI, THEOREM.

The rectangle under the semi-axis major and the square of an ordinate to it, is equal to the rectangle under half the parameter and the difference of the squares of the semi-axis major and of the abscissa.

For (Pr. 9) - - - - - $a^2y^2 = b^2(a^2 - x^2)$,

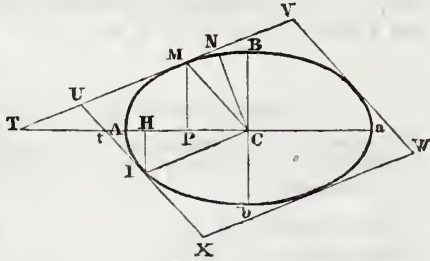
and (Pr. 20) - - - - - $b^2 = \frac{1}{2}ap$;

therefore, eliminating b , - $ay^2 = \frac{1}{2}p(a^2 - x^2)$

PROPOSITION XXII, THEOREM.

Every parallelogram of which the sides are parallel to two conjugate diameters, and are tangents to the curve, is equal to the rectangle of the two axes.

Let CI and CM be two semi-conjugate diameters, and let Aa be the axis major and Bb the axis minor, and let the two adjacent sides UV and UX of the parallel-



ogram UVWX touch the curve at M and I. Produce Aa to cut UX in t and VU produced in T. Draw the ordinates of the axes HI, PM; also draw CN perpendicular to UV, cutting UV in N.

Let Ct = t, TC = r, CN = p, and CI = n.

Since (Pr. 16, Cor.) $z^2 = a^2 - x^2$
and (Pr. 9) $b^2(a^2 - x^2) = a^2y^2$,
and (Pr. 15) $a^2 = tz$;

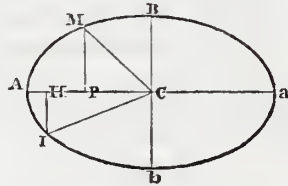
and, by similar Δ s $\begin{cases} tCI, CTM & ty = ry \\ TCN, CIH & ry = pn \end{cases}$
therefore, eliminating r, t, x, y, z, $ab = pn$.

Multiply the two first equations, and we have $bz = ay$; multiply this and the three remaining equations, and we have $ab = pn$.

PROPOSITION XXIII, THEOREM.

The sum of the squares of any two conjugate diameters is equal to the sum of the squares of the two axes.

Let CM, CH be two semi-conjugate diameters on the same side of the axis minor Bb; then $CM^2 + CH^2 = CA^2 + CB^2$.



For draw the ordinates MP and HI to the axis major Aa, and let MC = m, and CH = n.

By (Pr. 16, Cor. 2) $\begin{cases} a^2 = x^2 + z^2 \\ b^2 = y^2 + \gamma^2 \end{cases}$

and since (Ge. Th. 35) $\begin{cases} CM^2 = CP^2 + PM^2, m^2 = x^2 + y^2 \\ CI^2 = CH^2 + HI^2, n^2 = z^2 + \gamma^2 \end{cases}$
therefore, eliminating x, y, z, $m^2 + n^2 = a^2 + b^2$

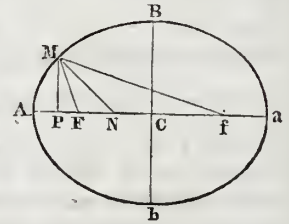
For adding the two first equations together, we obtain $a^2 + b^2 = x^2 + z^2 + y^2 + \gamma^2$; and adding the two last together, we have $m^2 + n^2 = x^2 + z^2 + y^2 + \gamma^2$; therefore $m^2 + n^2 = a^2 + b^2$.

PROPOSITION XXIV, THEOREM.

The rectangle under the axis major, and that part of it between the intersection of the normal and the nearest focus, is equal to the rectangle under the least radius vector and the distance between the focii.

Let FN = u, fM = v, and FM = w; then will fN = 2e - u.

By (Pr. 4, Cor.) $\begin{cases} a + \frac{ex}{a} = v \\ a - \frac{ex}{a} = w \end{cases}$



and by (Ge. Th. 53) $\frac{v}{w} = \frac{2e - u}{u}$

whence, eliminating v, $w, u = e - \frac{e^2x}{a^2}$.

The elimination is obtained by dividing the first equation by the second; and multiplying the result by the third, and reducing, we find $u = e - \frac{e^2x}{a^2}$.

PROPOSITION XXV, THEOREM.

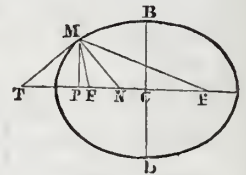
The rectangle under the square of the semi-axis major and the sub-normal is equal to the rectangle under the square of the semi-axis minor and the abscissa.

Now, PF = x - e. Let PN = w; consequently $w = u + x - e$,

and by (Pr. 24) $u = e - \frac{e^2x}{a^2}$

also by (Pr. 3, Cor.) $e^2 = a^2 - b^2$;

therefore, eliminating e, u, v, $a^2w = b^2x$.



The elimination is obtained by substituting $a^2 - b^2$ from the third for e^2 in the second, and we obtain $u = e - x + \frac{b^2x}{a^2}$, or by transposition, $u + x - e = \frac{b^2x}{a^2}$, and consequently $w = \frac{b^2x}{a^2}$; therefore $a^2w = b^2x$.

PROPOSITION XXVI, THEOREM.

The rectangle under the abscissa and the subtangent is equal to the difference of the squares of the semi-axis major and of the abscissa.

For PT being = s (see Notation, p. 1), and PN = w, and since TMN is a right angled Δ , and PM $\perp$ TN (see Fig. Pr. 25), the Δ s TPM and MPN are similar.

Then, by similar Δ s TPM, MPN, $sw = y^2$
and (Pr. 9) $a^2y^2 = b^2(a^2 - x^2)$
and by (Pr. 25) $b^2x = a^2w$;
wherefore, eliminating b, w, and y, $sx = a^2 - x^2$.

The resulting equation is found by simply multiplying the three given equations as they stand, and rejecting the common factors.

CURVES OF THE FIRST ORDER.

OF THE HYPERBOLA.

DEFINITIONS (continued from pages 1 and 2).

Def. 8. A line drawn through the centre perpendicularly to the transverse axis, and limited at each extremity by the circumference of a circle described from the vertex with a radius equal to the distance of the focus from the centre of the figure, is called the *conjugate axis*.

Def. 9. Any straight line drawn through the centre and terminated by the opposite curves, is called a *diameter*.

Def. 10. A diameter which is parallel to a tangent at the extremity of another diameter, is called the *conjugate* of that diameter.

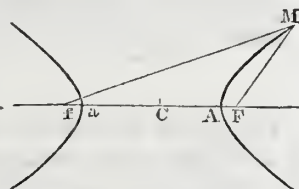
Def. 11. The distance of the centre from either focus, is called the *excentricity*.

Def. 12. A straight line drawn through the centre, continually approaching nearer to the curve than any assigned distance, but never meeting it, is called an *asymptote*.

PROPOSITION I, THEOREM.

The difference of the two radius vectors in the hyperbola is equal to the transverse axis.

Let Aa be the transverse axis, M any point in the curve, F the nearer and f the remoter focus.

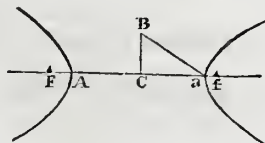


For (Def. 7, p. 1) $\begin{cases} fM - FM = fA - AF = fa + aA - AF \\ fM - FM = Fa - af = FA + Aa - af \end{cases}$
therefore $fa + aA - AF = FA + Aa - af$;
consequently $2af = 2AF$,
and $af = AF$;
therefore $fM - FM = fA - AF = fa - af = aA$

PROPOSITION II, THEOREM.

The square of the excentricity is equal to the sum of the squares of the two semi-axes.

Let CB be the semi-conjugate axis; then, since $aB = Cf$ (Def. 8, p. 9) the excentricity, and since $aB^2 = aC^2 + CB^2$ (Ge. Th. 35), $e^2 = a^2 + b^2$.

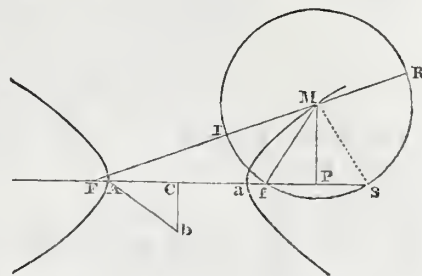


PROPOSITION III, THEOREM.

Half the sum of the two radius vectors is equal to the product of the excentricity and the abscissa divided by the semi-axis major.

$$\frac{1}{2}(FM + fM) = \frac{CF \times CP}{CA}$$

From M, with the radius Mf, describe the circle IfSR, cutting FM at I, and Aa produced at S, and produce FM to R. Let FR = v.



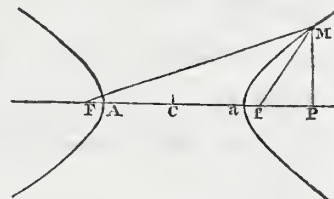
Now $SF = 2CF + 2Pf = 2CP = 2x$; and since $FI = 2a$ (Pr. 1); and since $Ff = 2e$; and since $FR \cdot FI = Ff \cdot FS$ (Ge. Th. 42) $2av = 4e$ and therefore, dividing $4a \dots \dots \dots \frac{v}{2} = \frac{ex}{a}$

Coroll. $FM = \frac{ex}{a} + a$, and $fM = \frac{ex}{a} - a = \frac{ex - a^2}{a}$

PROPOSITION IV, THEOREM.

The rectangle of the square of the semi-axis major and the square of an ordinate is equal to the rectangle of the square of the semi-axis minor, and of the difference of the squares of the abscissa and the semi-axis major.

Here $FP = CF + CP = e + x$.
Let $FM = v$, and $e + x = w$; therefore $FP = w = e + x$.



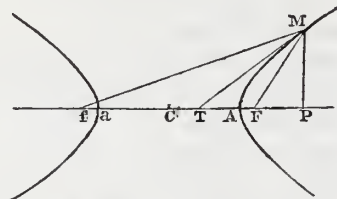
Then, since $PM^2 = FM^2 - FP^2$ (Ge. Th. 35, Cor.) $y^2 = v^2 - w^2$ and since $FM = \frac{ex}{a} + a$ (Pr. 3, Cor.) $v^2 = (\frac{ex}{a} + a)^2$ and since $w = e + x \dots \dots \dots w^2 = (e + x)^2$ also, since (Pr. 2) $e^2 = a^2 + b^2$; whence, eliminating $e, v, w \dots \dots \dots a^2y^2 = b^2(x^2 - a^2)$

For, substituting the expansions of $(\frac{ex}{a} + a)^2$ and $(e + x)^2$ from the second and third equations for their equals v^2, w^2 in the first, we shall have $y^2 = \frac{e^2}{a^2}(x^2 - a^2) - x^2 + a^2$. In this equation substitute $a^2 + b^2$ from the fourth for e^2 , and we have $y^2 = \frac{a^2 + b^2}{a^2}(x^2 - a^2) - x^2 + a^2 = \frac{b^2}{a^2}(x^2 - a^2)$, or multiplying by a^2 , $a^2y^2 = b^2(x^2 - a^2)$.

Coroll. Whence $b^2 = \frac{b^2x^2}{a^2} - y^2$.

PROPOSITION V, THEOREM.

The square of the semi-axis major is equal to the rectangle of these two parts of the line of the axis, the one between the centre and the intersection of the tangent, and the other between the centre and the ordinate.



Let $fM = u$, $FM = v$,
 $fT = w$, $FT = z$; then
 will $w + z = 2e$.

And, since $\frac{fM}{FM} = \frac{fT}{FT}$ (Ge. Th. 53) ----- $uz = vw$,

and since (Pr. 3) ----- $\frac{2ex}{a} = u + v$,

and since (Pr. 3, Cor.) ----- $v = \frac{ex - a^2}{a}$

also, since ----- $w + z = 2e$;

whence, eliminating u , v , w , ----- $a^2 = x(z - e)$

For to each side of the first equation add zv , and it will become by ordering $z(u + v) = v(w + z)$. Multiply this and the second, third, and fourth equations together, and we obtain $xz = ex - a^2$; whence $a^2 = x(e - z)$.

Coroll. 1. Hence $z = e - \frac{a^2}{x}$.

Coroll. 2,

Hence $PT = FT + FP = (e - \frac{a^2}{x}) + (x - e) = \frac{x^2 - a^2}{x}$.

Coroll. 3,

Hence $AT = TP - AP = \frac{x^2 - a^2}{x} - (x - a) = \frac{a(x - a)}{x}$.

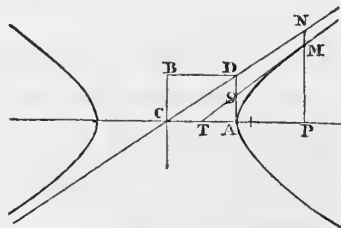
Coroll. 4,

Hence $CT = CA - AT = a - \frac{a(x - a)}{x} = \frac{a^2}{x}$.

PROPOSITION VI, PROBLEM.

Given the two axes in position and magnitude, to find the position of the asymptotes.

Draw AD perpendicular to AC , and BD parallel to AC ; and through the points C and D draw the straight line CN , CN will be an asymptote to the curve.



For let s = the sub-tangent PT (see Notation, p. 1), $AT = z$, and $AS = v$.

By similar triangles TPM , TAS , ----- $s^2v^2 = y^2z^2$,

and by co-ordinates (Pr. 4) ----- $a^2y^2 = b^2(x^2 - a^2)$;

and since $AT = z = \frac{a(x-a)}{x}$ (Pr. 5, Cor. 3) - $x^2z^2 = a^2(x-a)^2$;

and since $PT = s = \frac{x^2 - a^2}{x}$ (Pr. 5, Cor. 2), $(x^2 - a^2)^2 = s^2x^2$;

wherefore, eliminating s , y , z , we obtain ----- $v = b\sqrt{\frac{x-a}{x+a}}$

If we now suppose x infinite, the quantity $\frac{x-a}{x+a}$ will not differ from unity; we shall therefore have $v = b$, and because

$CT = \frac{a^2}{x}$ (Pr. 5, Cor. 4), therefore since x by hypothesis

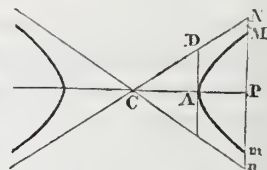
is infinitely great, $\frac{a^2}{x}$ will be infinitely small; wherefore, the

tangent at an infinite distance will pass through the centre C , and therefore the asymptote will pass through the points C and D .

The above elimination is simply obtained by multiplying the given equations, and dividing out the common factors.

PROPOSITION VII, THEOREM.

If an ordinate be produced to meet each asymptote, the rectangle of the two segments of the line between the asymptotes, as separated by either branch of the curve, is equal to the square of the semi-axis minor.



For produce the ordinate PM at both ends to meet the asymptotes in N and n , and the other branch of the curve in m .

By similar Δ s CAD , CPN , ----- $PN = \frac{bx}{a}$;

therefore $NM = PN - PM = \frac{bx}{a} - y$,

and ----- $nM = PN + PM = \frac{bx}{a} + y$;

therefore ----- $NM \times Mn = \frac{b^2x^2}{a^2} - y^2$;

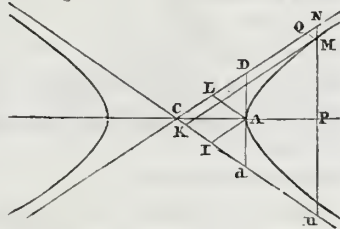
and since (Pr. 4, Cor.) ----- $b^2 = \frac{b^2x^2}{a^2} - y^2$;

wherefore ----- $NM \times Mn = b^2 = AD^2 = CB^2$.

PROPOSITION VIII, PROBLEM.

To find the equation of the co-ordinates as relates to the asymptotes.

Draw AL and MQ parallel to the asymptote Km, cutting CN in L and Q; also draw AI parallel to CN, cutting Cn in I; then,



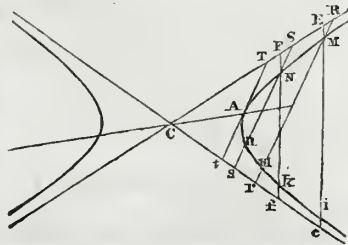
by similar Δ s $\begin{cases} \text{NMQ, DAL} & \text{--- NM} \times \text{AL} = \text{MQ} \times \text{AD} \\ \text{nMK, ADL} & \text{--- nM} \times \text{DL} = \text{MK} \times \text{AD} \end{cases}$
and by (Pr. 7) $\text{AD}^2 = \text{nM} \times \text{MN}$;
whence, eliminating AD, $\text{nM} \times \text{NM}, \text{AL} \times \text{DL} = \text{MK} \times \text{MQ}$.

But, because AL and DL are equal, the result is $\text{AL}^2 = \text{MK} \cdot \text{MQ}$; that is, by making $\text{AL} = a$, $\text{MK} = \text{CQ} = x$ the abscissa, and making $\text{QM} = y$ the ordinate, we shall have $a^2 = xy$, which is the equation of the co-ordinates as it relates to the asymptotes.

PROPOSITION IX, THEOREM.

If any two parallel lines be drawn so as to cut two connected branches and their asymptotes, the product of the two segments of each line, contained between the asymptotes and separated by either of the branches of the curve, are equal.

For let the two parallels Rr, Ss, terminated by the asymptotes in R, S, r, s, cut the hyperbola in the points M, N, m, n, and let the lines Ee, Ff be drawn through the points M, N, perpendicular to the axis, to cut the connecting branch of the curve in i, k, and the asymptotes in E, F, e, f, and we shall have,



by similar Δ s $\begin{cases} \text{RME, SNF} & \text{--- RM} \times \text{NF} = \text{ME} \times \text{SN} \\ \text{rMe, sNf} & \text{--- Mr} \times \text{Nf} = \text{Me} \times \text{sN} \end{cases}$

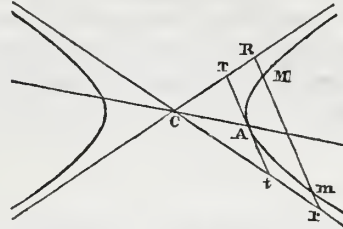
by (Pr. 7) $\begin{cases} \text{ME} \times \text{Me} = b^2 \\ b^2 = \text{FN} \times \text{Nf} \end{cases}$

therefore $\text{RM} \times \text{Mr} = \text{SN} \times \text{Ns}$

and therefore, also, $\text{rm} \times \text{mR} = \text{sn} \times \text{nS}$.

PROPOSITION X, THEOREM.

If a straight line be drawn between the asymptotes to cut two branches of the curve, the parts of the straight line without the curve are equal.



By (Pr. 9) $\begin{cases} \text{RM} \times \text{Mr} = \text{TA} \times \text{At} \\ \text{TA} \times \text{At} = \text{rm} \times \text{mR} \end{cases}$

and since $\begin{cases} \text{Mm} + \text{mr} = \text{Mr} \\ \text{Rm} = \text{RM} + \text{Mm} \end{cases}$

therefore, by multiplication, $\text{RM}(\text{Mm} + \text{mr}) = \text{rm}(\text{RM} + \text{Mm})$
and this, by actual multiplication, becomes

$$\text{RM} \cdot \text{Mm} + \text{RM} \cdot \text{mr} = \text{rm} \cdot \text{RM} + \text{rm} \cdot \text{Mm};$$

and throwing out the common quantities, we have

$$\text{RM} \cdot \text{Mm} = \text{rm} \cdot \text{Mm};$$

that is, by dividing by the common factor Mm,

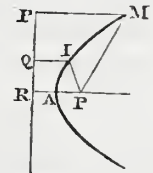
$$\text{RM} = \text{rm}.$$

CURVES OF THE FIRST ORDER.

OF THE PARABOLA.

DEFINITIONS (continued from Cor. Def. 7, p. 1).

Def. 8. The line RP which is at the same distance from any point of the curve, as the focus is from that point, is called the *directrix*.



Def. 9. A straight line, drawn through the focus perpendicular to the directrix, is called the *axis*.

Def. 10. The point of the curve cut by the perpendicular drawn to the directrix, is called the *vertex*.

Def. 11. A straight line drawn through the focus perpendicular to the axis and terminated by the curve, is called the *parameter*, or *latus rectum* of the axis.

Def. 12. Every straight line parallel to the axis, is called a *diameter*.

Def. 13. The point where a diameter meets the curve, is called the *vertex* of that diameter.

Def. 14. A straight line parallel to a tangent, and limited by a diameter passing through the point of contact and by the curve, is called an *ordinate* to that diameter.

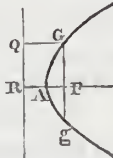
Def. 15. The part of the straight line of a diameter limited by its vertex and an ordinate, is called the *abscissa*.

Def. 16. A chord drawn from the focus parallel to a tangent at the extremity of any diameter, is called the *parameter* of that diameter.

PROPOSITION I, THEOREM.

The latus rectum of the axis is equal to four times the distance of the focus from the vertex.

Let RQ be the directrix, and Gg the latus rectum; and let QN be perpendicular to RQ: then GQ = GF; therefore QG = $\frac{1}{2}$ Gg; and consequently, because QG = RF, RF = $\frac{1}{2}$ Gg; wherefore AF = AR = $\frac{1}{4}$ Gg, or 4AF = Gg.

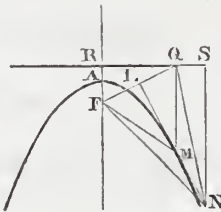


PROPOSITION II, THEOREM.

The line bisecting the angle made by the radius vector, and a line from that point of the curve where the radius vector meets it perpendicular to the directrix, is a tangent to the curve.

Let ML bisect the angle FNQ, and meet FQ in L. In LM take any point N, and draw NS perpendicular to the directrix.

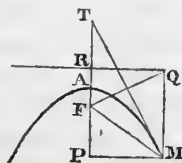
Then, since MF = MQ and $\angle FML = \angle QML$, FL = QL, and ML is perpendicular to FQ; hence NF = NQ; consequently NF is greater than NS and N not in the curve, which therefore lies wholly on one side of NL.



PROPOSITION III, THEOREM.

The subtangent of the axis is double of the abscissa.

Because the parallels TP and QM are joined by TM, the alternate angles PTM and QMT are equal: but because the $\angle FMQ$ is bisected, the $\angle QMT = \angle FMT$; therefore the $\angle FMT = \angle PTM$; and therefore FM = FT. But PR = AP + AR = AP + AF; and since FT = FM, and FM = PR, therefore FT = PR; consequently PF = TR; and since AF = AR, therefore PF + AF = TR + AR. But PF + AF = AP, and TR + AR = AT; whence AP = AT, and consequently AP + AT = twice AP.



PROPOSITION IV, THEOREM.

The rectangle under the parameter and the abscissa of the axis is equal to the square of the ordinate.

That is, $Gg \cdot AP = PM^2$.

Let AP = x, PM = y, AF = f,

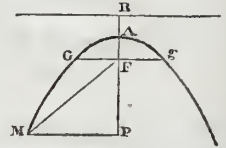
Gg = p; then will

FP = AP - AF = x - f,

FM = PR = x + f.

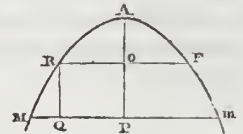
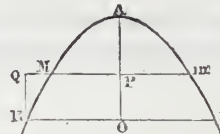
Since (Ge. Th. 35) $FP^2 + PM^2 = FM^2$, $(x-f)^2 + y^2 = (x+f)^2$ wherefore, by multiplication and reduction, $-y^2 = 4fx$; and since (Pr. 1) $4f = p$, therefore, eliminating f, $-y^2 = px$.

This will be simply found by actual multiplication and the rejection of the common terms x^2, f^2 , and transposing $2fx$.



PROPOSITION V, THEOREM.

The rectangle under the parameter and a perpendicular, from any point in the curve to a chord ordinately applied to the axis, is equal to the rectangle under the two segments of that chord.



That is, $p \cdot QR = QM \cdot Qm$; where p is the parameter.

Let AO = z, OR = γ , Qm = u, QM = v, and QR = w.

Then will $Qm = Pm + OR = y + \gamma = u$,

$QM = PM - OR = y - \gamma = v$,

$QR = AP - AO = x - z = w$.

Since (Pr. 3, Cor.) $p \cdot AO = OR^2$, and $p \cdot AP = PM^2$ $\begin{cases} px = y^2 \\ pz = \gamma^2 \end{cases}$ therefore $p(x - z) = (y - \gamma)(y + \gamma)$, or $pw = uv$

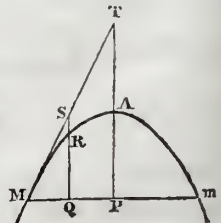
For, by subtracting the second equation from the first, we have $px - pz = y^2 - \gamma^2$, or $p(x - z) = (y + \gamma)(y - \gamma)$.

PROPOSITION VI, THEOREM.

The ratio of the two parts of a chord perpendicular to the axis, on each side of a parallel to the axis, is equal to the ratio of the two parts of that parallel between the chord and a tangent at the extremity of that chord on each side of the curve.

That is, $\frac{MQ}{Qm} = \frac{RS}{RQ}$

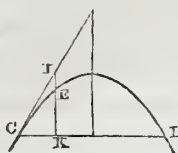
Let Qm = u, QM = v, QR = w, and QS = s; then will MQ = Mm - Qm = $2y - u$, and RS = QS - QR = $s - w$; and since AT = AP = x, PT = 2x; and since PM = y, Mm = 2y.



By similar Δ s MQS, MPT - - - - - $2xv = sy$
 by (Pr. 4) - - - - - $y^2 = px$
 and by (Pr. 5) - - - - - $pw = uv$;
 therefore, eliminating p, v, x , - - - - - $2wy = su$.
 Now, subtracting uw from each side, - $w(2y - u) = u(s - w)$,
 and dividing each side by uw , - - - - - $\frac{2y - u}{u} = \frac{s - w}{w}$

PROPOSITION VII, THEOREM.

The rectangle of the parameter, and the portion of a line parallel to the axis between the curve and a tangent, is equal to the square of that part, a chord drawn through the point of contact perpendicular to the axis between that point and the prolongation of the line which is parallel to the axis.



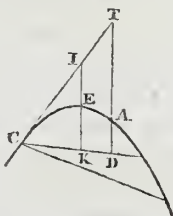
That is, $p \times \text{IE} = \text{CK}^2$.

Since (Pr. 6) $\frac{EI}{EK} = \frac{KC}{KL} \dots \dots \dots EI \cdot KL = EK \cdot KC$

and (Pr. 4) $p \cdot EK = CK \cdot KL$
therefore, eliminating EK, KL , $p \cdot EI = CK^2$.

PROPOSITION VIII, THEOREM.

The ratio of two lines parallel to the axis, intercepted between the curve and a tangent, is equal to the ratio of the squares on the tangent from the point of contact to the intersection of each of the parallels.



That is, $\frac{A^T}{EI} = \frac{C^T \mathbf{T}^2}{CI^2}$.

For (Pr. 7) - - - - - $\begin{cases} p \cdot EI = CK^2 \\ CD^2 = p \cdot AT \end{cases}$

and by similar Δ s TCD, ICK, ... $CT^2 \cdot CK^2 = CD^2 \cdot CI^2$
wherefore, eliminating p , CD, CK, . . . $CT^2 \cdot EI = CI^2 \cdot AT$,

and dividing by $CI^2 \cdot EI$ - - - - - $\frac{CT^2}{CI^2} = \frac{AT}{EI}$.

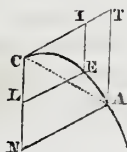
PROPOSITION IX, THEOREM.

The ratio of the abscissas of any diameter is equal to that of the squares of the ordinates.

That is, CL, CN being the abscissas, LE, NA their corresponding ordinates,

$$\frac{NA^2}{LE^2} = \frac{CN}{CL}.$$

Draw the tangent CT, which will be parallel to NA, and draw EI and AT parallel to NC.

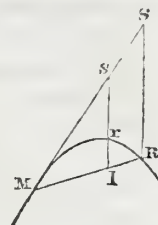


Let $CL = IE = x$, $CN = AT = z$, $LE = CI = y$, and $NA = CT = \gamma$.

$$\text{Since (Pr. 8) } \frac{AT}{EI} = \frac{CT^2}{CI^2} \dots\dots\dots \frac{z}{x} = \frac{y^2}{y^0}.$$

PROPOSITION X, THEOREM.

A line parallel to the axis, terminated by a double ordinate and a tangent at the extremity of that ordinate, is divided by the curve in a ratio equal to that which the line divides the double ordinate.



Then $\frac{rs}{r!} = \frac{1M}{1R}$.

Let $MS = v$, $sr = x$, $SR = z$, $sl = m$,
 $Ms = n$, $MR = t$, and $MI = u$.

Then $IR = MR - MI = t - u$, and $rl = ls - rs = m - x$.

Then, by similar Δ s RSM, lsM, $z_n = mv$.

and by (Pr. 8) $\frac{z}{r} = \frac{v^2}{n^2} \dots \dots \dots xv^2 = zn^2,$

and by similar Δ s, lMs, RMS, - - - - - $nt = uv$;

therefore, eliminating $n, v, z, \dots \dots \dots t_x = mn,$

or, subtracting ux from both sides, - - - $(t - u)x = (m - x)u$.

Coroll. Hence the subtangent is, in all cases, double of the abseissa; for when $t = 2u$, the expression $x(t - u) = u(m - x)$ becomes $2x = m$, sr.MS = sl.Ms.

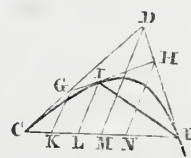
PROPOSITION XI, THEOREM.

If two tangents meeting each other be divided each into two parts, so that the ratio of the parts of the one may be equal to the ratio of the two parts of the other in a contrary order, the line drawn between the points of division will touch the curve.

Let CD be divided in G , and DE in

H, so that $\frac{GD}{GC} = \frac{HE}{HD}$, the line GH

will touch the curve in the point I, and

$$\frac{IH}{IG} \text{ will be } = \frac{GD}{GC} = \frac{HE}{HD}.$$


For through the points G, I, D, H, draw the diameters GK, IL, DM, HN, as also the lines CI, EI, which will be double ordinates to the diameters GK, HN; therefore the diameters DM, GK, HN will bisect the chords CE, CI, EI, joining the points of contact.

Now $KM = CM - CK = \frac{1}{2}CE - \frac{1}{2}CL = \frac{1}{2}LE = LN = NE$,
and $MN = ME - NE = \frac{1}{2}CE - \frac{1}{2}LE = \frac{1}{2}CL = CK = KL$.

Let $MN = CK = KL = u$, and $KM = LN = NE = v$;

then $\frac{u}{v} = \frac{GI}{IH} = \frac{CG}{GD} = \frac{DH}{HE}$.

VARIOUS ORDERS OF LINES.

CURVES are distinguished into two kinds, viz.—*Algebraical* and *Transcendental*.

Algebraical Curves are those in which the relation of the abscisses to the ordinates can be defined by a common algebraical expression.

Transcendental Curves are those in which the abscissa or ordinate depends on some transcendental quantity; as an arc, sine, eosine, logarithm, &c.

Lines are divided into various orders, according to the degree of the equation expressing the conditions of the locus.

The *locus* of a simple equation is called a *line of the first order*.

The *locus* of a quadratic equation is called a *line of the second order*.

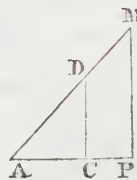
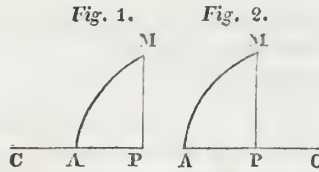
The *locus* of a cubic equation is called a *line of the third order*; and so on.

LINES OF THE FIRST ORDER.

Let C be a fixed point, PM an ordinate to the abscissa CP; then CP being $= x$, and $PM = y$, the equation which regulates the length of the ordinate PM may be $ay = c + bx$. Now, in order to ascertain the distance of the point A where the locus AM cuts the abscissa from the fixed point C, y or PM must be $=$ zero or nothing, and consequently $c + bx = 0$, and therefore $x = -\frac{c}{b}$; which indicates that, when y is zero, x must be set towards the left hand at the distance of $\frac{c}{b}$; but when $x = 0$, $ay = c$, and consequently $y = \frac{c}{a}$.

Therefore, if in the line AP we fix the point C, and make the distance $CA = \frac{c}{b}$, and draw CD at a given angle to AC, and make $CD = \frac{c}{a}$, then CD will be the ordinate at the point C.

Since $AP = AC + CP$, $AP = \frac{c}{b} + x = \frac{c + bx}{b}$; therefore we shall have $\frac{c}{b}$ and $\frac{c + bx}{b}$ respectively for the two distances AC, AP, and $\frac{c}{a}$, y for the corresponding ordinates CD, PM.



Now $\frac{c}{b} \times y = \frac{c + bx}{b} \times \frac{c}{a}$, or $ay = c + bx$, the proposed equation. It therefore appears that the locus AM is a straight line, for the ratio of AC, AP is equal to that of CD, PM.

LINES OF THE SECOND ORDER.

Let the equation of the co-ordinates be $y^2 = A + Bx + Cx^2$. Now in order to ascertain in terms of the given quantities A, B, C, where the locus AM cuts the abscissa, we must, as before, make $y = 0$, and consequently $A + Bx + Cx^2 = 0$: the solution of this quadratic equation gives $x = \frac{-B \pm \sqrt{(B^2 - 4AC)}}{2C}$, which indicates that y may be $=$ zero in two different places; therefore the locus of this equation must either be one continued line or two separate lines, as it will meet the abscissa in two different points.

Since $y^2 = A + Bx + Cx^2$, we shall have $y = \pm \sqrt{A + Bx + Cx^2}$; from which it is evident that the greater x is, the greater also must be the value of y , and therefore there can be no real value of y between the two values of x ; whence the curve can never return to itself: the equation, therefore, belongs to two separate curves.

In the equation $x = \frac{-B \pm \sqrt{(B^2 - 4AC)}}{2C}$, when both values of x are affirmative, the vertices of both curves are on the right hand of the origin C of the abscissa; if both negative, they are on the left; and if one be affirmative and the other negative, the affirmative value is on the right of C, and the negative on the left. The value of y to any value of x between the two vertices is imaginary.

When $B^2 - 4AC$ is negative, the axis of the curve changes the direction assumed for it, and takes the parallelism of the ordinates; for in this case no value of y can ever become zero.

In order to exemplify what has now been advanced, let the locus of the curve be the equation $y^2 = 3x^2 + 8x + 4$. By making $y = 0$, we shall have, from the equation $3x^2 + 8x + 4 = 0$, the two values of x , $-\frac{2}{3}$ and -2 ; which indicate that the vertex of the one curve is $\frac{2}{3}$ of unity to the left hand of the assumed point C, and that the vertex of the other curve is 2 in the same direction; therefore the distance between the vertices is $1\frac{1}{3}$.

In order to calculate the ordinates, since the vertices of both curves are towards the left, we shall begin with the greatest negative value of x , and proceed progressively by adding unity, which will of course diminish the negative values, and increase the affirmative ones.

Let $x = -5$; then $y = \pm \sqrt{3x^2 + 3x + 4} = \sqrt{39} = 6.24$
 $x = -4$ - - - $y = 4.47$
 $x = -3$ - - - $y = 2.64$
 $x = -2$ - - - $y = 0$
 $x = -1$ - - - $y = \text{imaginary}$
 $x = -\frac{2}{3}$ - - - $y = 0$
 $x = 0$ - - - $y = 2$
 $x = 1$ - - - $y = 3.87$
 $x = 2$ - - - $y = 5.65$
 $x = 3$ - - - $y = 7.41$
 &c. &c.

In *Plate I, Fig. 1, Appendix*—Draw the straight line DE at pleasure, and in it assume the point C for the origin of the abscissa. From a scale of equal parts, subdivided into tenths, take $\frac{2}{3}$ or .66, and set the extent towards the left from C to A; also from the same point C, set two parts from C to a: from C, set off C1, C2, C3, &c. each way to the right and left of C, extending to the points 1, 2, 3, &c.

Through the points 1, 2, 3, &c. draw perpendiculars.

In the curve on the right, beginning at zero or the origin of the curve, make the ordinates CM, 1M', 2M'', &c. respectively equal to 2, 3.87, 5.65, &c., and the same distances downwards: and through the points M, M', M'', &c. draw the curve. In the left hand curve, make the ordinates 3m, 4m', 5m'', &c. respectively equal to 2.64, 4.47, 6.24, &c., and repeat the same distances downwards; and through the points m, m', m'', &c. draw a curve which will represent the locus of the equation as required.

Let $y^2 = 3x^2 \pm 6x + 4$ be proposed as another example: but since, making $y = 0$, the equation $3x^2 \pm 6x + 4 = 0$ has no real root, therefore every value of y is possible. Here it might be proper to enquire what might be the value of x , when y is the least possible; but as this would lead us to the differential calculus, we shall drop this enquiry, and begin at pleasure, making the first negative value of x , 3, and ascending to its positive values, by adding unity to every last value. The several values of y in the equation $y = \sqrt{3x^2 \pm 6x + 4}$ corresponding to the assumed values of x , are as stated in the following table:—

When $x = -3$, $y = 7.0$
 $x = -2$, $y = 5.28$
 $x = -1$, $y = 3.605$
 $x = \mp 0$, $y = 2.0$
 $x = 1$, $y = 1.0$
 $x = 2$, $y = 2.0$
 $x = 3$, $y = 3.605$
 $x = 4$, $y = 5.28$
 $x = 5$, $y = 7.0$
 &c. &c.

The curve drawn according to these values of x and y , is exhibited in *Fig. 2, Plate I, Appendix*.

Let it now be required to trace the locus of the equation $y^2 = A + Bx - Cx^2$. To ascertain, in terms of the given quantities A , B , C , the points where the locus AM cuts the abscissa; by making $A + Bx - Cx^2 = 0$, we shall find $x = \frac{B \pm \sqrt{B^2 + 4AC}}{2C}$. But since $y = \pm \sqrt{A + Bx - Cx^2}$,

it is evident that all the possible values of y will be contained between the two values of x , when $y = 0$, and therefore the curve will return to itself.

As a practical example, let it be required to trace the locus of the equation $y^2 = 4 - x - \frac{1}{2}x^2$.

Making $y = 0$, we shall find $x = -4$, or $+2$; therefore one extremity of the axis is 4 equal parts on the left hand of the origin of the abscissa, and the other extremity 2 equal parts on the right of it; and consequently the whole axis is equivalent to 6 equal parts.

It will therefore be proper to find the values of y , beginning the first when $x = -4$, and $y = 0$; therefore, making x successively $-4, -3, -2, -1, \mp 0, 1, 2$, in the equation $y = \sqrt{4 - x - \frac{1}{2}x^2}$, and aggregating the quantities within the $\mp 0, 1, 2$, &c. radical sign, and extracting the root each time, we shall find the values of y agreeable to the assigned values of x in the following table:—

When $x = -4$, $y = 0$
 $x = -3$, $y = 1.58$
 $x = -2$, $y = 2.0$
 $x = -1$, $y = 2.12$
 $x = \mp 0$, $y = 2.0$
 $x = +1$, $y = 1.58$
 $x = +2$, $y = 0$

Draw the straight line DE (*Fig. 3*), in which assume C for the origin of the abscissa. From a scale of equal parts, subdivided into tenths, take 4, and set the extent on the left from C to A; again, take 2, and set it from C to a on the right; mark the parts on each side of C at 1, 2, &c.: through these points draw perpendiculars; then set the ordinates on the perpendiculars on each side of the axis from extremities of their corresponding abscissas, and trace the curve through the points of distance.

The curve will return to itself in all cases when the sign of the term which contains the square of x is negative, whatever may be the signs of the other two terms.

Equations which have the sign of the term containing the square of x negative, can only be one of the four following forms, viz.:—

$Py^2 = A + Bx - Cx^2$
 or $Py^2 = A - Bx - Cx^2$
 or $Py^2 = -A + Bx - Cx^2$
 or $Py^2 = -A - Bx - Cx^2$

It is evident in the last of these equations that x cannot have any affirmative values, for then both values of y will be negative, and consequently impossible.

Let it be required to trace the locus of the equation $Py^2 = A + Bx$.

Making $y = 0$, the quantity x can only have one value, from the equation $A + Bx = 0$; therefore the curve cannot cut the abscissa in more points than one; hence it is evident there can neither be a continued curve, nor two opposite ones.

By way of illustration let $y^2 = 3 + 2x$, or $y = \pm \sqrt{3 + 2x}$.

Making $y = 0$, we shall have $x = -\frac{3}{2} = -1.5$; from which it appears that the apex of the curve is 1.5 on the left of the origin; therefore, making $x = -1$, $x = -\frac{1}{2}$, $x = 0$, $x = 1$, $x = 2$, &c. in the equation $y = \pm \sqrt{3 + 2x}$, aggregating the quantities within the radical sign, and extracting the root each time, we shall find the values of y agreeable to the assigned values of x in the following table:—

$$x = -1, y = \sqrt{1} = 1.00$$

$$x = 0, y = \sqrt{3} = 1.73$$

$$x = 1, y = \sqrt{5} = 2.23$$

$$x = 2, y = \sqrt{7} = 2.64$$

$$x = 3, y = \sqrt{9} = 3.00$$

&c.

&c.

Draw the straight line DE (*Plate I, Fig. 4, Appendix*), and in it take any point C; take 1.5 from the scale, and set the extent from C in the line DE towards the left hand to the point A, which will be the vertex of the curve: set off one part from the origin on the left side of it, and three parts to the right, numbering them 1, 2, 3. Through the several points of division draw perpendiculars as in the former examples, and upon them set the ordinates on each side of the axis, and trace the curve as before.

We shall now show that all lines of the second order are curves of the first order, so that every quadratic equation is the locus of an ellipse, hyperbola, or parabola.

For let it be required to find the nature of the curve which is the locus of the equation $y^2 = A \pm Bx - Cx^2$.

Divide the equation by C, and $\frac{y^2}{C} = \frac{A}{C} \pm \frac{B}{C}x - x^2$; for $\frac{A}{C}$ substitute mn ; and for $\frac{B}{C}$ substitute $m - n$, and $\frac{y^2}{C} = mn \pm (m - n)x - x^2 = (m + x)(n - x)$: multiply both sides of this equation by C, and $y^2 = C(m + x)(n - x)$; for C substitute $\frac{b^2}{(m + n)^2}$, and $y^2 = \frac{b^2}{(m + n)^2}(m + x)(n - x)$.

In *Fig. 5, Plate I, Appendix*, let AC = m , Ca = n , CP = x , PM = y , and BC = b .

Then will Aa = CA + Ca = $m + n$

AP = AC + CP = $m + x$

aP = Ca - CP = $n - x$

and since $(m + n)y^2 = b^2(m + x)(m - x)$, Aa²·PM' = Bb²·AP·Pa.

But if C be the middle of AC, AP·Pa will be = AC² - CP²; therefore, in the equation derived from the algebraic expression now found, for AP·Pa substitute AC² - CP², and for Aa and Bb substitute their halves CA and CB, and AC²·PM² = BC²(AC² - CP²): now, therefore, because AC² is greater than AC² - CP², BC² is greater than PM²; and when AC² and AC² - CP² are equal, BC² will be = PM², and this can only be when the point P coincides with the middle point C; therefore, since the equation of the curve is AC²·PM² = BC²(AC² - CP²), and since PM coincides with BC at the centre, the curve is an ellipse (Curves of the First Order, Pr. 5).

In the same manner it may be demonstrated, that if the equation be of the form $y^2 = -A + Bx - Cx^2$, by substituting mn for $\frac{A}{C}$, and $m - n$ for $\frac{B}{C}$ as before, and $\frac{b^2}{(m - n)^2}$ for C, the curve is an ellipse.

The origin of the abscissa for the locus of the equation $y^2 = A \pm Bx - Cx^2$ is within the axis; and the origin of the abscissa for the locus of the equation $y^2 = -A + Bx - Cx^2$ is in the prolongation of the axis.

The same demonstration also extends to the hyperbola, by observing when the absolute number A is affirmative, the parts of the binomial in the denominator of the fraction $\frac{b^2}{(m \pm n)^2}$ substituted for C, have contrary signs, and both parts of the binomial $m \pm n$ substituted for $\frac{B}{C}$ have the sign +; and the contrary when the absolute term is negative. These circumstances are the very reverse in the ellipse.

Lastly, let it be required to find the nature of the locus of the equation $y^2 = A + Bx$.

Divide both sides by B, and we have $\frac{y^2}{B} = \frac{A}{B} + x$; for $\frac{A}{B}$ substitute m , and we have $\frac{y^2}{B} = m + x$, and consequently $y^2 = B(m + x)$; for B substitute $\frac{b^2}{m + n}$, and $y^2 = \frac{b^2}{m + n}(m + x)$, or $(m + n)y^2 = b^2(m + x)$.

In *Plate I, Fig. 5, Appendix*, let AC = m , CP = n , PM = y , and PM' = b .

Then will AP' = AC + CP' = $m + n$

AP = AC + CP = $m + x$

therefore AP'·PM² = AP·P'M².

From the properties demonstrated of each of the three curves, all the others may be derived in the very same manner as is usually done from the cone.

DIRECTIONS TO THE BINDER.

Order of the Letter-press.

1. Preface.
2. Memoir.
3. Primary Problems.
4. Builder and Workman's Director.
5. Appendix.
6. Directions to the Binder.

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Portrait opposite to Title-page.

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N.B.—There are two of *Plate 1*: that which is to be inserted opposite to page 88 contains fifteen Diagrams; the other, which contains only eight, is to be inserted among the Miscellaneous Plates at the end of the Work.

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DIRECTIONS TO THE BINDER.

The remaining miscellaneous Plates to be placed at the end of the Work in the following order, viz. :—

CURVE LINES.

1. Ellipse, *Plate 2*, containing 8 Diagrams.
2. Hyperbola, *Plate 1*, containing 6 Diagrams.
3. Parabola, *Plate 2*, containing 8 Diagrams.

These Three Plates have been superseded by those which have been inserted as directed opposite the proper pages.

GENERAL PRINCIPLES.

4. Formation of the Edges of Ribs, Brackets, or Handrails, to range in prismatic and cylindric Surfaces, *Plate 1*.

This Plate will be generally understood from what has been described of *Plate 2* opposite to p. 154, Primary Problems. The intention of this, as well as the following Plate, is to show the application of Solid Trigonometry to Carpentry and Joinery. By considering the properties of the Right Angled Trehedral, the bare inspection of these figures will render them sufficiently clear to be understood.

EXERCISES in SOLID TRIGONOMETRY.

5. Oblique Trehedral.

In consequence of the Letter-Press having been printed at the same time the plates were engraven, which was necessary to hasten the publication of the Work, several errors in the references have been unavoidably passed over. Unless the plates are engraved previously to the printing of the Letter-press, it is hardly possible that a work can be free from errors, it being necessary in reading the proof sheets that the plates should be read at the same time. It is however to be hoped, that, the following corrections being made, others that may be found will be of less consequence.

ERRATA.

Article 61, Plane Geometry, p. 11, is deficient; instead of saying "*contained between two radii*," it should have been said, *between two radii at right angles*.

In the enunciation of Problem vii, p. 120, Primary Problems, for "*trace*" read *seat*.

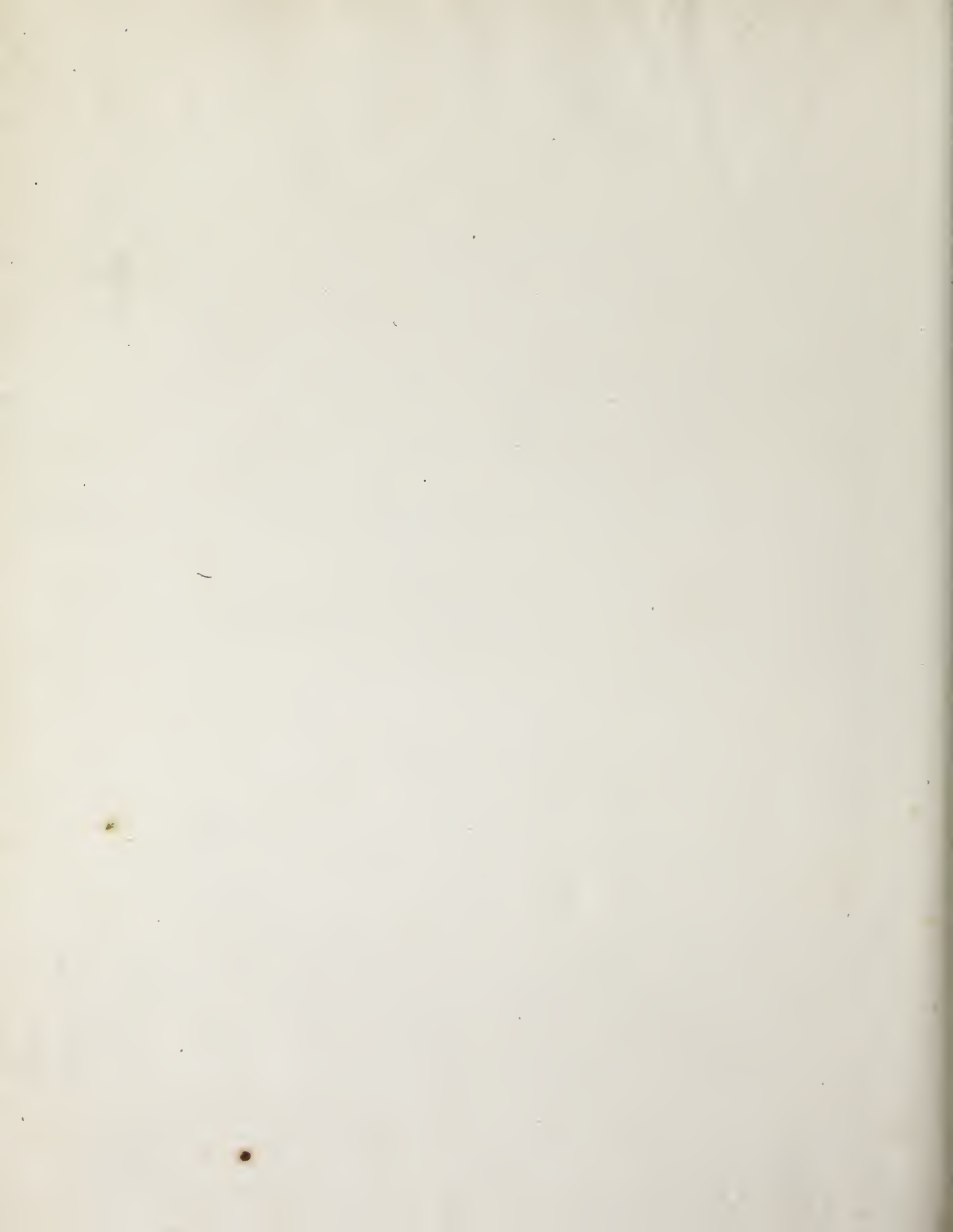
In the equation of the enunciation of Problem xxiii, p. 132, Primary Problems, instead of " $(p^2 - h^2)$," insert $(p^2 - h^2)y^2$.

In the same page, viz. 132, in the 6th line from the bottom, instead of "*Plate 1*," insert *Plate 3*.

In page 133, 14 lines from the bottom, for "*Plate 1*" read *Plate 3*; and in the 5th line from the bottom, for "*G*" read *O*; and in the bottom line, instead of "*Plate 2*" read *Plate 4*.

In page 134, the 3d line from the top, instead of "*Plate 1*" read *Plate 3*, as before; and in the 12th line from the top, instead of "*Plate 3*" read *Plate 5*; and in the 22d line from the top, instead of "*Plates 4 and 5*," read *Plates 6 and 7*.



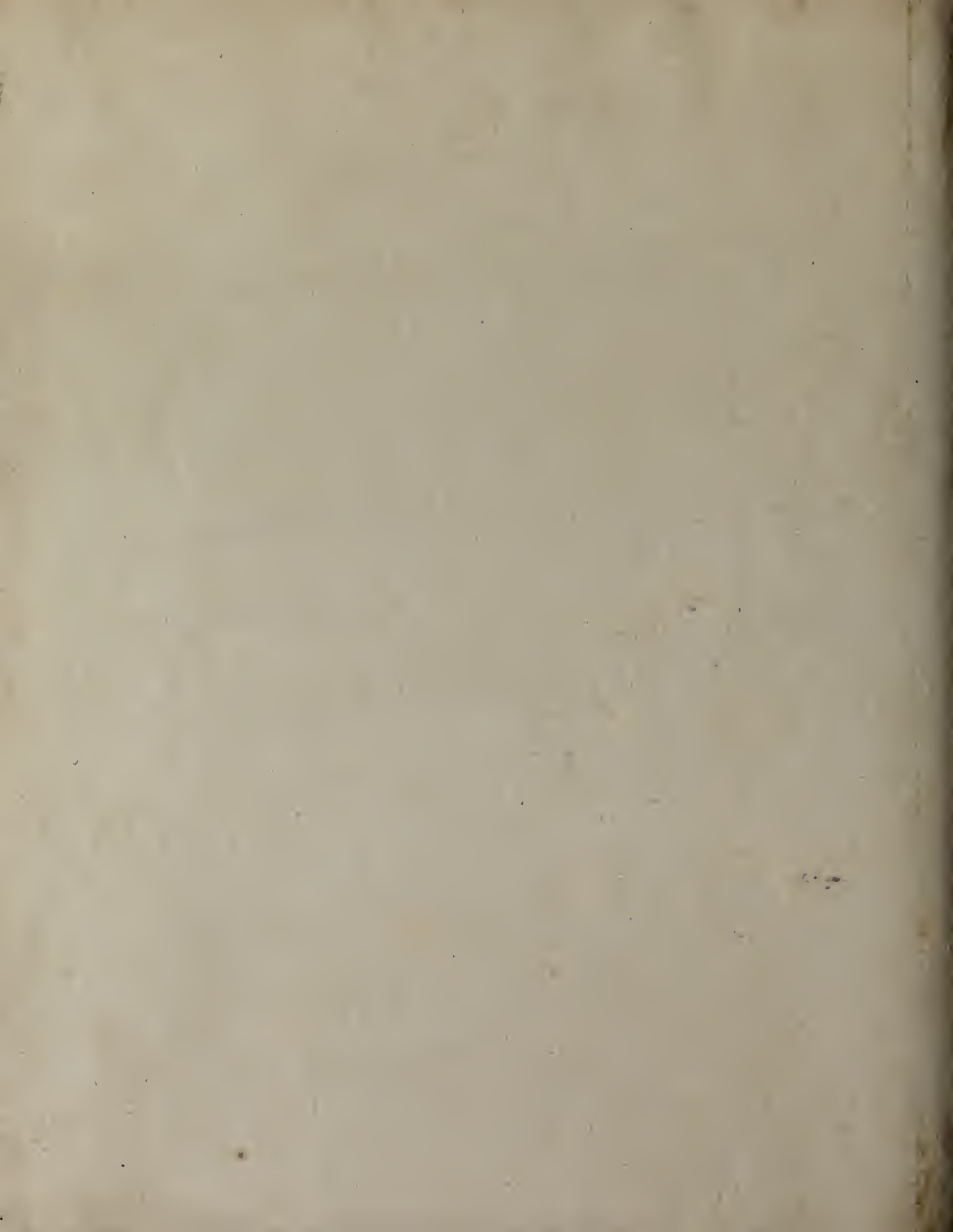


Collated complete

fronts, th, A^2 , B- Aa^4 ; a-c⁴; a-i² in 2/2 (Lipudis)

with 139 plates as listed in "positions"

at end



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